A shear-driven mixing parameterisation for ocean climate models.

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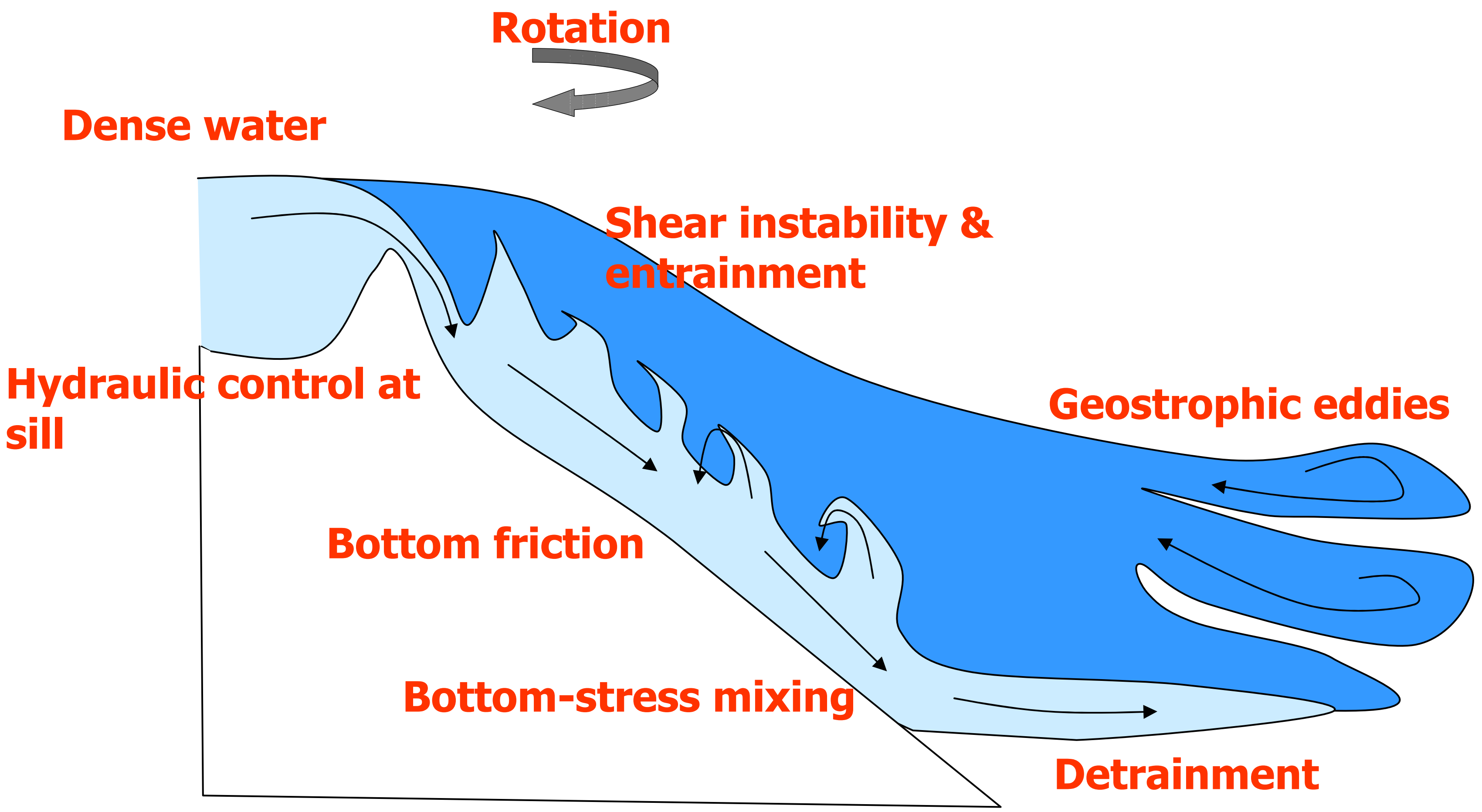
Part of the Gravity Current Entrainment Climate Process Team:

- Study processes important for climate prediction to improve representation in climate models

- Collaborations between different institutions

- Observations, process studies, regional modelling, climate centres

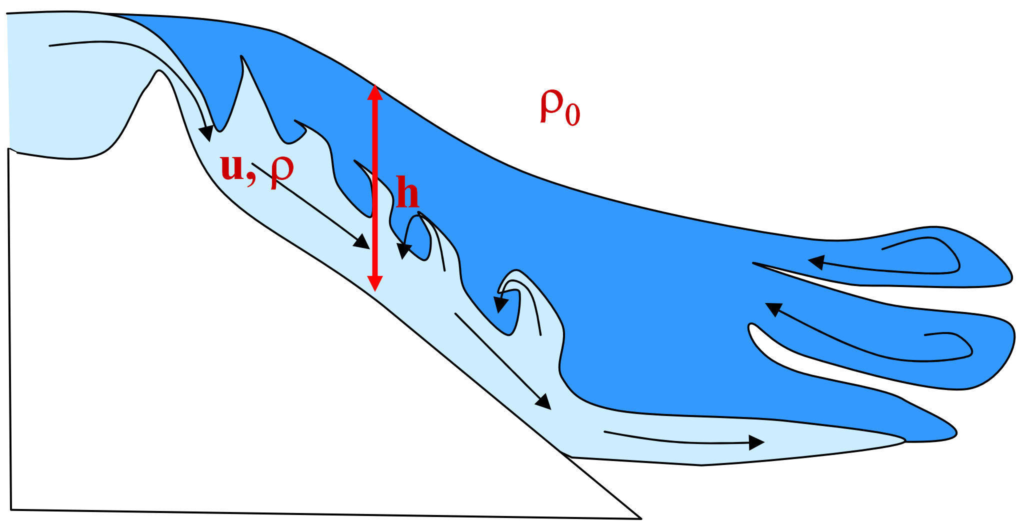
Critical Processes in Gravity Currents



Parameterising Gravity Current Entrainment

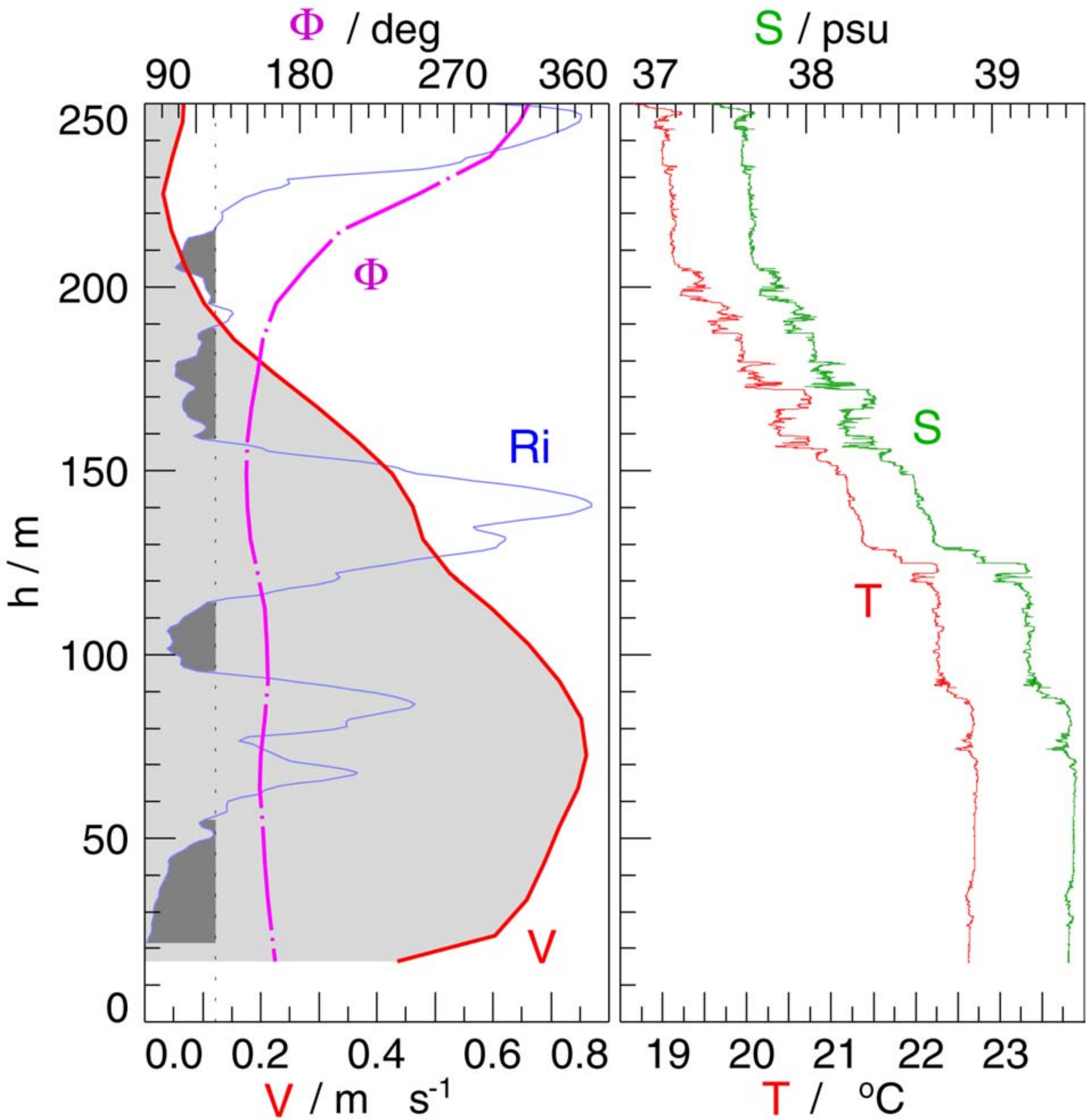
Generally parameterised in terms of a bulk Richardson number Ri_b or a local gradient Richardson number Ri_g - ratio of potential energy to kinetic energy.

$$Ri_b = \frac{gh\Delta\rho/\rho_0}{\Delta u^2} \qquad Ri_g = \frac{-g\rho_z/\rho_0}{u_z^2} = \frac{N^2}{S^2}$$



Linear theory - there is critical Ri_c=0.25 above which there is no mixing.

Shear-driven mixing of stratified turbulence



Observed profiles from Red Sea plume from RedSOX (Peters and Johns, '05)

Actively mixing interfacial layer

Shear param. appropriate here.

Well-mixed bottom boundary layer

(see Legg et al. 2006)

Existing Parameterisations

Z-Coordinates - Diffusivity κ= κ(Ri) specifies entrainment as a diffusion of density

$$\frac{D\rho}{Dt} + w\rho_z = \frac{\partial}{\partial z}\left(\kappa \frac{\partial \rho}{\partial z}\right)$$

• Interior part of KPP: κ(Ri) = 0.005(1-min{1,Ri/Ri_c}) (Large et al, 1994)

• Pacanowski and Philander (1996) also has κ(Ri)

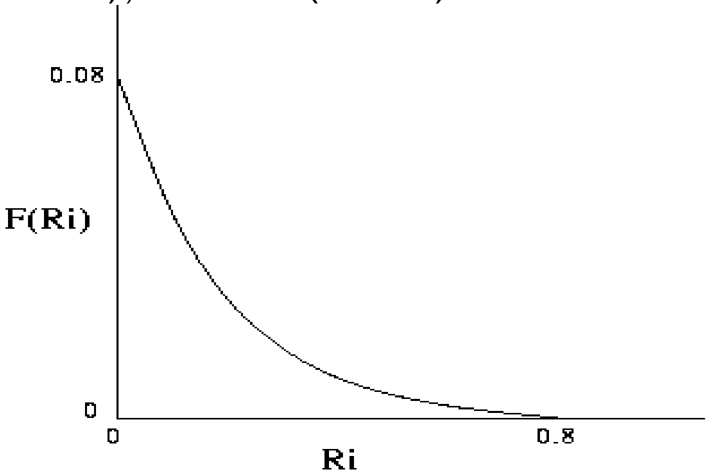
• More sophisticated second order turbulence models

Isopycnal Coordinates - an increase in layer thickness is applied

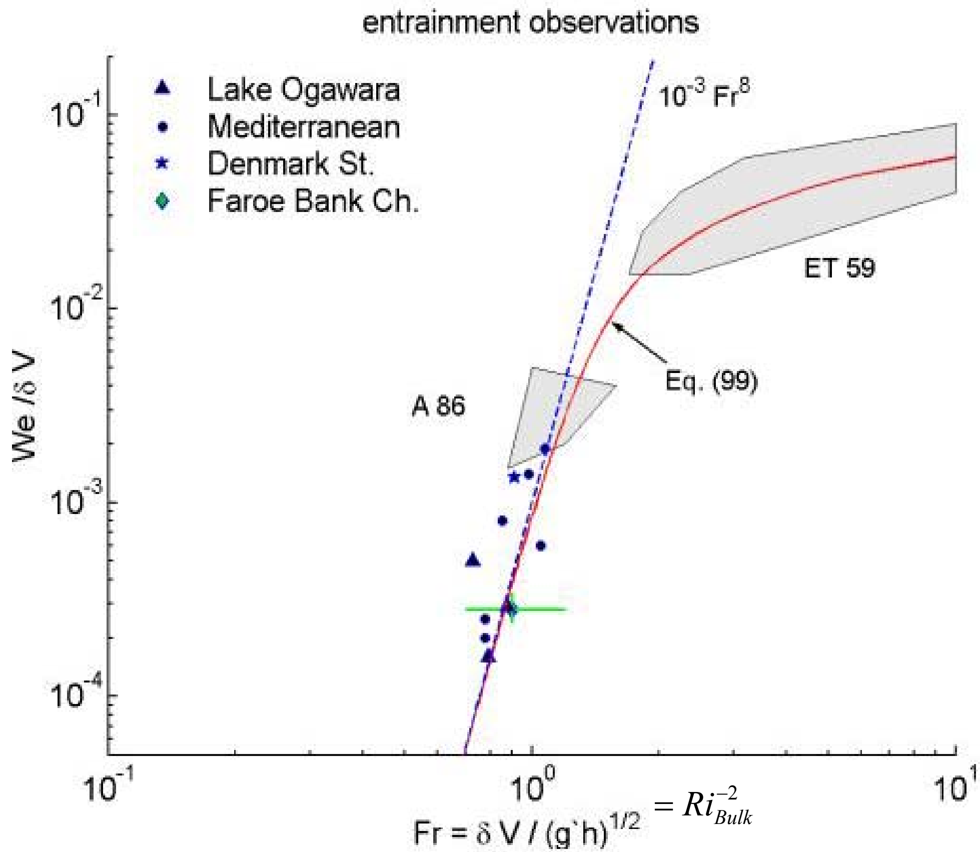
• Hallberg (2000), based on Ellison and Turner (1959), Turner (1986)

$$\frac{Dh}{Dt} + h\nabla.\mathbf{u} = \Delta U F\left(Ri_b\right)$$

$$\frac{D}{Dt}\left(-\frac{\partial z}{\partial \rho}\right) - \frac{\partial z}{\partial \rho}\nabla.\mathbf{u} = \frac{\partial w^*}{\partial \rho} = 2\left\|\frac{\partial \mathbf{u}}{\partial \rho}\right\| F\left(Ri\right)$$



Parameterizing overflow entrainment: observations of bulk entrainment in oceanic and laboratory gravity currents (J. Price)



A bulk entrainment law applies, provided the Reynolds number is not quite small.

$$Ent = \Delta U \; F(Ri_{Bulk})$$

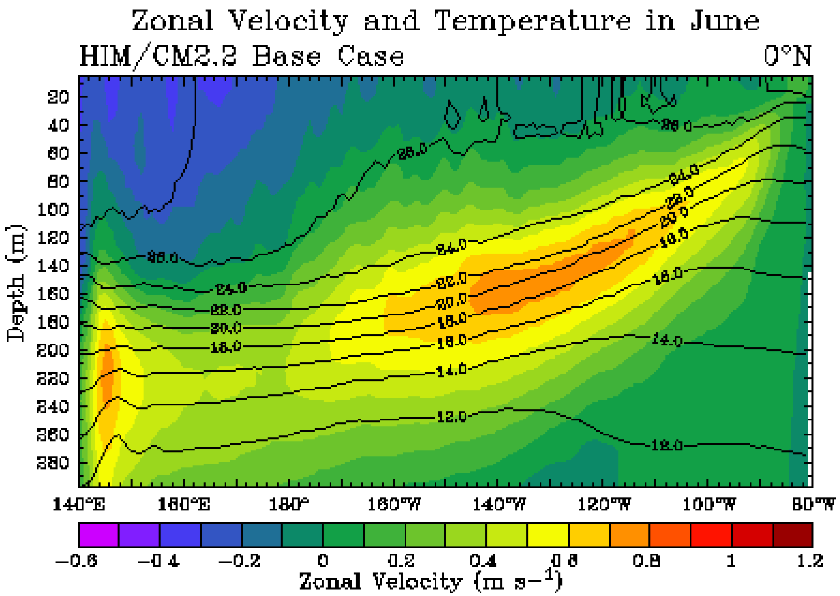
Failure and Success of Existing Shear Mixing Parameterizations

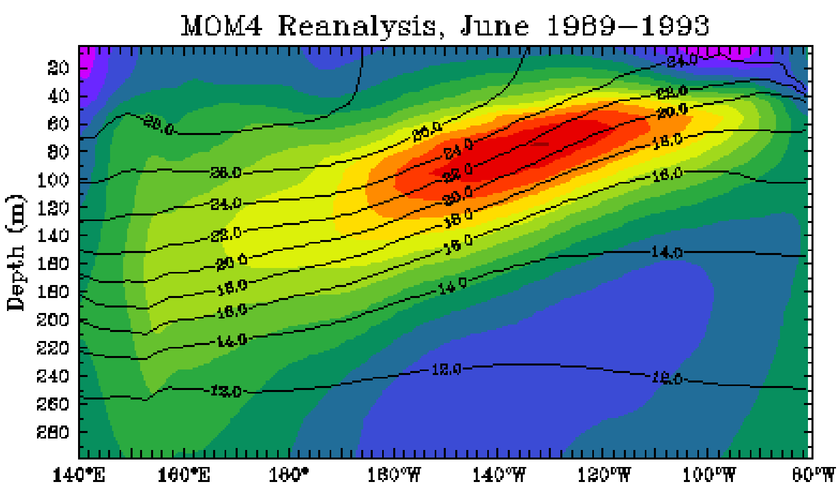
- A universal parameterization can have no dimensional "constants".
  - KPP's interior shear mixing (Large et al., 1994) and Pacanowski and Philander (1982) both use dimensional diffusivities.
- The same parameterization should work for all significant shear-mixing.
  - In GFDL's HIM-based coupled model, Hallberg (2000) gives too much mixing in the Pacific Equatorial Undercurrent or too little in the plumes with the same settings.
- To be affordable in climate models, must accommodate time steps of hours.
  - Longer than the evolution of turbulence.
  - Longer than the timescale for turbulence to alter its environment.
- 2-equation (e.g. Mellor-Yamada, k-ε, or k-w) closure models may be adequate.
  - The TKE equations are well-understood, but the second equation (length-scale, or dissipation rate, or vorticity) tend to be ad-hoc (but fitted to observations)
  - Need to solve the vertical columns implicitly in time for:
    1. TKE
    2. Dissipation/vorticity/length-scale
    3. Stratification (T & S)
    4. (and 5.) Shear (u & v)
  - Simpler sets of equations may be preferable.
  - Many use boundary-layer length scales (e.g. Mellor-Yamada) and are not obviously appropriate for interior shear instability.

However, sensible results are often obtained by any scheme that mixes rapidly until the Richardson number exceeds some critical value. **(e.g., Yu and Schopf, 1997)**

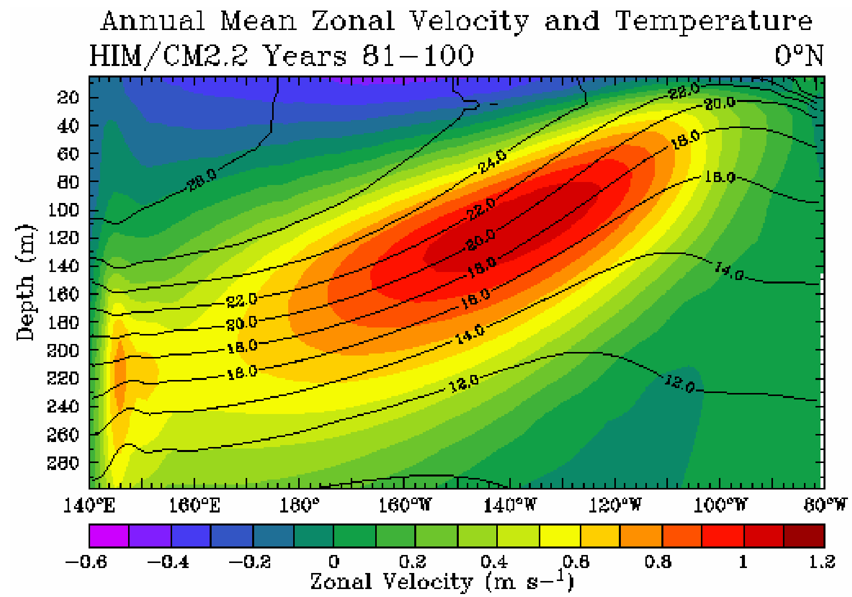
Impact of Entrainment Parameterization on EUC

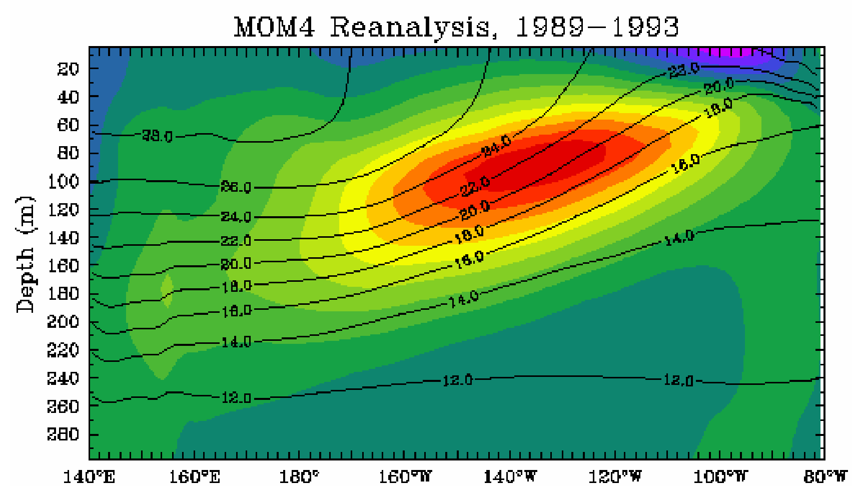
June Pacific EUC with
$Ri_{crit} = 0.8$ and $E_o = 0.1$ (Original values)





Annual Mean Pacific EUC
$Ri_{crit} = 0.2$ and $E_o = 0.005$





Link between z-coordinate and isopycnal parameterisations

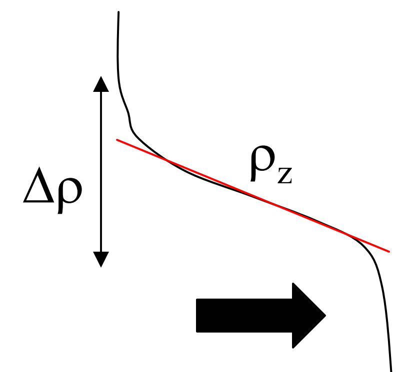
The diffusion of density can be linked to the increase in layer thickness by

$$\frac{D}{Dt}\left(-\frac{\partial z}{\partial \rho}\right)\Bigg|_{\rho}-\frac{\partial z}{\partial \rho}\nabla.\mathbf{u}\Big|_{\rho}=\frac{\partial w^{*}}{\partial \rho}=\frac{1}{\rho_z}\frac{\partial}{\partial z}\left(\frac{1}{\rho_z}\frac{D\rho}{Dt}\bigg|_{z}\right)=\frac{1}{\rho_z}\frac{\partial}{\partial z}\left(\frac{1}{\rho_z}\frac{\partial}{\partial z}\left(\kappa\frac{\partial \rho}{\partial z}\right)\right)$$

With the parameterisation from Hallberg (2000) this gives:

$$\text{change in layer thk }=\frac{1}{\rho_z}\frac{\partial}{\partial z}\left(\frac{1}{\rho_z}\frac{\partial}{\partial z}\left(\kappa\frac{\partial \rho}{\partial z}\right)\right)=2\left\|\frac{\partial \mathbf{u}}{\partial \rho}\right\|F(Ri)$$

Problems - undefined diffusivity if stratification is zero
- layer thickness never decreases



Constant stratification - $\frac{\partial^2 \kappa}{\partial z^2}=-2\|U_z\|F(Ri)$

New Theory

$S=|U_z|$

$$\boxed{\frac{\partial^2\kappa}{\partial z^2}-\frac{\kappa}{L_d^2}=-2SF(Ri)}$$

$$L_d=\min\left(\frac{\lambda Q^{1/2}}{N},\frac{\lambda Q^{1/2}}{f},L_{wall}\right)$$

Properties:

•F(Ri) decreases with Ri till $F(Ri_c)=0$

•$\kappa \sim S\,L^2$ with a length scale which is a combination of the width of the low Ri region (where F(Ri)>0) and the decay length scale L_D

•Decays exponentially away from low Ri region with a length scale which is the minimum of:

- buoyancy length scale (over which stratification affects TKE)
- Ekman length scale (over which rotation affects TKE)
- or decays linearly over the distance to boundary L_{wall}

•Produces smooth diffusivity

Lars Umlauf has pointed out to us that

$$\frac{\partial^2 \kappa}{\partial z^2} - \frac{\kappa}{L_D^2} = -2SF(Ri)$$

can also be written as

$$\frac{1}{\kappa}\frac{D\kappa}{Dt} = \frac{1}{\kappa}\left[\underbrace{\frac{\partial}{\partial z}\left(\kappa \frac{\partial \kappa}{\partial z}\right)}_{\text{'diffusion' of diffusivity}} \underbrace{- \left(\frac{\partial \kappa}{\partial z}\right)^2 - \frac{\kappa^2}{L_D^2}}_{\text{sink of diffusivity over different length scales}} + \underbrace{2F(Ri)\kappa S}_{\text{'source' of diffusivity}}\right] = 0$$

Need TKE budget

Vertical transport — Buoyancy sink

$$\boxed{\frac{\partial}{\partial z}\left((\kappa+\nu_0)\frac{\partial Q}{\partial z}\right)+\kappa\left(S^2-N^2\right)-Q(c_N N+c_S S)=0}$$

Shear production — Dissipation

Assumptions:

- Q reaches steady state faster than background flow is evolving so no DQ/Dt term
- Assume Pr = 1 (for now)
- Include non-local diffusion of TKE
- Parameterisation of dissipation as $\varepsilon=Q(c_N N+c_S S)$

Limits

Homogeneous turbulence (constant shear and stratification)

- $\kappa \propto S\, L_b^2$ and $$2\lambda^2 F(Ri) = \frac{Ri\left(c_N Ri^{1/2} + c_S\right)}{1 - Ri}$$

- Only has a solution for a unique value of $Ri<Ri_c$, ie other steady state solutions have non-local balances.
- Compare to homogeneous turbulence literature (eg Baumert and Peters, 2000) where full equilibrium only occurs for single flux and gradient $Ri<Ri_c$.

Ellison and Turner limit:

- Assume large TKE
- Reduces to form similar to Hallberg (2000)

$$\frac{\partial^2 \kappa}{\partial z^2} = -2\left\| U_z \right\| F(Ri)$$

Unstratified limit:

Evolves to a nearly parabolic diffusivity and constant shear over the interior, with large velocity shears near the edges. There are problems near the boundary, but the parameterisation can be modified to obey the law-of-the wall.

$$\frac{\partial^2 \nu}{\partial z^2} - \frac{\nu}{L_d^2} = -2F_0 \left| \frac{\partial u}{\partial z} \right|$$

$$\nu \left| u_z \right| = u^{*2}$$

$$\nu \approx \sqrt{2F_0}\, u^* z$$

$$\boxed{\frac{\partial^2 \kappa}{\partial z^2} - \frac{\kappa N^2}{\lambda^2 Q} = -2SF(Ri)}$$

Parameters

$$\boxed{\frac{\partial}{\partial z}\left((\kappa + \nu_0)\frac{\partial Q}{\partial z}\right) + \kappa\left(S^2 - N^2\right) - Q\left(c_N N + c_S S\right) = 0}$$

Parameter constraints:

$$\lambda = \frac{L_d}{L_b}$$

- Length scale ratio of decay scale to buoyancy scale. Not clear what length scale L_d is, but expect $\lambda \sim 0.6\text{-}1.0$

$$F(Ri) = F_0 \frac{1 - Ri / Ri_c}{1 + \alpha Ri / Ri_c}$$

- $F(0) \sim 0.1$ from measurements of layer growth for unstratified turbulent mixing. Consistent with Xu et al (2006) parameterisation.

- $F(Ri_c) = 0$, $Ri_c = 0.25\text{-}0.3$

- The shape α is unknown, though $\alpha > -1$

$$\left(\frac{L_{oz}}{L_b}\right)^2 = c_N + c_S Ri^{-1/2}$$

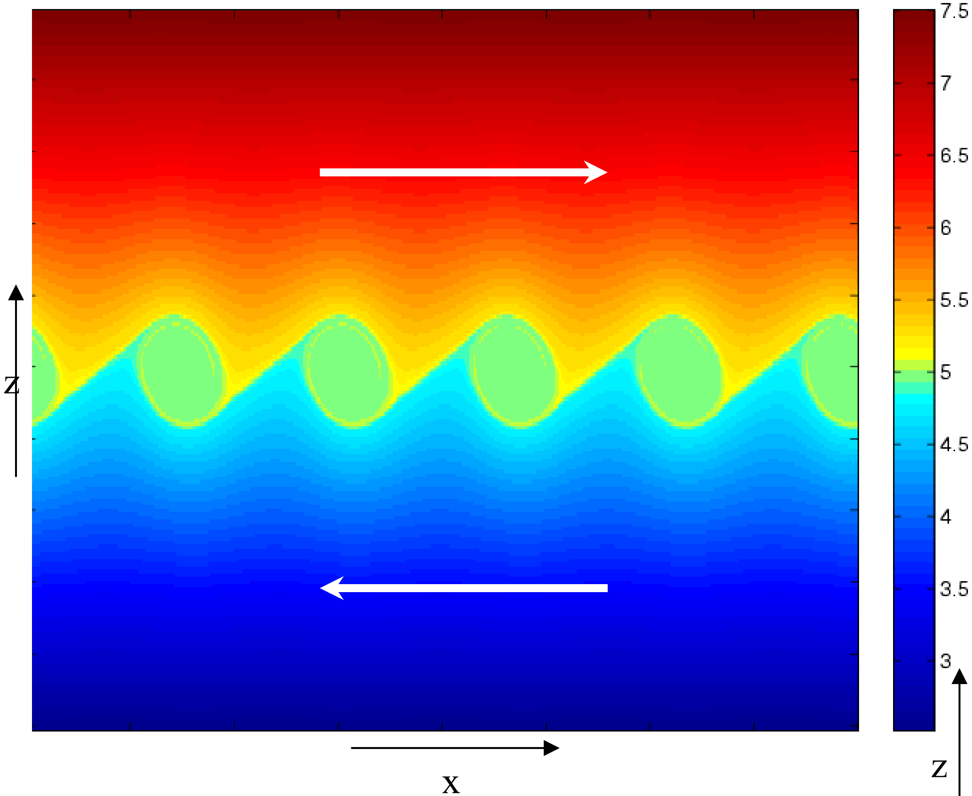
- Dissipation can be written in terms of the ratio of the Ozmidov and buoyancy length scales. Literature – ratio is constant or depends on $Ri^{-1/2}$. Various values from literature: $\sim O(0.1)$

Simulations of shear-driven stratified turbulence

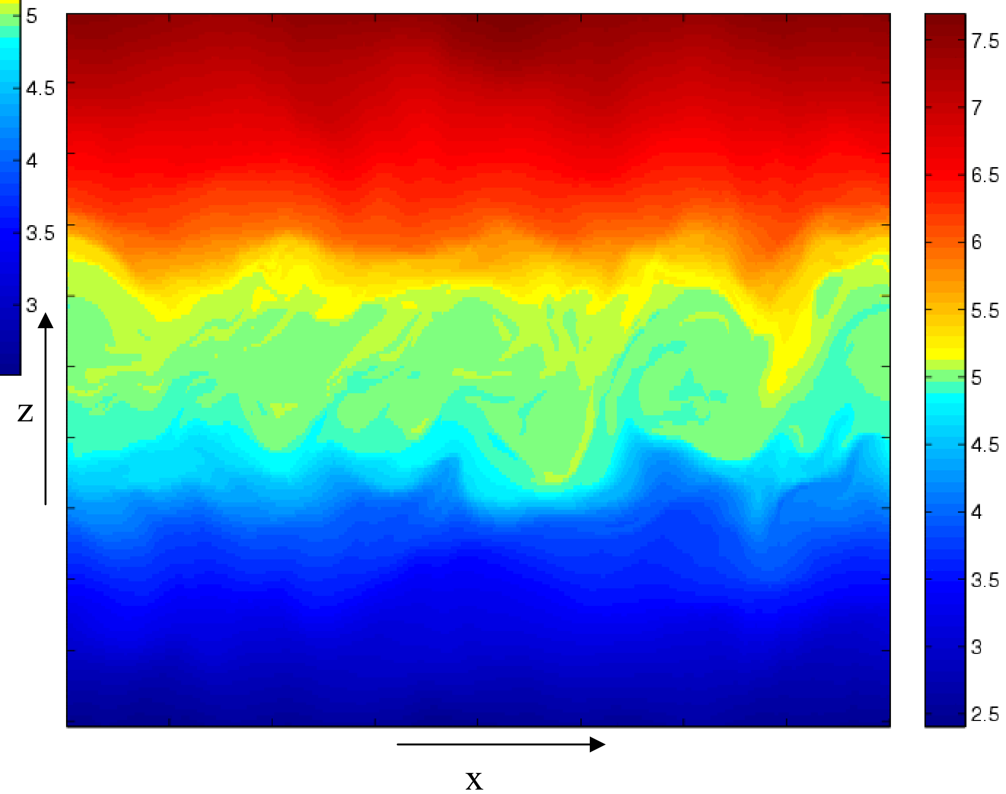
- High resolution numerical simulations (MITgcm)
- Non-hydrostatic
- 2m x 2m x 2.5m with grid size ~ 2.5mm in centre
- Background $\kappa=10^{-6}$ m^2s^{-1}, $\nu=2.5\times10^{-6}$ m^2s^{-1}
- Cyclic domain in x,y
- Constant initial stratification
- Shear and jet velocity profiles
- Forced so that a statistically steady state can be reached
- All profiles are spatially averaged in x and y and time averaged

Shear results

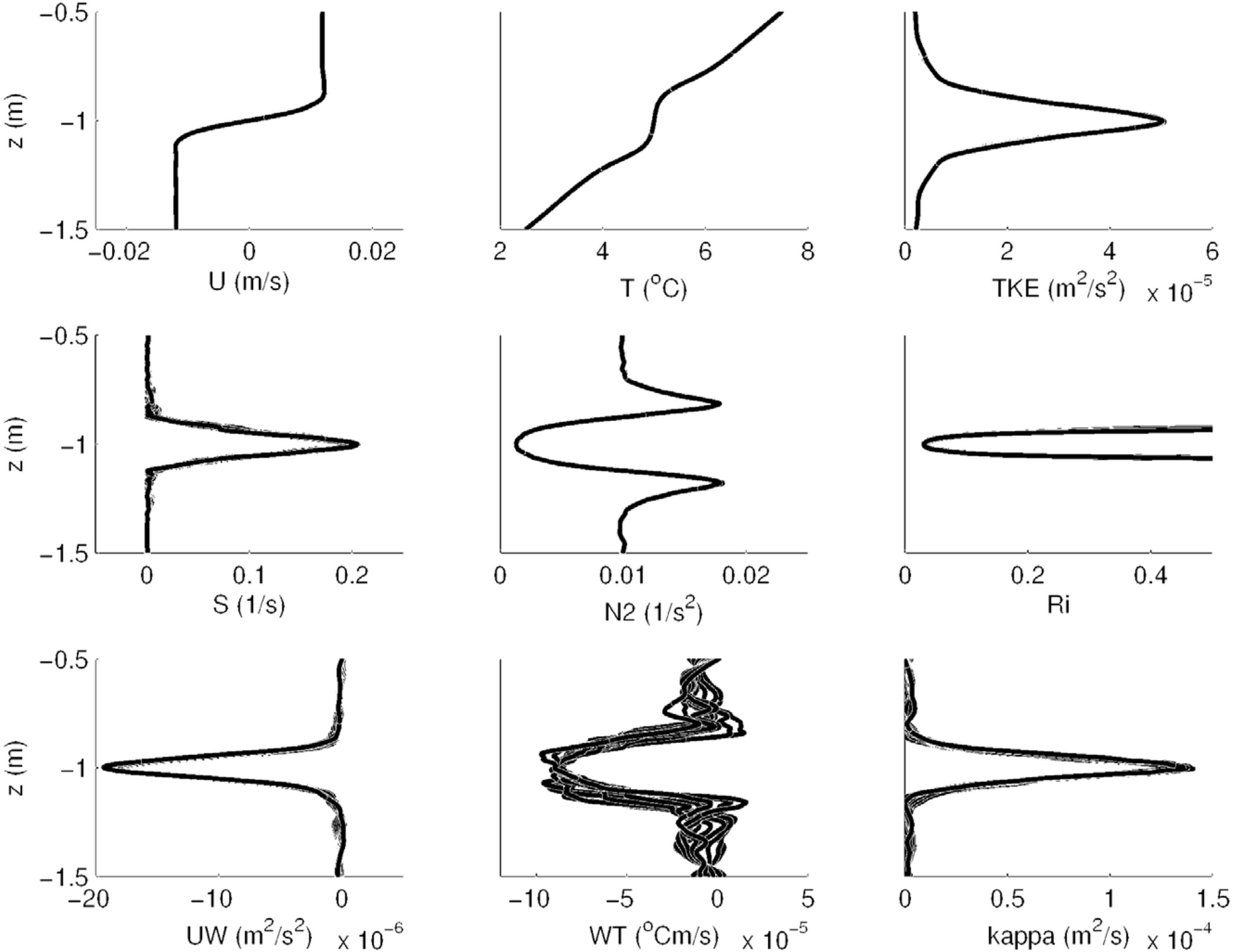
Kelvin-Helmholtz instability

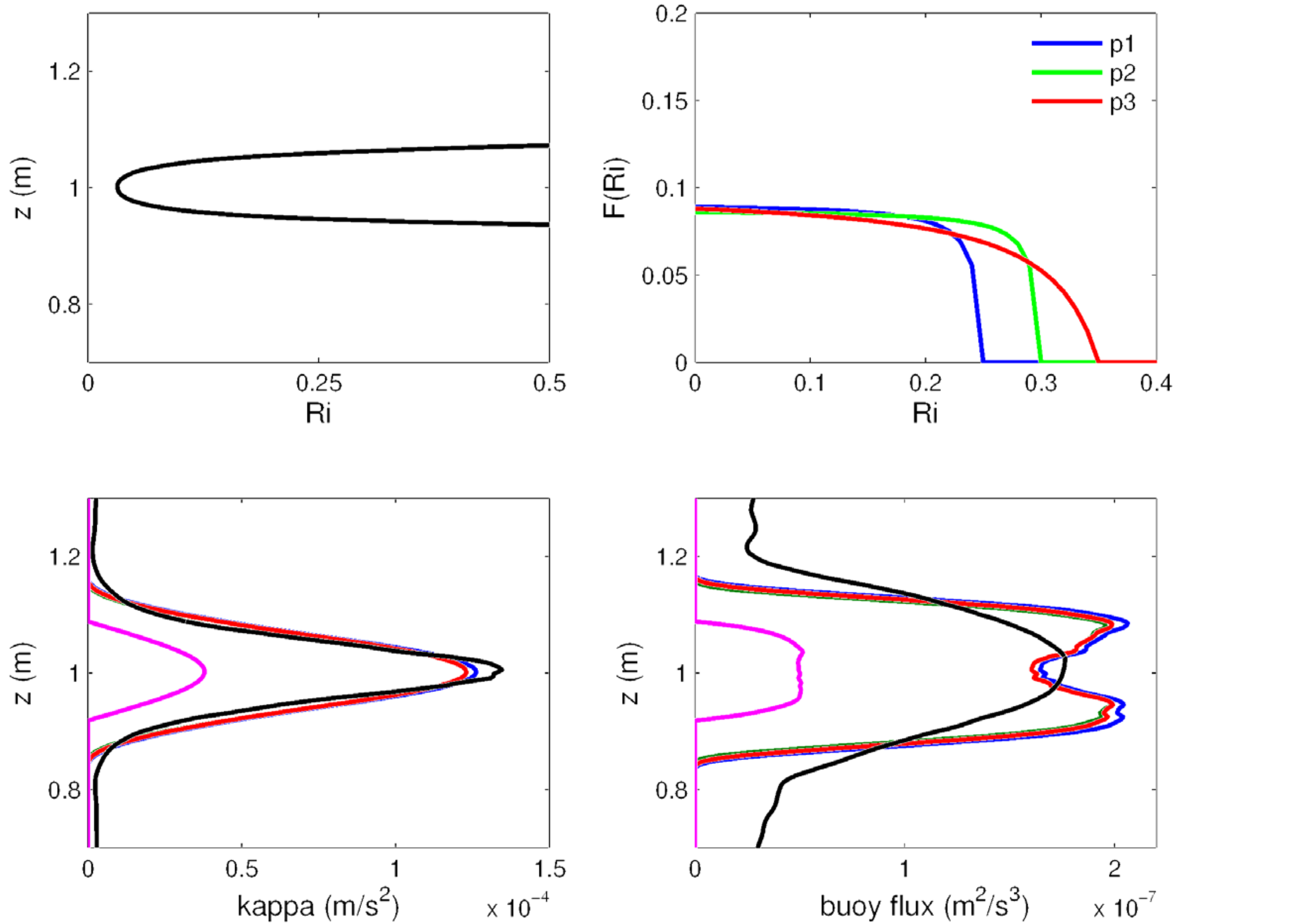


3D stratified turbulence



Shear results



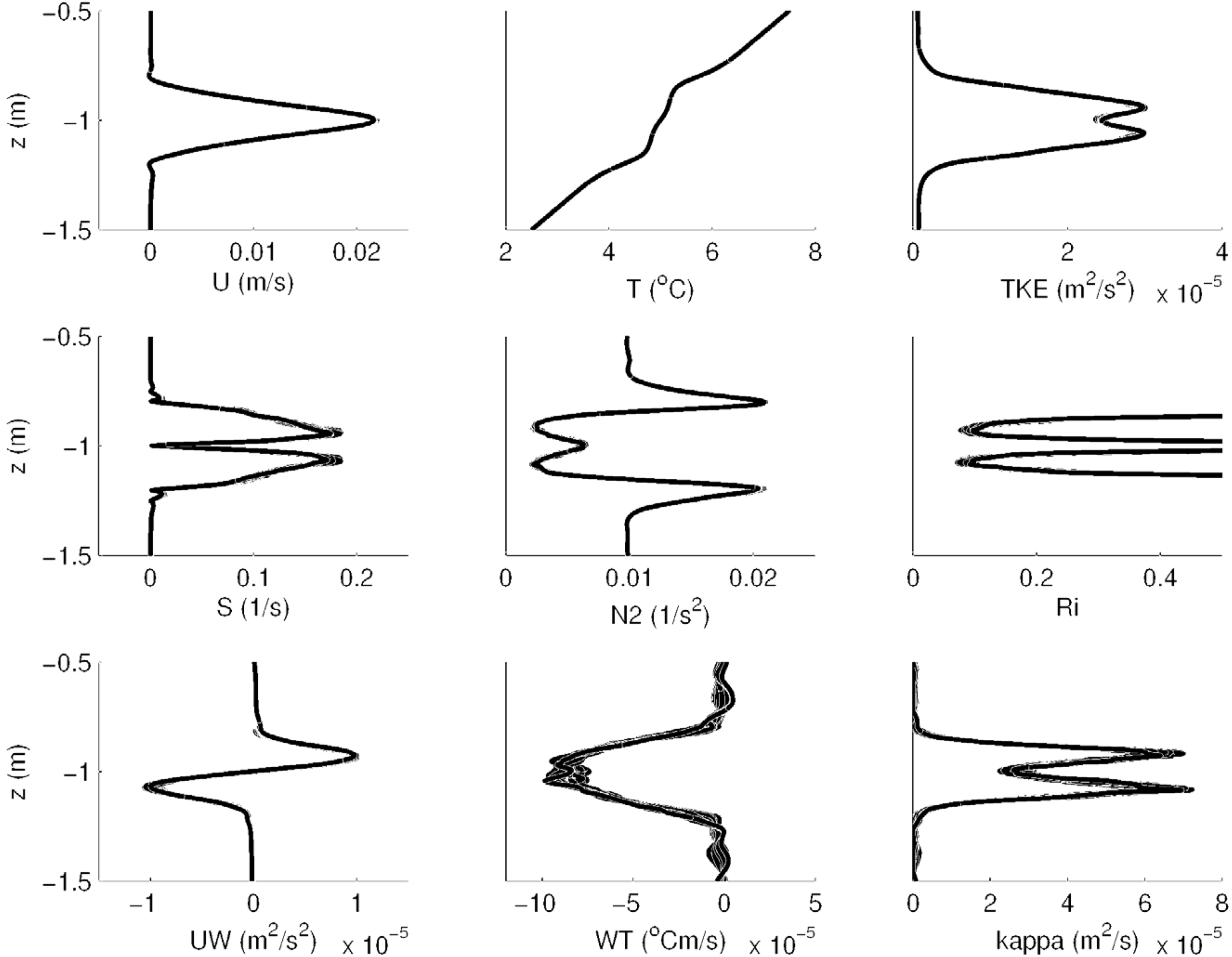


DNS data

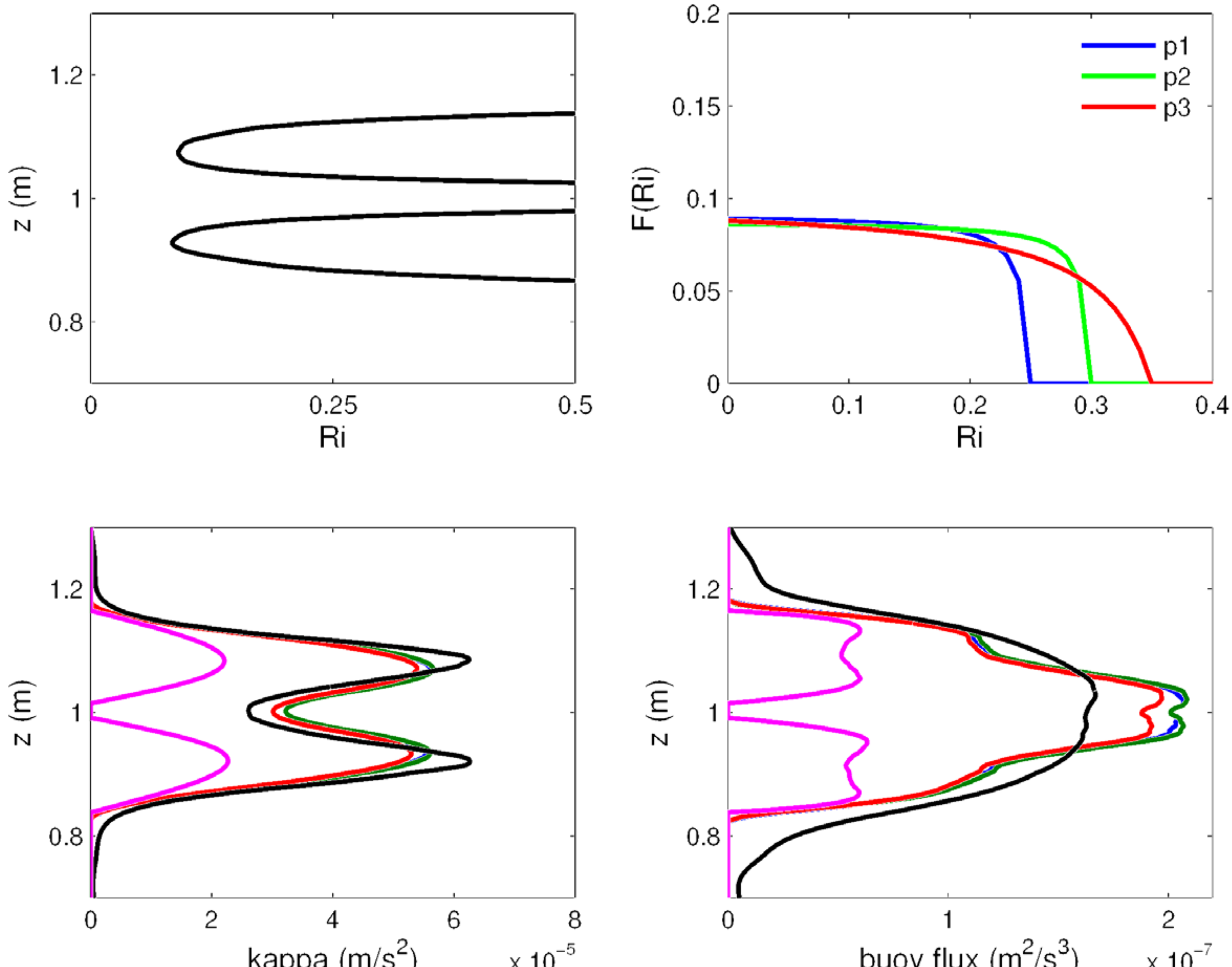
ET parameterisation

New parameterisation
$Ri_{Cr} = 0.25$, $c_N = 0.30$, $c_S = 0.11$, $\lambda = 0.85$
$Ri_{Cr} = 0.30$, $c_N = 0.25$, $c_S = 0.11$, $\lambda = 0.79$
$Ri_{Cr} = 0.35$, $c_N = 0.24$, $c_S = 0.12$, $\lambda = 0.80$

Jet results



z (m)
−0.5
−1
−1.5
0
0.01
0.02
U (m/s)
2
4
6
8
T (°C)
0
2
4
TKE (m²/s²) x 10⁻⁵
0
0.1
0.2
S (1/s)
0
0.01
0.02
N2 (1/s²)
0
0.2
0.4
Ri
−1
0
1
UW (m²/s²) x 10⁻⁵
−10
−5
0
5
WT (°Cm/s) x 10⁻⁵
0
2
4
6
8
kappa (m²/s) x 10⁻⁵



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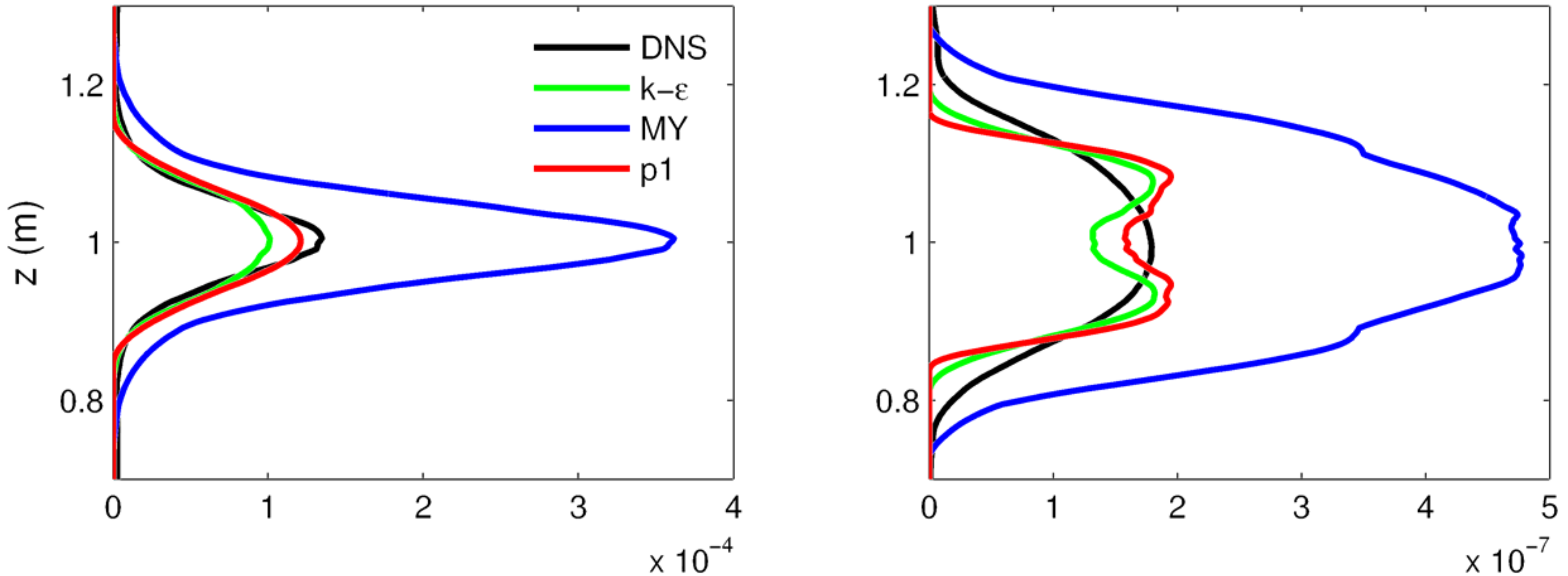
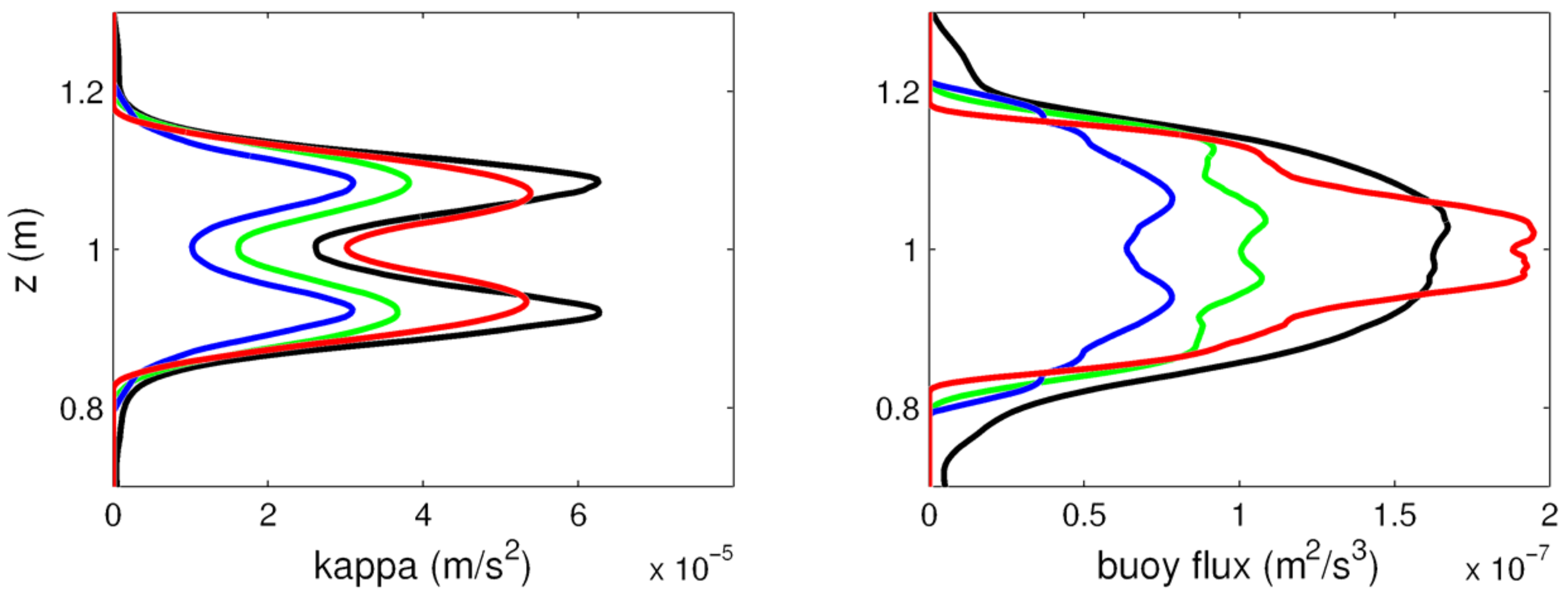
Comparison to two equation turbulence models

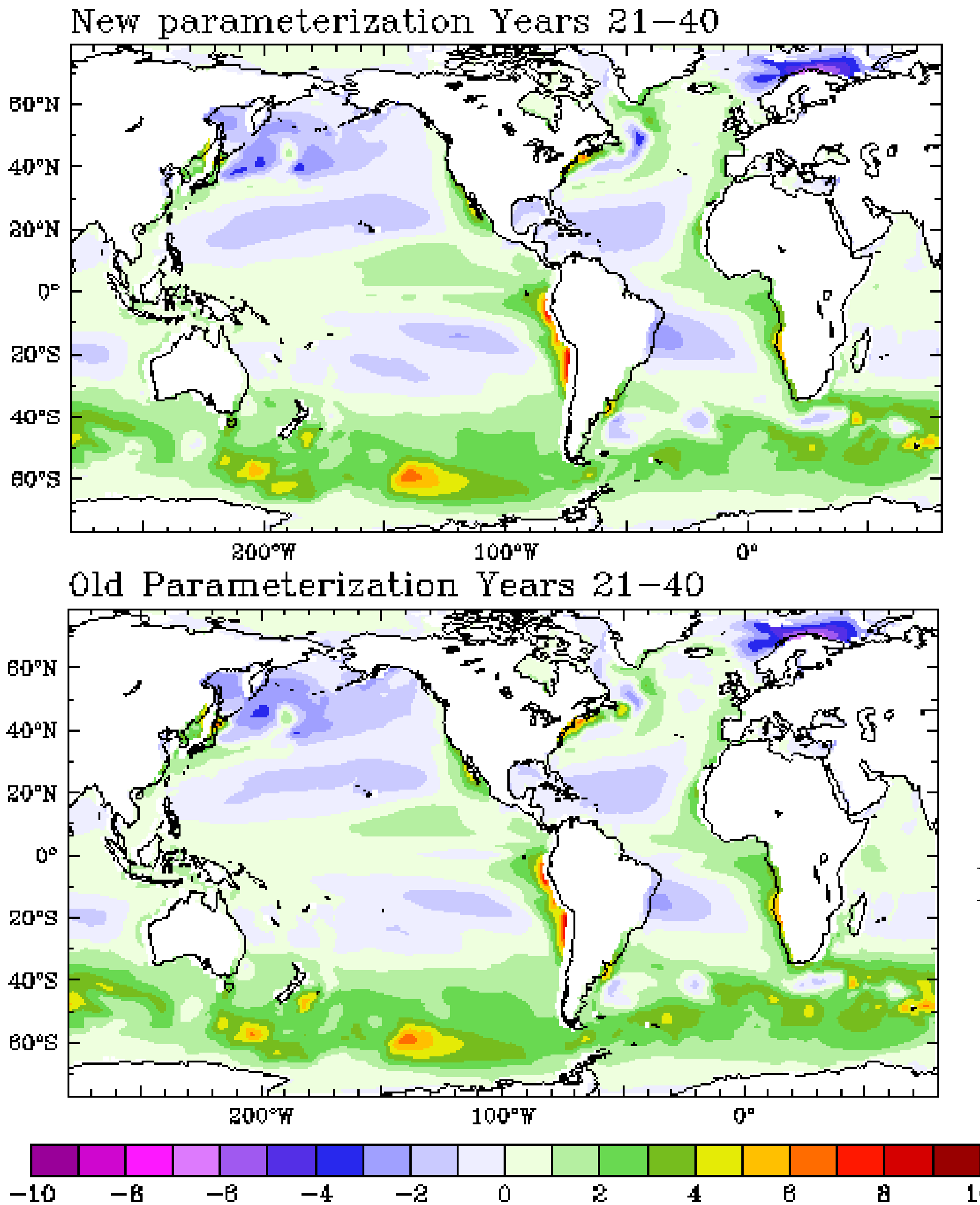
Shear results



Jet results



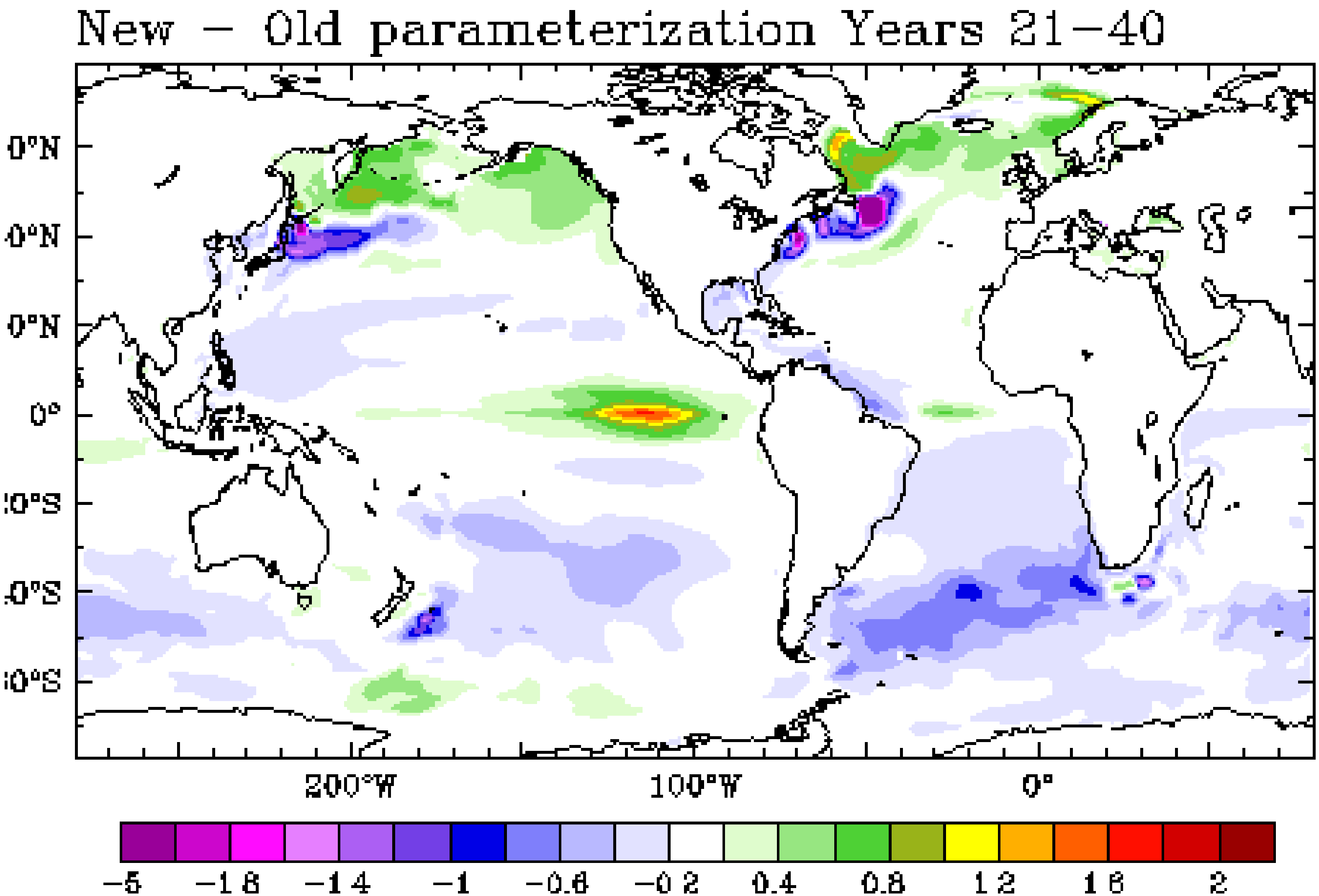
Coupled Model Annual Mean SST Anomalies



RMS = 1.52°C

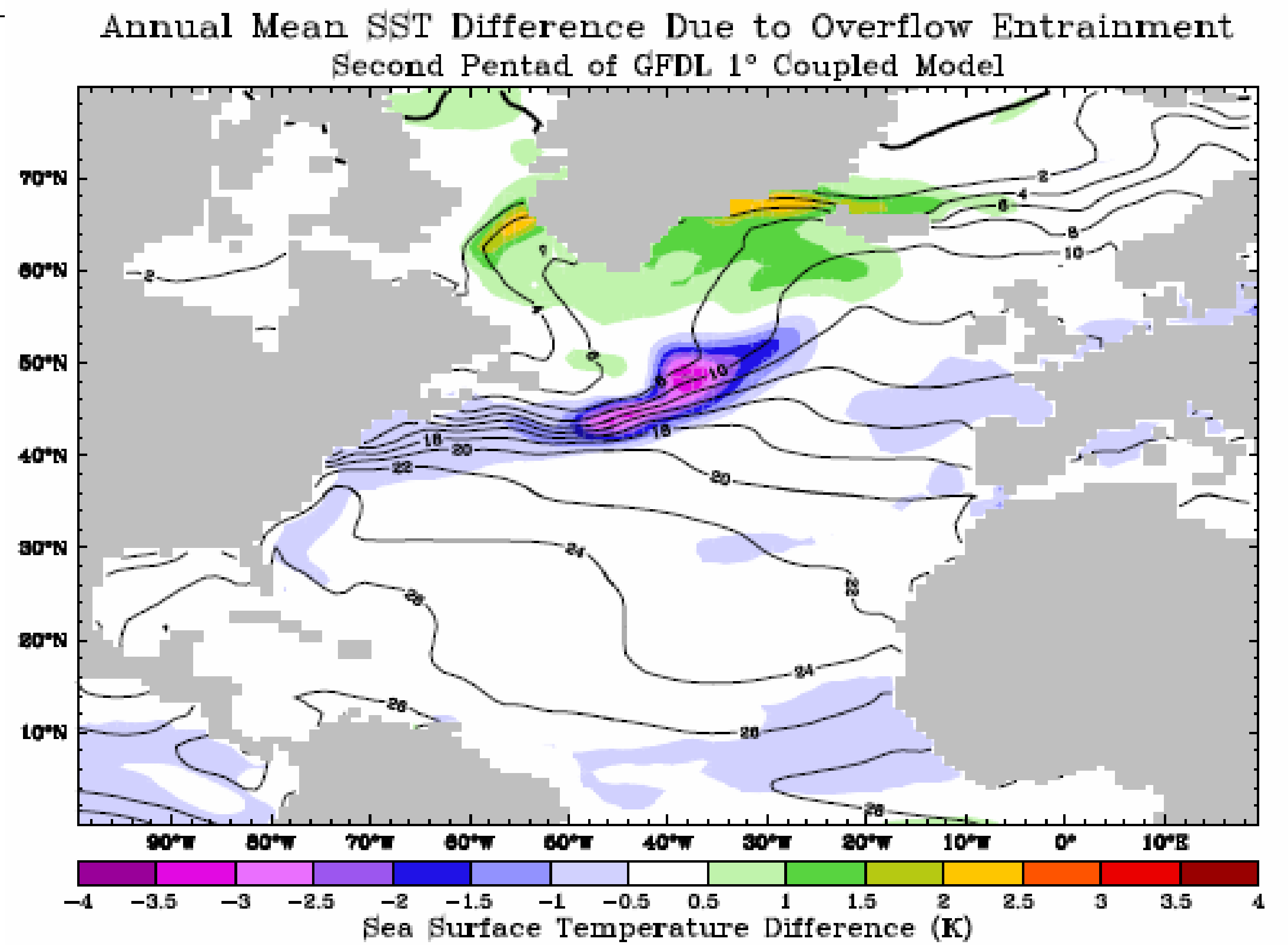
RMS = 1.59°C

Annual Mean Coupled Model SST Errors (°C)



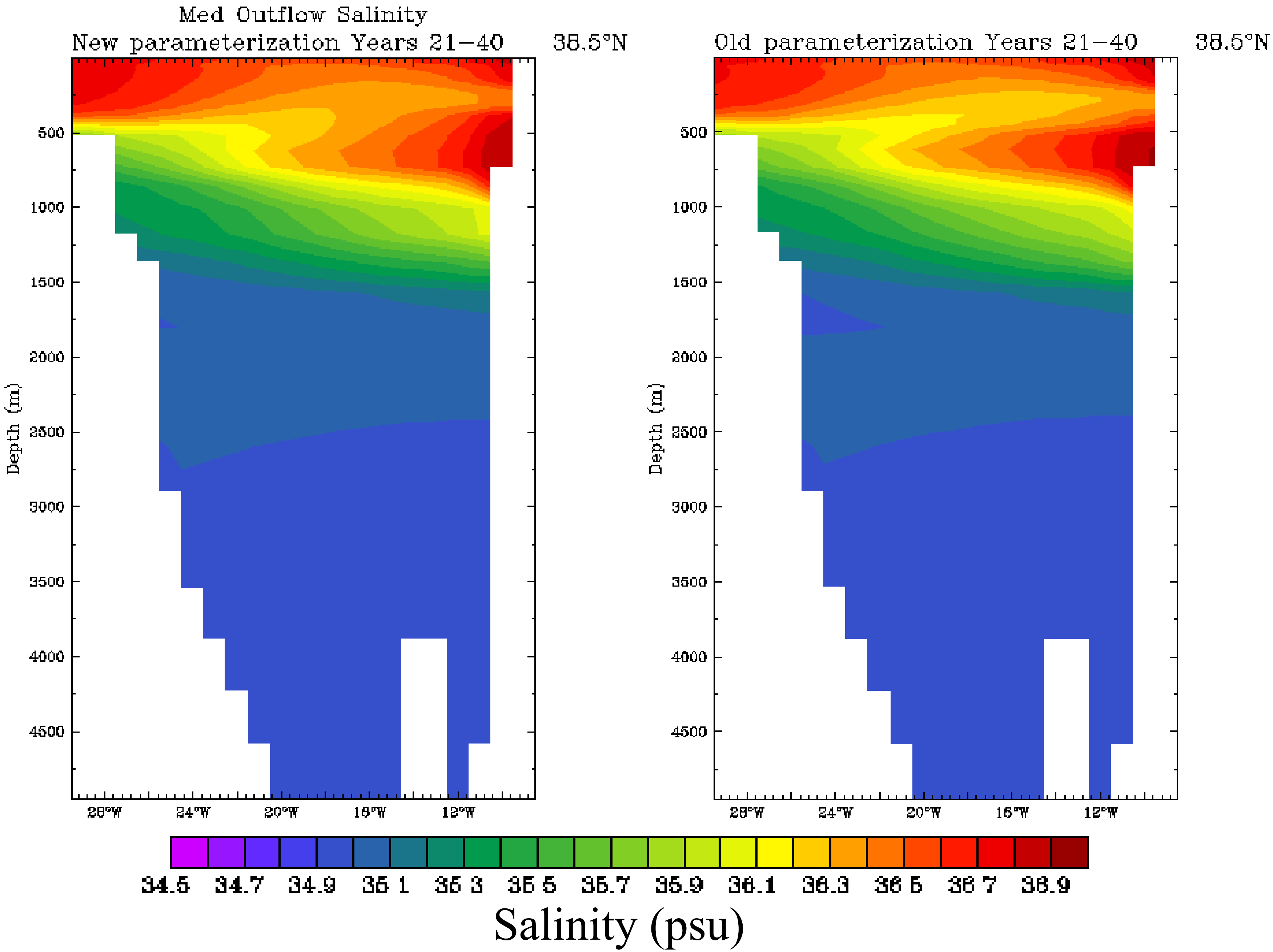
Annual Mean Sea Surface Temperature Difference (°C)

Results: Impact of shear-driven mixing parameterization on climate

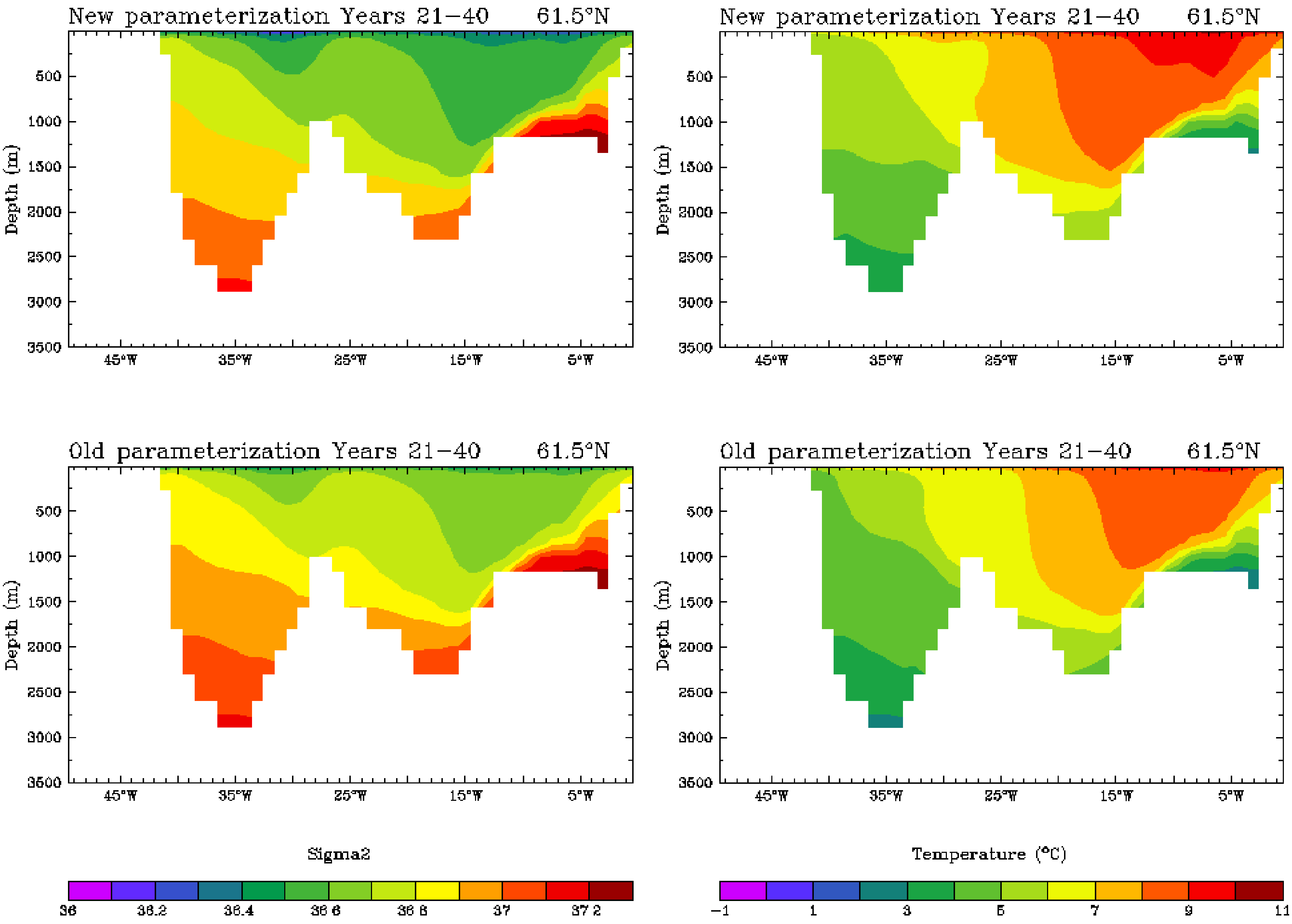


In coupled simulations using Hallberg Isopycnal Model, with entrainment in Nordic overflows SSTs are warmer near entrainment site, and cooler to south, due to change in location of Gulf Stream induced by DWBC transport changes.

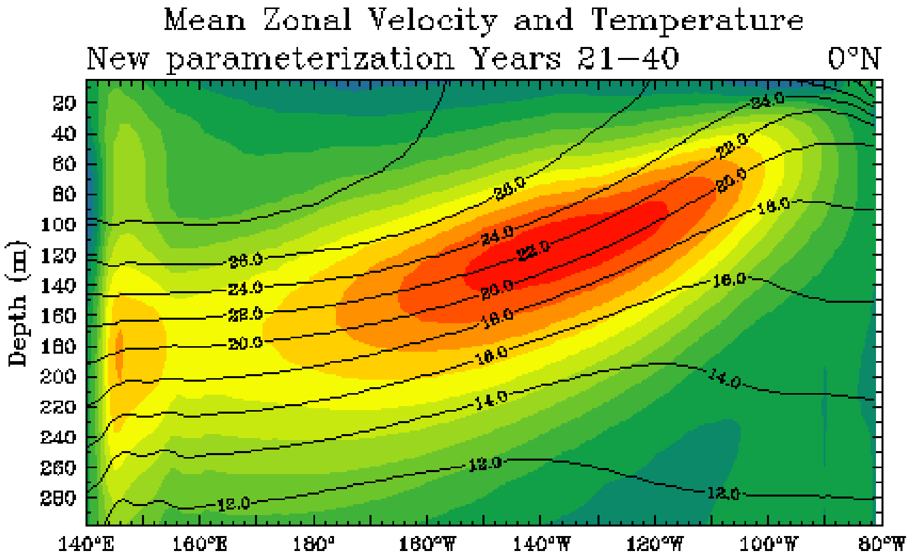
Mediterranean Overflow



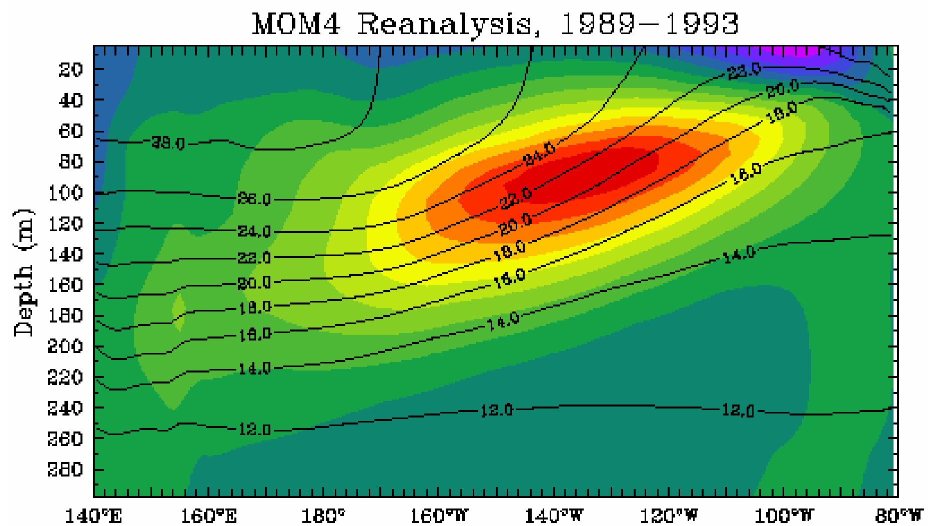
North Atlantic Overflows

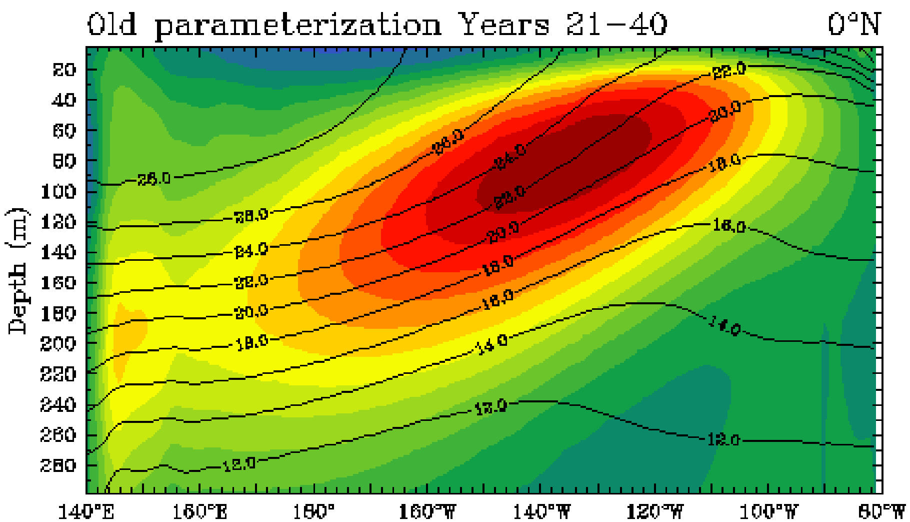


Pacific Equatorial Undercurrent

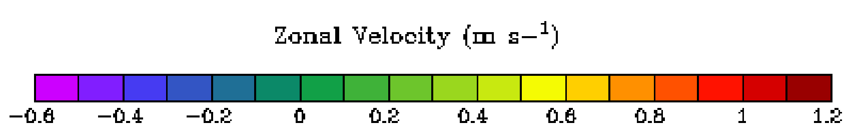


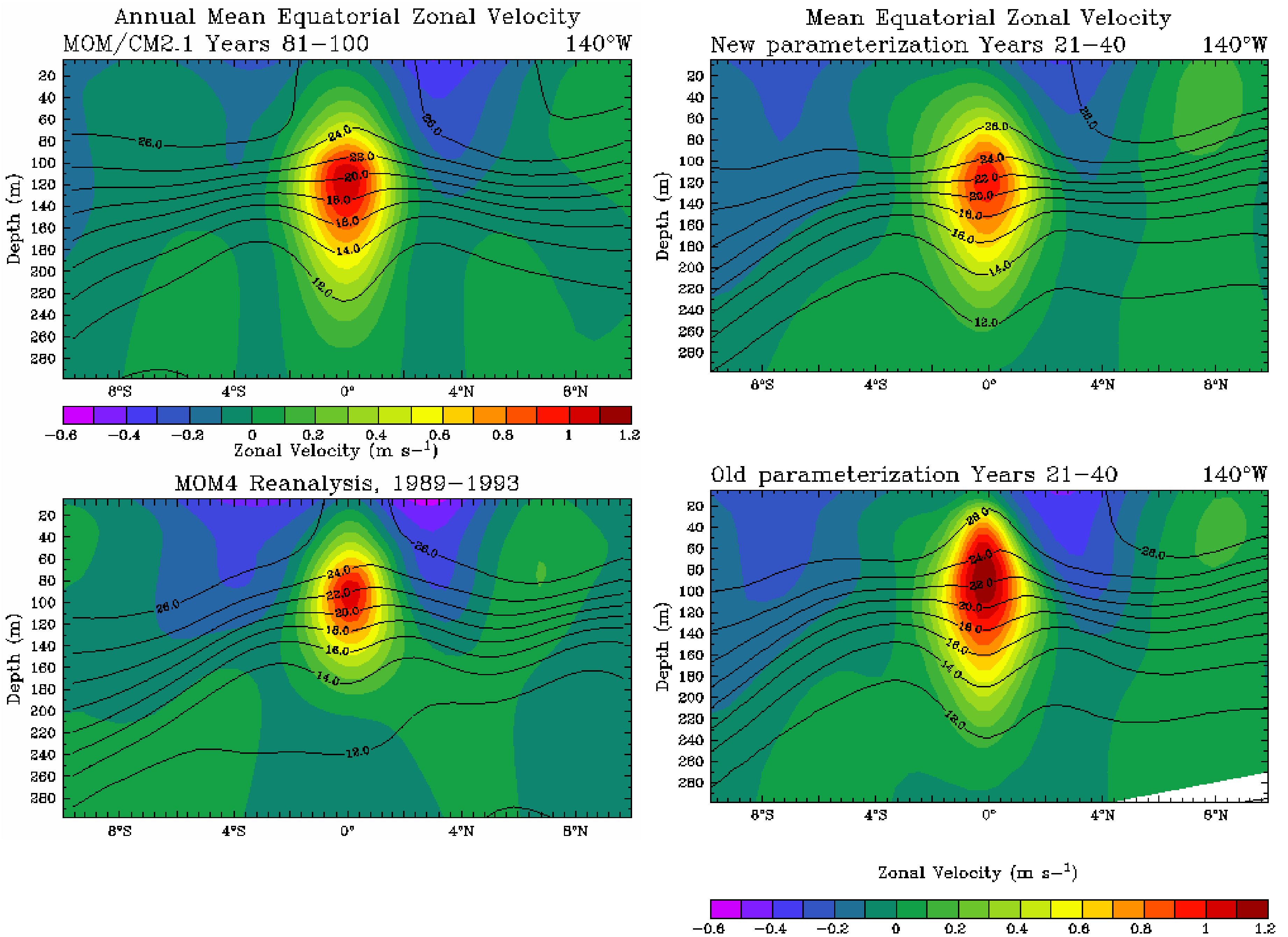
New parameterisation





Modified ET param





Annual Mean Equatorial Zonal Velocity
MOM/CM2.1 Years 81–100
140°W
Depth (m)
Zonal Velocity (m s−1)
MOM4 Reanalysis, 1989–1993
Mean Equatorial Zonal Velocity
New parameterization Years 21–40
Old parameterization Years 21–40
8°S
4°S
0°
4°N
8°N

Conclusions

- Diffusivity based on a local Ri cannot capture the buoyancy flux for a stratified jet - need non-local terms
- Can link isopycnal parameterisations of mixing to a turbulent diffusivity so a parameterisation can be used for z and density level models.
- New shear-driven mixing parameterisation compares well to DNS results
- New parameterisation gives sensible solutions in particular limits
- Implicit numerical solution is robust and efficient
- Initial results of mixing in the overflows and EUC from a coupled global simulation are good.

Future/continuing work

- Analysis of mixing in global climate model.
- Comparing to high resolution gravity currents from collaborators in Miami.
- Sensitivity tests using idealised DOME test case