Resolution Dependent Relative Dispersion Statistics in a Hierarchy of Ocean Models

A.C. Haza , A. C. Poje*, T. M. Özgökmen, M. G. Magaldi , Z. D. Garraffo**

RSMAS / University of Miami
*CUNY, Staten Island, NY
**CCS / University of Miami

LOM 2009 – Miami, Florida

Layout

- Importance of Relative Dispersion in ocean models.
- RD study from 3 numerical models.

  - Spectral operator in a 2D turbulence model

  - Buoyant coastal Jet (ROMS) at different resolutions.

  - Gaussian filter in the Gulf Stream region (HYCOM 1/12° Atlantic simulation)
- Scaling by the Okubo-Weiss parameter
- Summary

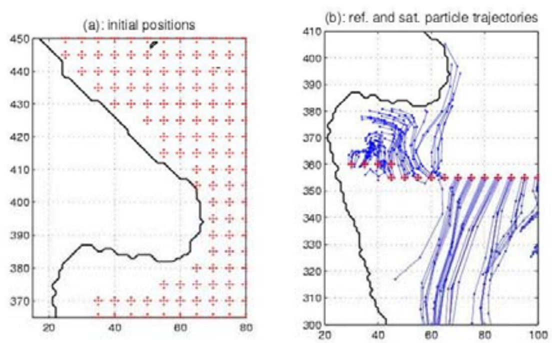
Importance of relative dispersion. 1: FSLE maps

FSLE maps or **local** Finite Scale Lyapunov Exponent is defined as:

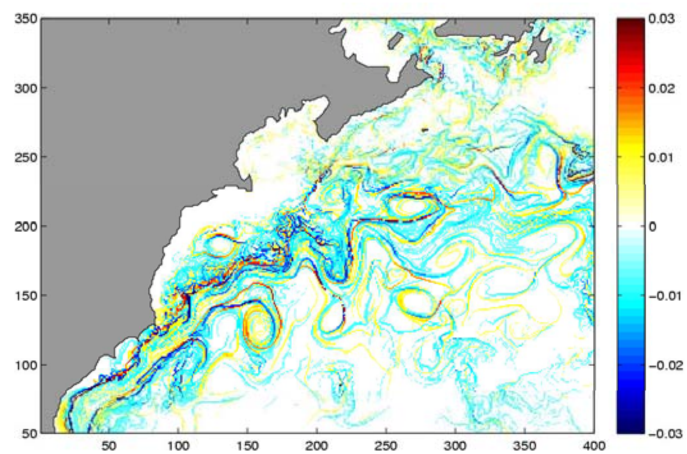
$$\Lambda(\mathbf{x}, t, \delta_i, r) = \frac{1}{\tau}\, log\ r$$

where τ is the time required for particle pairs to separate from δi to r x δi .

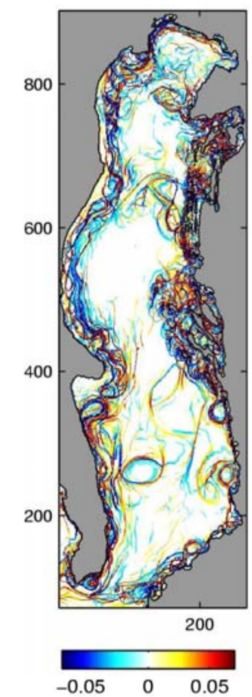
Type of launch configuration:



Gulf Stream (MICOM)



Adriatic Sea (NCOM)



Help define the natural transport barriers of the flow

- targeting drifter launches.
- visualize transport of pollutants & fish larvae.. Etc.

Importance of relative dispersion. 2: Dispersion regimes

Statistics of RD defined as:

$$\boxed{D^2(t) =< |\mathbf{x}^{(1)}(t) - \mathbf{x}^{(2)}(t)|^2 >}$$

Relative dispersion
(for all particle-pairs)

$$K(t) = \frac{1}{2}\frac{dD^2}{dt} = \langle \mathrm{D}(t, \mathrm{D}_0) \cdot \delta \mathbf{v}(t, \mathrm{D}_0) \rangle$$

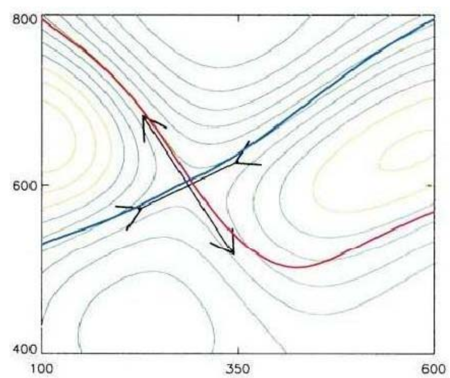
Relative diffusion

$$\boxed{\lambda(\delta) = \frac{ln(\alpha)}{< \tau(\delta) >}}$$

FSLE value for the scale δ
(<τ> = averaged time for all particle pairs to separate from distances δ to αδ)

Applications: define the dispersion regimes at all scales

➢ Investigate conditions for an **exponential** regime at small scales.



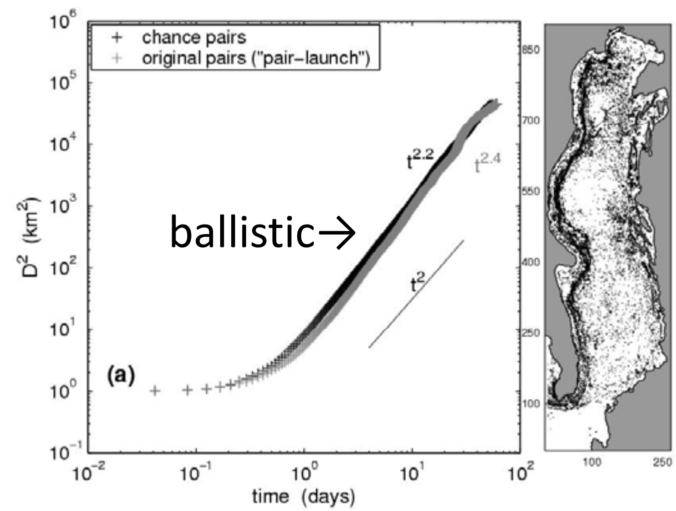
← **hyperbolic region** **(**particles separate at an **exponential** rate).
Poje et al., JPO 2002.

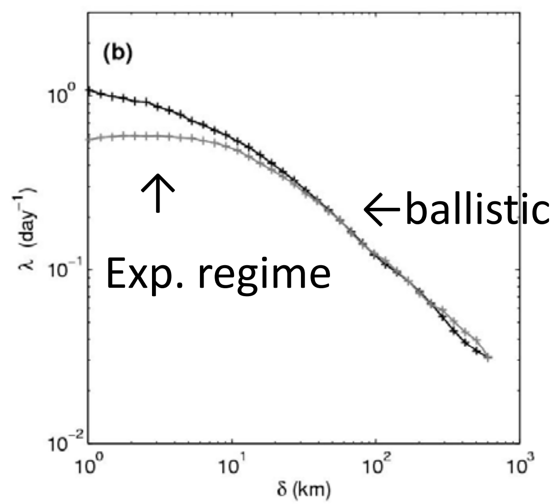
Relative dispersion from numerical models

From previous study in the Adriatic Sea with NCOM (horizontal grid = 1km):

-R.D. (δ > 2xRd) well captured (governed by eddies and jets), follows power laws in time and space.
- R.D. (δ < 2xRd) displays an exponential regime, but is very sensitive to the type of launch configuration and FSLE method of computation.

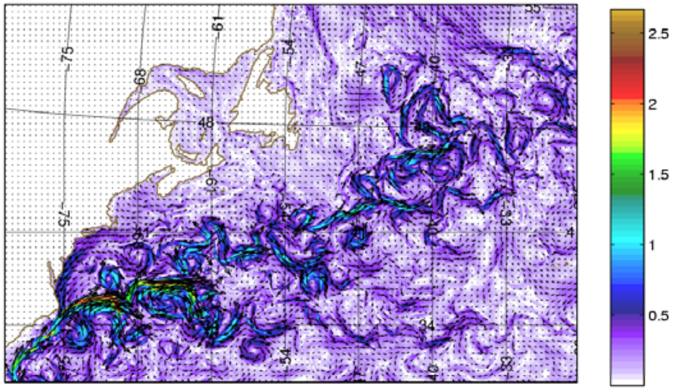
Adriatic Sea / NCOM





Relative dispersion from numerical models

HYCOM 1/12° surface velocities



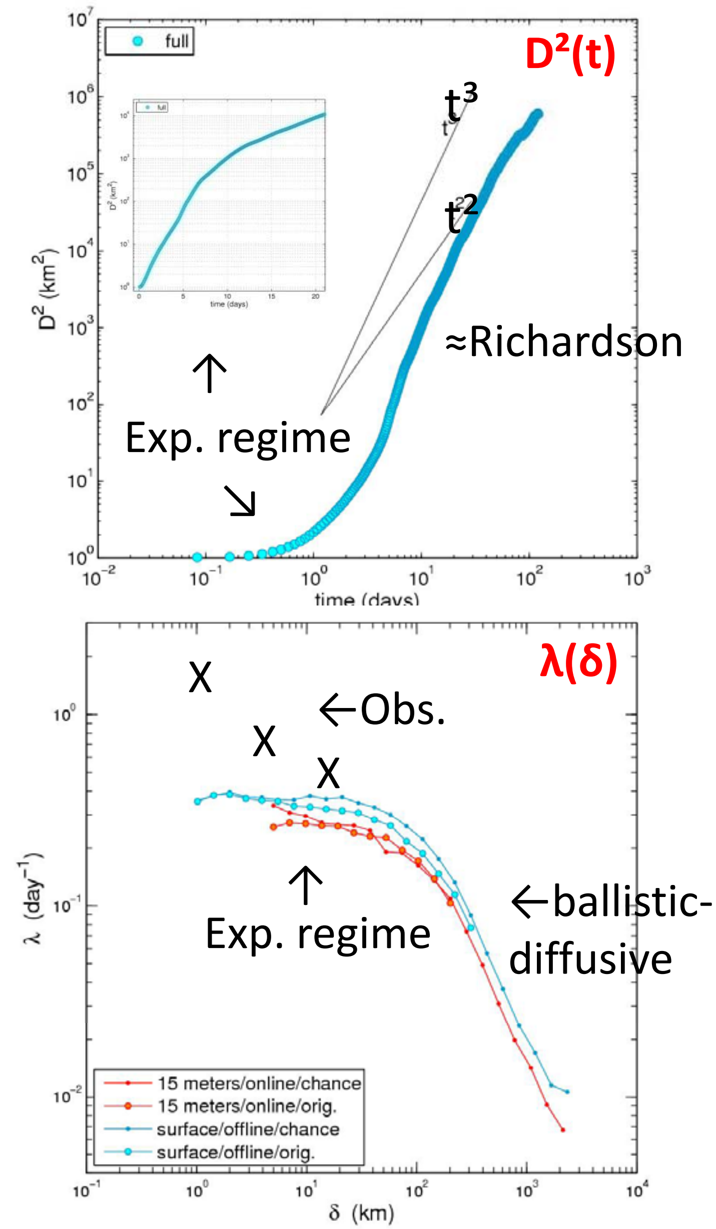
HYCOM configuration:

- 1/12° horizontal grid, 32 vertical layers.
- Atlantic basin, 28°S to 80°N.
- Particles launched between Cape Hatteras and the Grand Banks.

***Comparison with latest observations (Lumpkin et al., 2009)*:**

- RD (δ > 2xRd = 100km) is correct.
- RD (δ < 100km) exponential in the model only.

➢ ***Knowledge of the small scale dispersion regime remains elusive.***

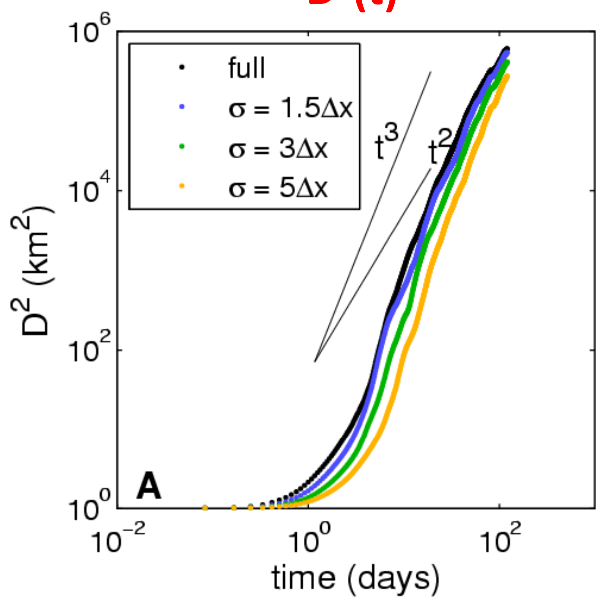


Reducing the resolution in HYCOM outputs and impact on the R.D.

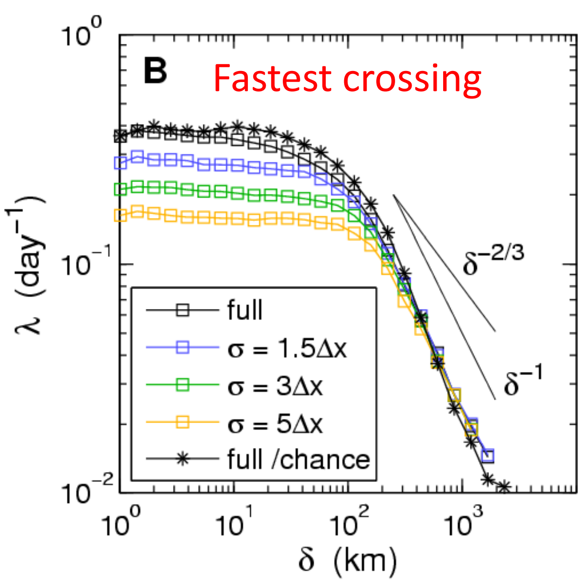
- Apply a **Gauss filter** on vel. outputs:

$$u^F(i_0, j_0, k) = \frac{\sum_{i,j} u(i,j,k) e^{\left[(i-i_0)^2+(j-j_0)^2\right]/\sigma^2}}{\sum_{i,j} e^{\left[(i-i_0)^2+(j-j_0)^2\right]/\sigma^2}}$$

D²(t)



λ(δ)



↗

← **Exponential regime extended with increasing smoothing.**

← **λmax (hyperbolicity) reduced .**

Barotropic 2D turbulence model

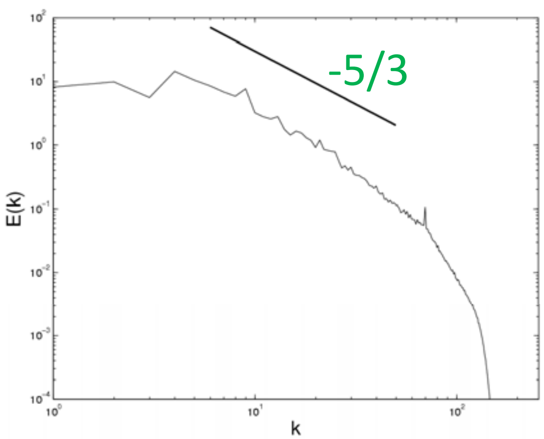
2D vorticity equation:

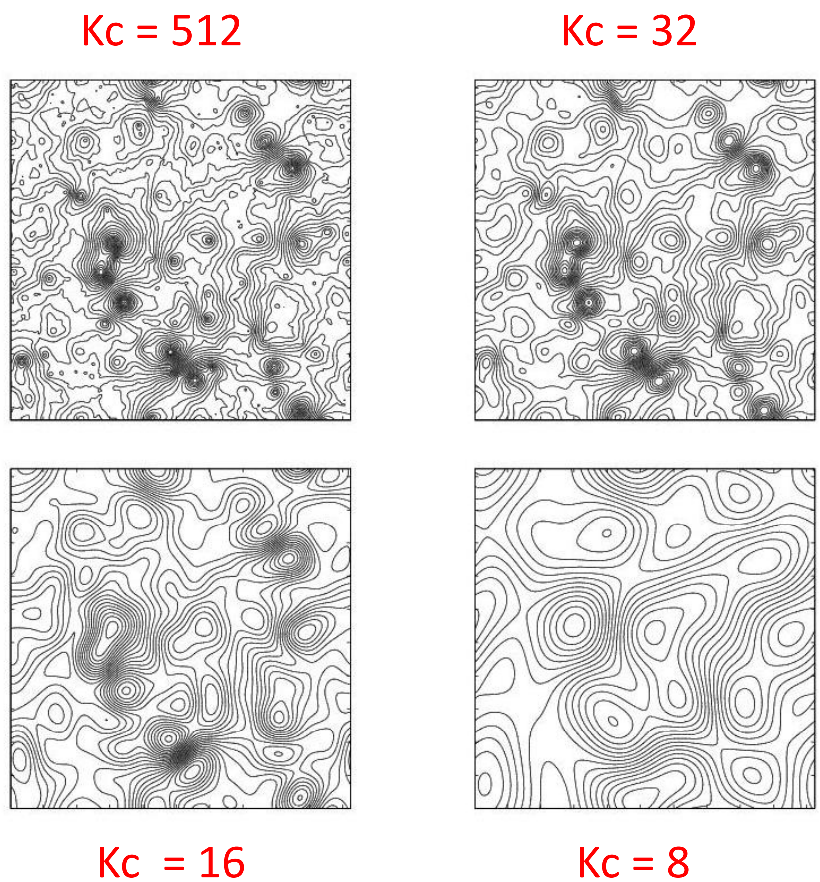
$$\begin{aligned}\frac{\partial \omega}{\partial t} + J(\psi, \omega) &= F(\omega) + D(\omega) \\ \omega &= \nabla^2 \psi\end{aligned}$$

The particles are advected with truncated spectral modes:

$$\boxed{\hat{\psi}(\mathbf{k}, t) = \begin{cases} \psi(\mathbf{k}, t) & \text{if } |\mathbf{k}| \leq k_c \\ 0 & \text{if } |\mathbf{k}| > k_c, \end{cases}}$$

KE spectrum for kf=70





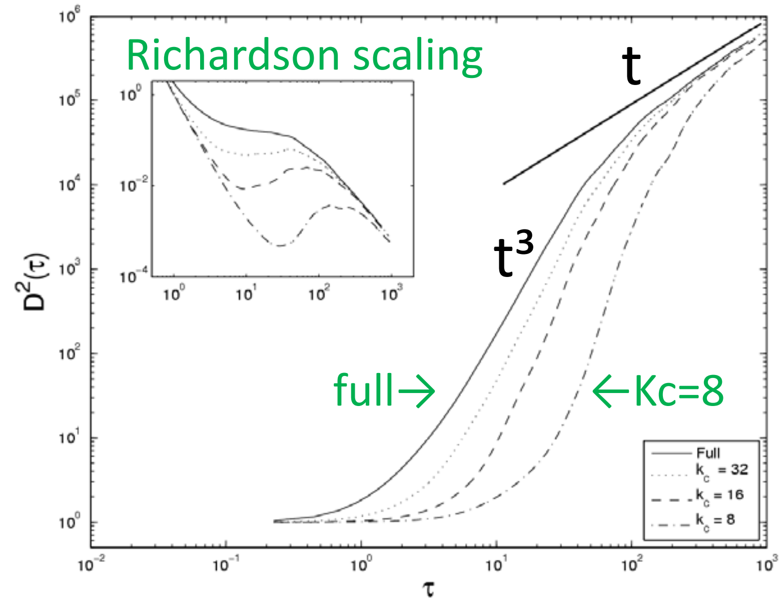
ψ snapshots for different k-cutoffs

D^2 & FSLE (2D turbulence model)

- Exponential regime extended.

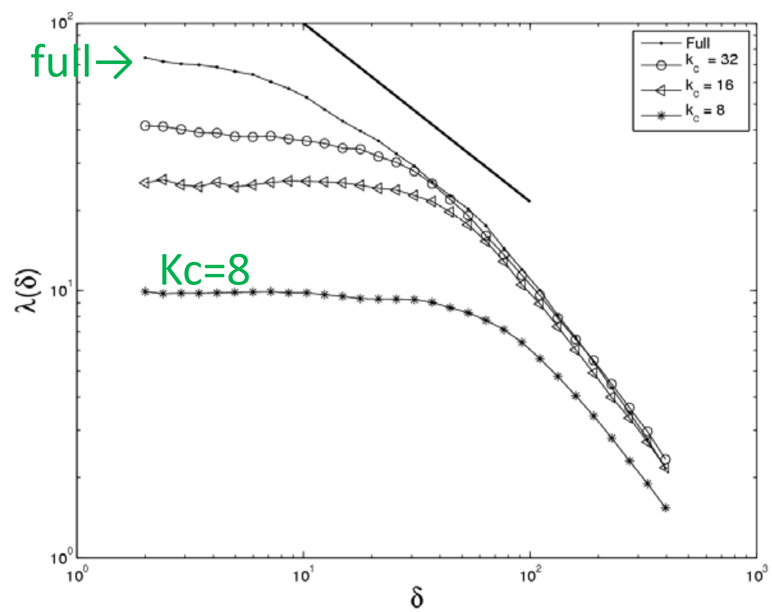
- Different power-law at intermediate times.

- Diffusive at longer times.



$D^2(t)$

- Exponential regime extended to larger scales.



$\lambda(\delta)$

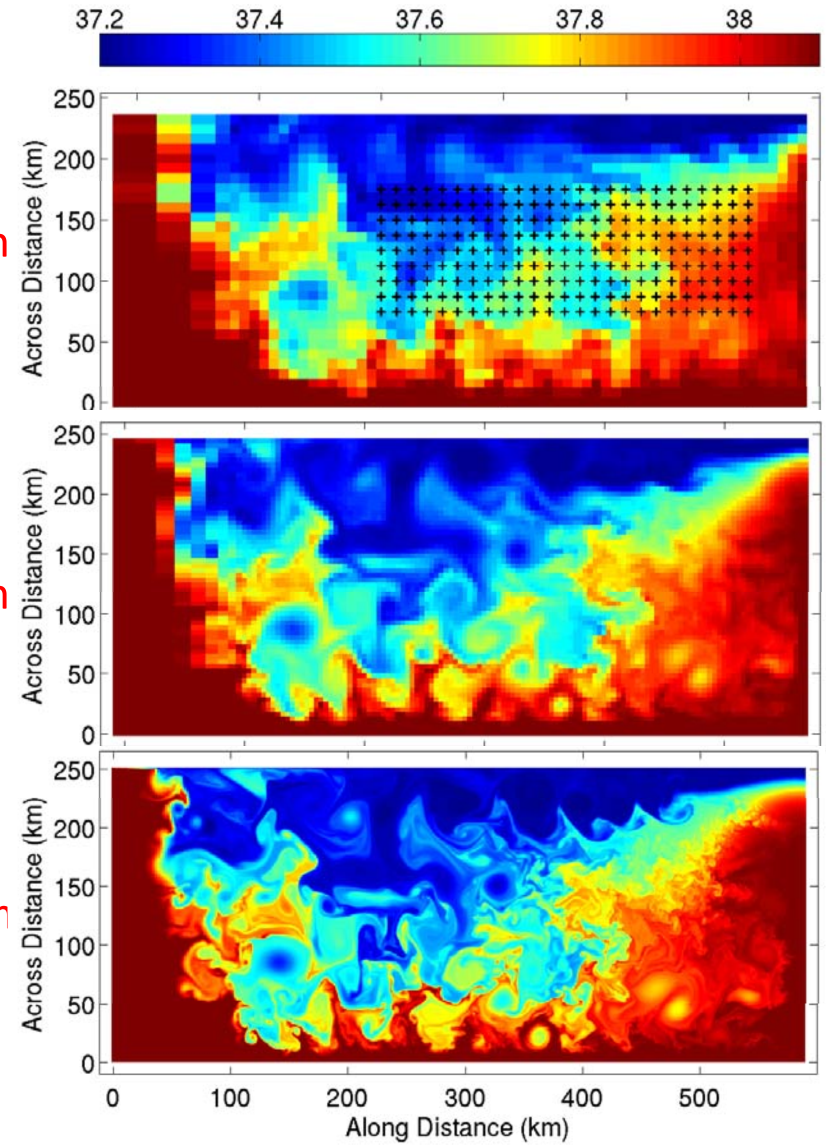
Buoyant coastal jet (ROMS)

Configuration:
- 580km x 258km, H=150m, 20 sigma layers.
-5 experiments of different horizontal Resolution: Δx = 8, 4, 2, 1, and 0.5km.
-main features: ***Jet and development of baroclinic instabilities.***

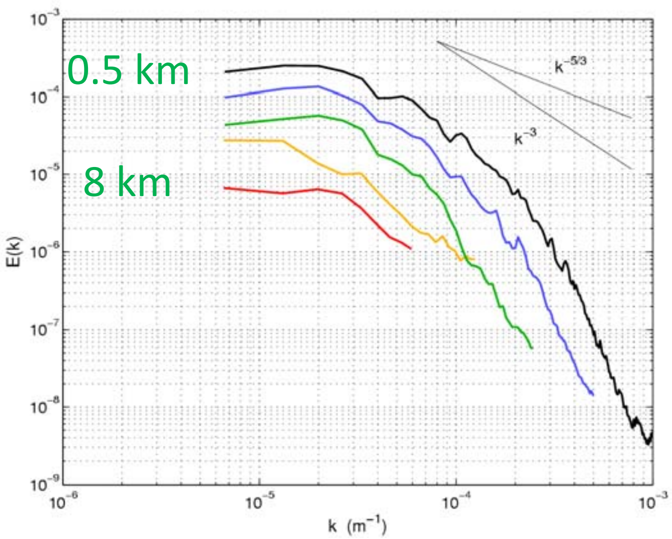
8 km

4 km

0.5 km

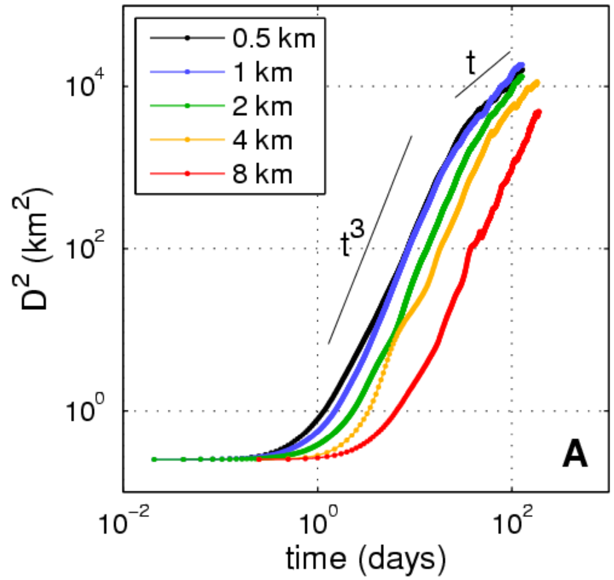


KE spectrum



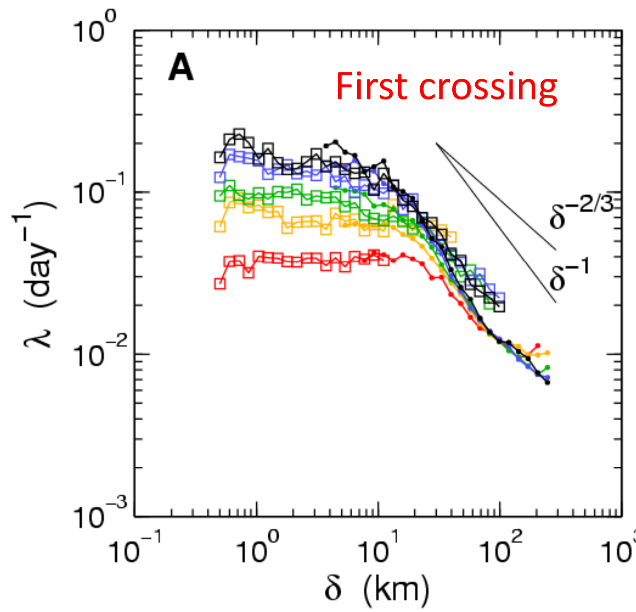
D^2 & FSLE (coastal jet ROMS)

- **Exponential regime** (extended with coarser resolutions).
- **Power law of 2.7** (~ Richardson)
- **Diffusive** at large scales, except for 8km run.



$D^2(t)$

- **Exponential regime extended** when resolution ↘.
- Power law between Richardson and ballistic.
- **Hyperbolicity ↘** when resolution ↘.



$\lambda(\delta)$

Okubo-Weiss parameter

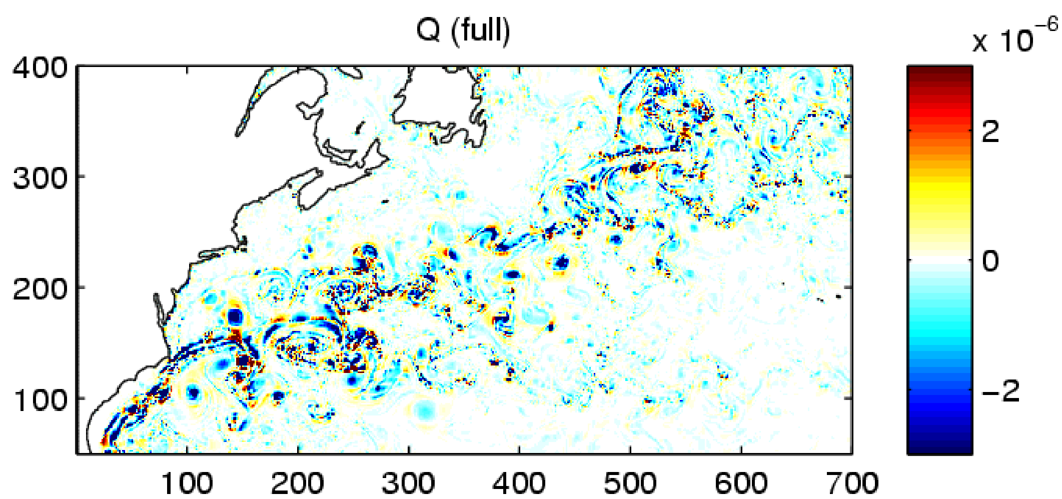
The resolution dependent Eulerian quantity:

$$Q^2 = S^2 - \omega^2 .$$

$$S^2 = S_n^2 + S_s^2, \quad S_n = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}, \quad S_s = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y},$$

where Sn, Ss are the normal and shear components of **strain**, and ω^2 is the **enstrophy**.

$Q^2 < 0$ ⇔ ***regions dominated by rotation*** (little divergence).
$Q^2 > 0$ ⇔ ***regions dominated by strain and deformation*** (exponential divergence)



HYCOM / Gulf Stream / units in 1/sec²

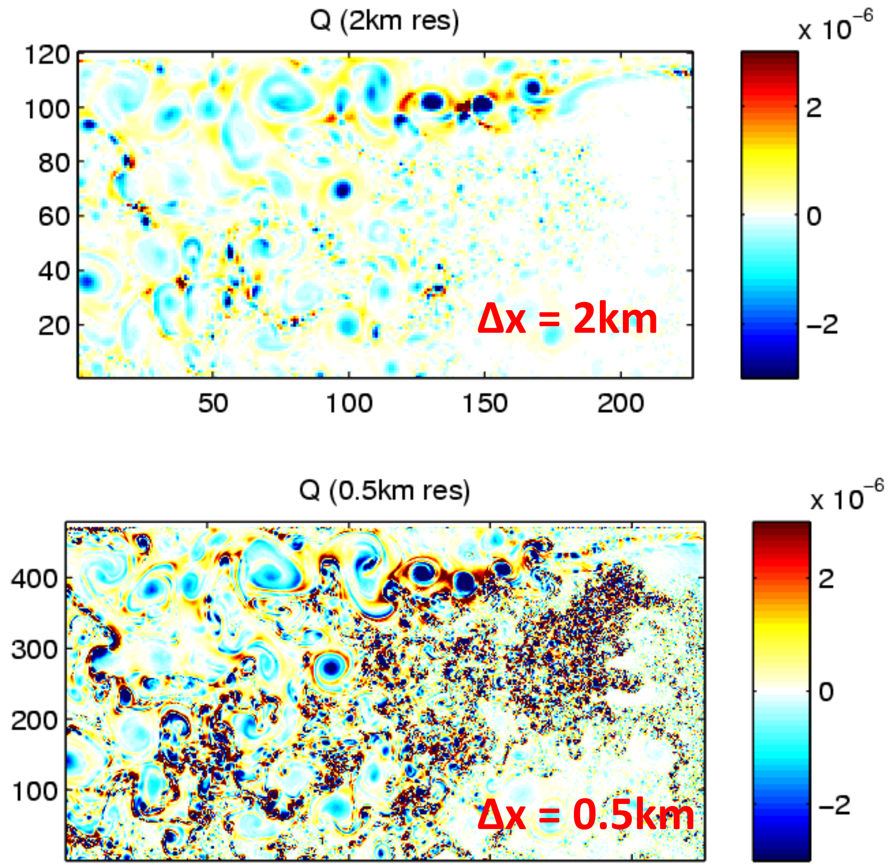
Okubo-Weiss parameter

Since $Q \sim \Delta U/\Delta x$, its magnitude increases dramatically with the horizontal resolution.

Therefore, we define the quantity:

$$\boxed{\overline{Q} = A^{-1} \int \sqrt{Q^2}\, dA}$$

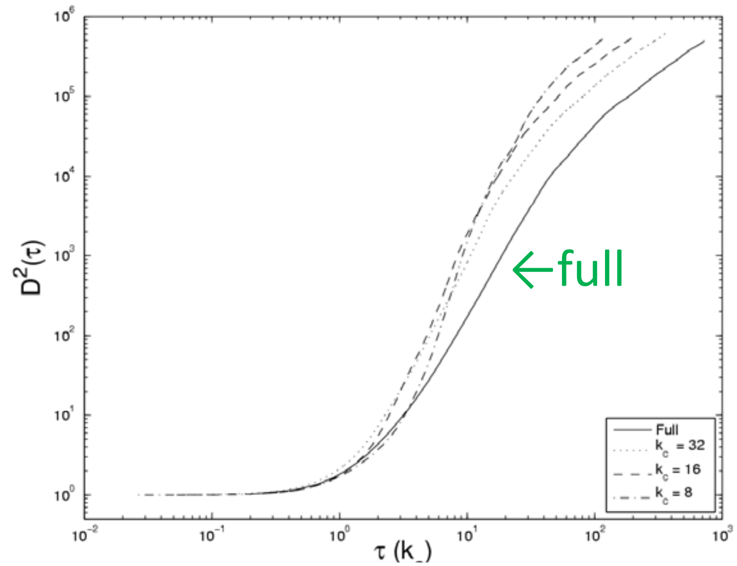
(with $Q^2 > 0$ only) is an Eulerian quantity similar to the average hyperbolicity of the model fields.



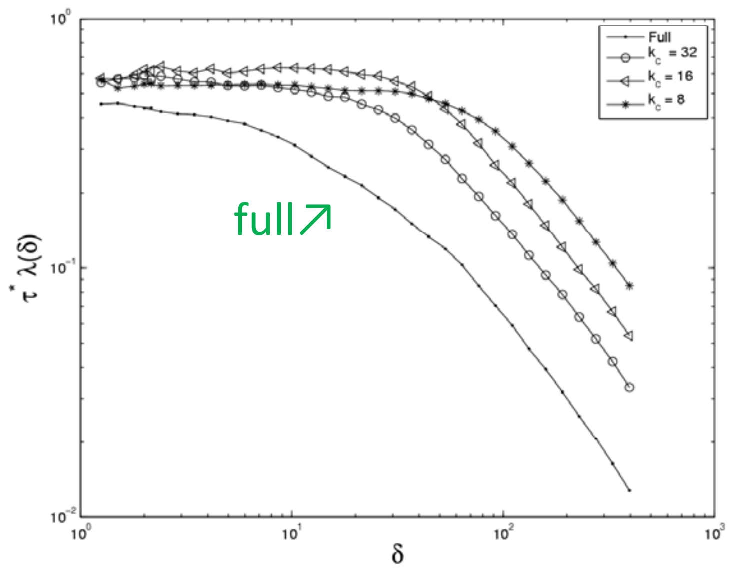
ROMS / coastal jet / (1/sec^2)

D^2 & FSLE scaled (2D turbulence)

- relatively faster R.D. than for the "full" case.



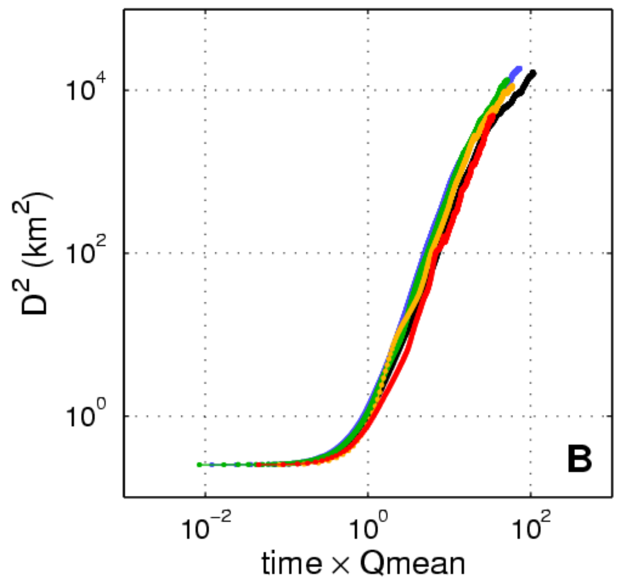
D²(t x Qm)



λ(δ)/Qm

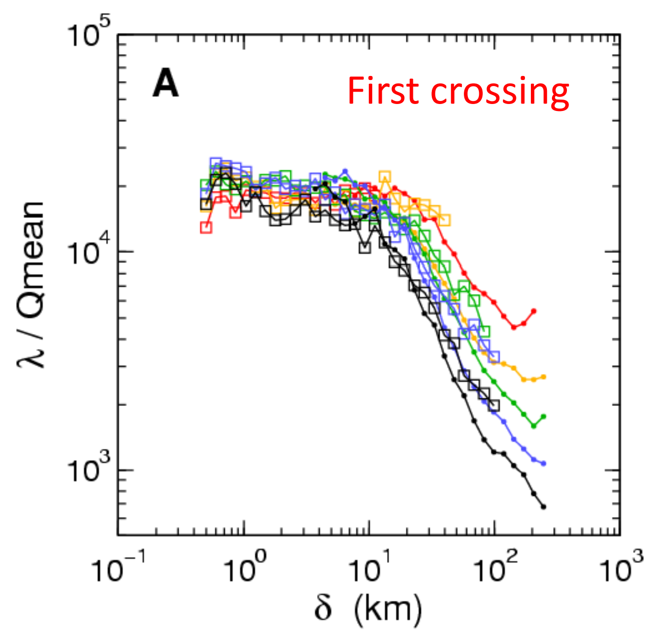
D^2& FSLE scaled by Qm >0 (ROMS)

Scaling by the hyperbolicity time does not change the slope
(unlike 2D turbulence experiment).



D^2(t x Qm)

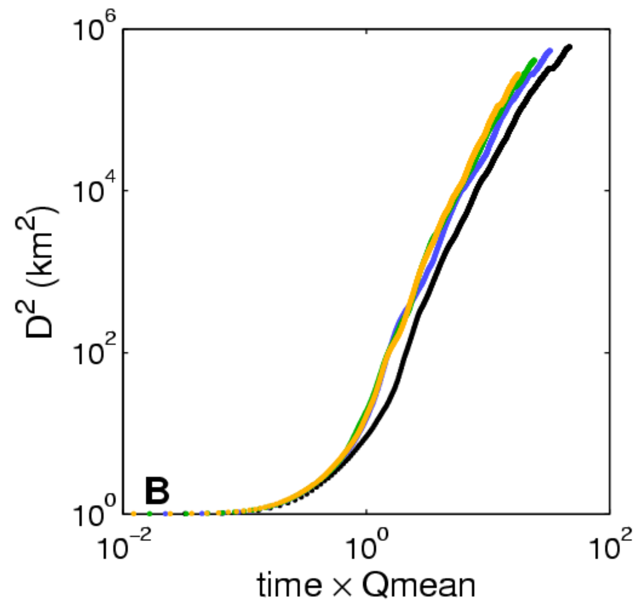
λmax/Qm collapse for all Simulations.



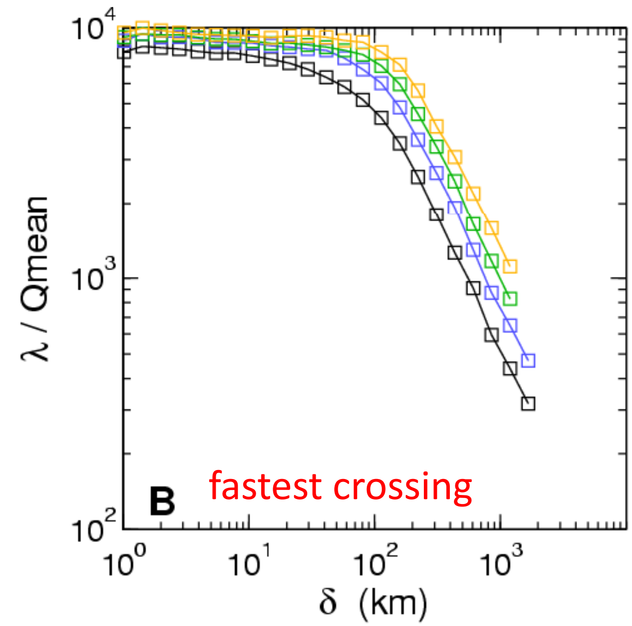
λ(δ)/Qm

D² & FSLE (HYCOM) scaled by Qm

- Same as ROMS/coastal jet exp.
- Still, the scaled R.D. of the full case is slightly slower/smaller as in the 2d turbulence experiment.



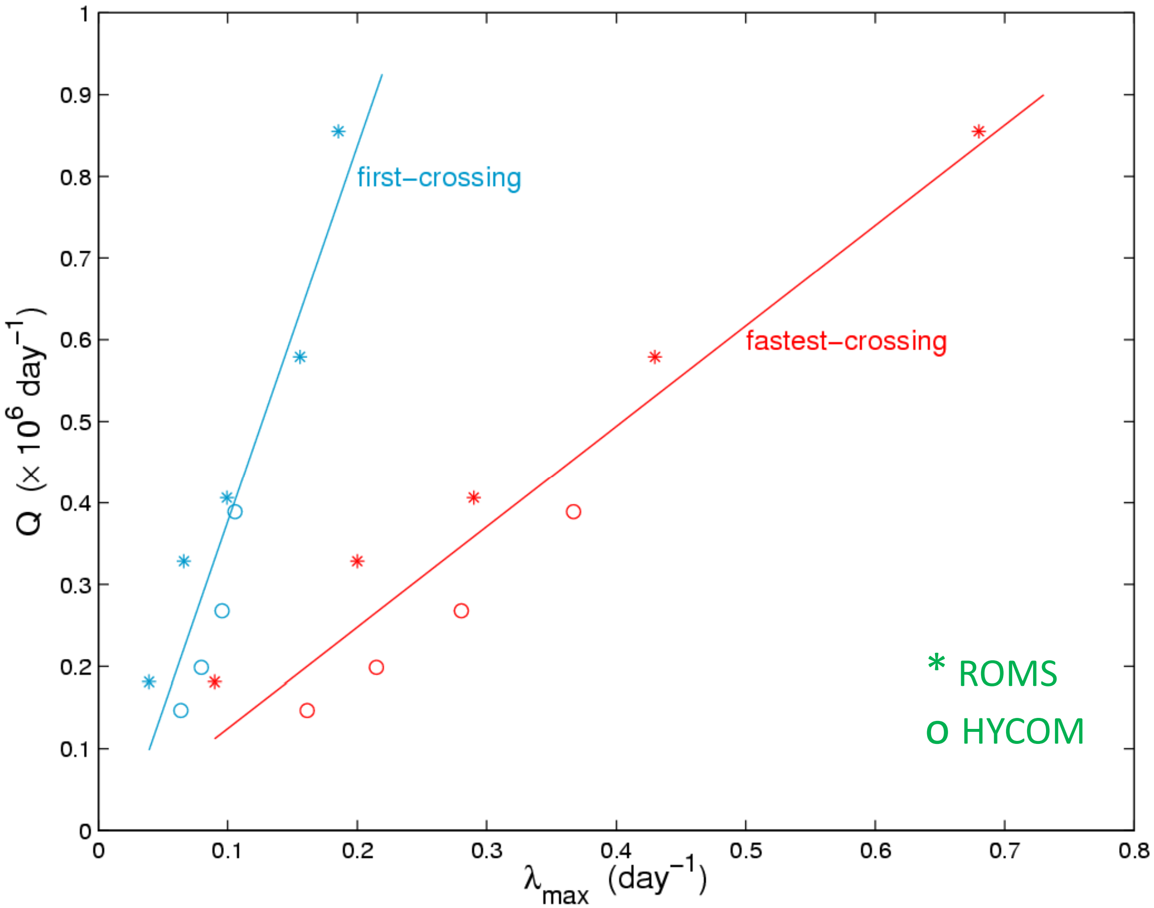
D^2(t x Qm)



λ(δ)/Qm

What sets the highest FSLE?

Qm > 0 vs **λmax** for ROMS & HYCOM experiments



Qm > 0 sets the exponential regions in between the coherent structures

➢ ***Sets the maximum separation at the small scales.***

Summary

- **Short scale dispersion:** defined by the spatial resolution of Eulerian measures of velocity gradients.
- **Large scale dispersion:** insensitive to resolution.
- **Intermediate scales:**

➢ resolution dependent, even at scales much larger than the grid-scale.

➢ Increase in the extent of the exponential separation regime..

A.C. Poje, A.C. Haza, T.M, Ozgokmen, M.G. Magaldi, Z.D. Garraffo. Submitted to Ocean Modelling, 2009.