Lessons learned…

(during 30 years of hybrid-coordinate modeling)

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Original motivation for this work

- Rossby was bothered by feature tracking problems on constant-height surfaces.
- Shuman believed that vertical advection terms ruin numerical stability.
- Everyone likes models that do a decent job resolving fronts.
- On fixed grids,
  - wave-induced vertical advection smears out vertical property contrasts;
  - horizontal transport & eddy mixing can have a false diapycnal component.

Lesson 1

Computers get faster over time.

Aside from that, modeling is a **zero-sum** game.
You gain a few points here, you lose some there.

For example:

“Isentropic models provide vertical resolution where it is needed most.”

Yes, but you pay a price for the resulting uneven grid spacing.

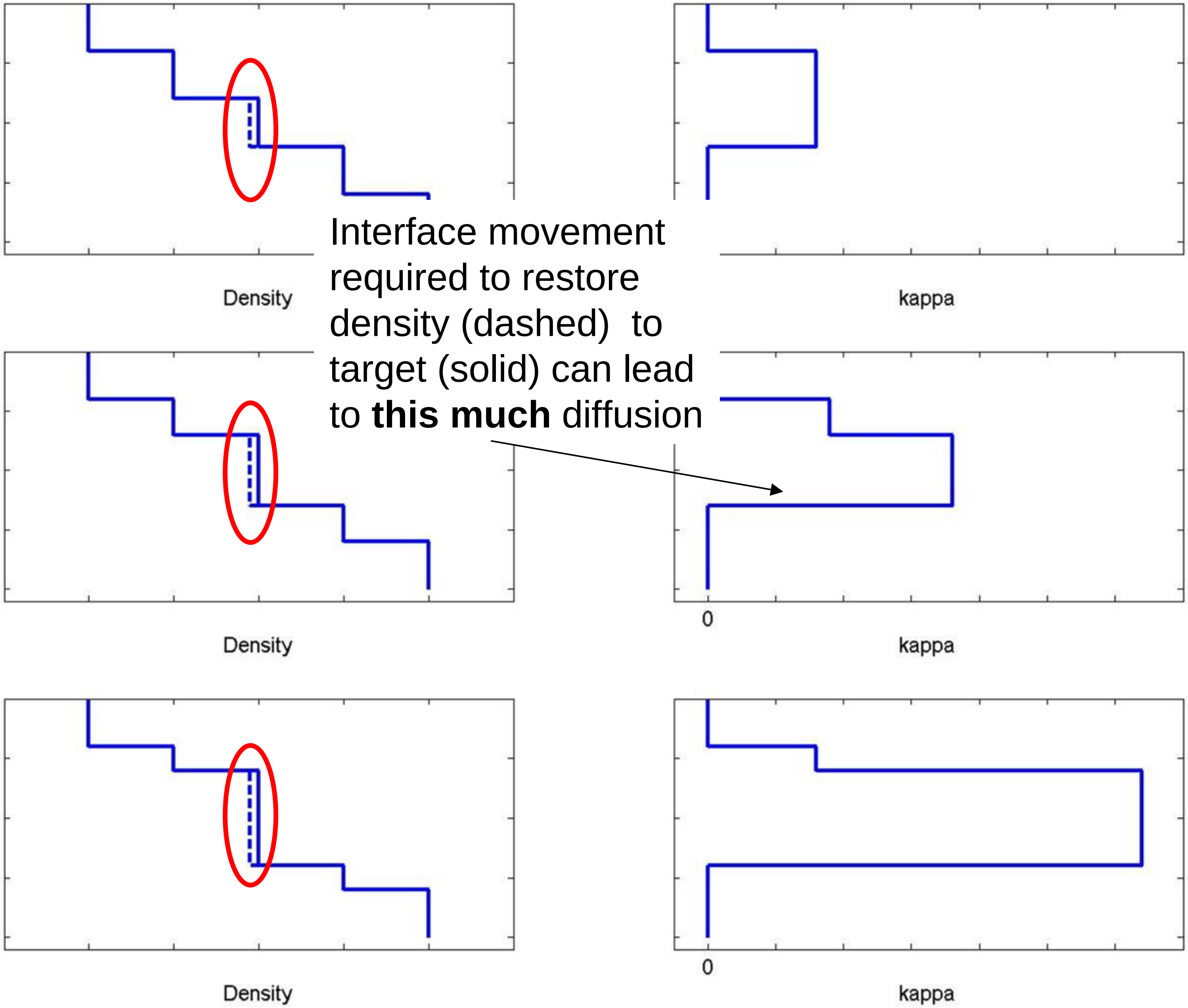
Diapycnal diffusivity implied by stratification trends

Start from
$$\left(\frac{\partial\theta}{\partial t}\right)_z = \frac{\partial}{\partial z}\left(\kappa\frac{\partial\theta}{\partial z}\right)$$

Switch role of **z,θ** as dependent/independent variables:
$$\left(\frac{\partial z}{\partial t}\right)_\theta = -\frac{\partial}{\partial\theta}\left(\kappa\frac{\partial\theta}{\partial z}\right)$$

Discretize:
$$2\,\frac{z^{n+1}_{k-1/2}-z^{n}_{k-1/2}}{\partial t} = -\frac{\left(\kappa_k\dfrac{\theta_{k+1}-\theta_{k-1}}{\overline{z}^{t}_{k+1/2}-\overline{z}^{t}_{k-1/2}}\right)-\left(\kappa_{k-1}\dfrac{\theta_{k}-\theta_{k-2}}{\overline{z}^{t}_{k-1/2}-\overline{z}^{t}_{k-3/2}}\right)}{\theta_k-\theta_{k-1}}$$

Now solve for **κ**


Interface movement required to restore density (dashed) to target (solid) can lead to **this much** diffusion
Density
kappa
Density
0
kappa
Density
0
kappa

Diapycnal diffusivity implied by stratification trends

Start from $\left(\frac{\partial\theta}{\partial t}\right)_z + \frac{\partial F}{\partial z} = 0$ where $F = -\kappa\frac{\partial\theta}{\partial z}$

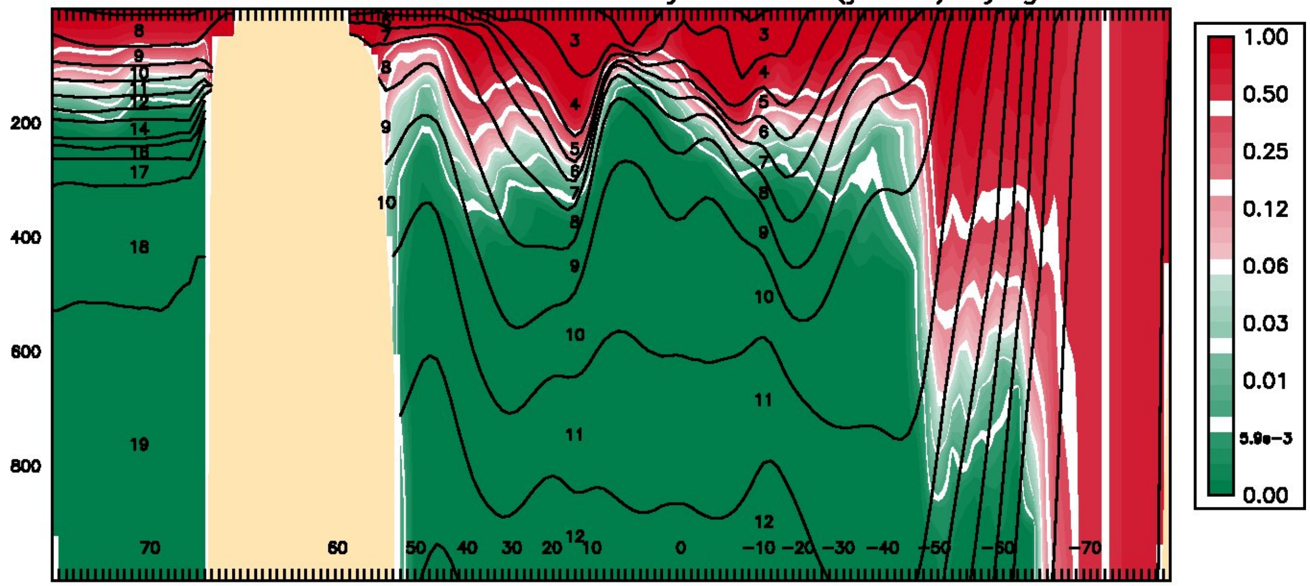
Integrate over z and discretize in time:

$$F(z) = \frac{1}{t_{n+1} - t_n}\left[\left(\int_{btm}^{z}\theta dz'\right)^{n+1} - \left(\int_{btm}^{z}\theta dz'\right)^{n}\right] + F_{btm}$$

Now solve for κ :

$$\kappa(z) = -F(z)\frac{\partial z}{\partial\theta}$$

merid.sec.167.00w tracer 1 yr 2.58 (jul.31) hybgn1



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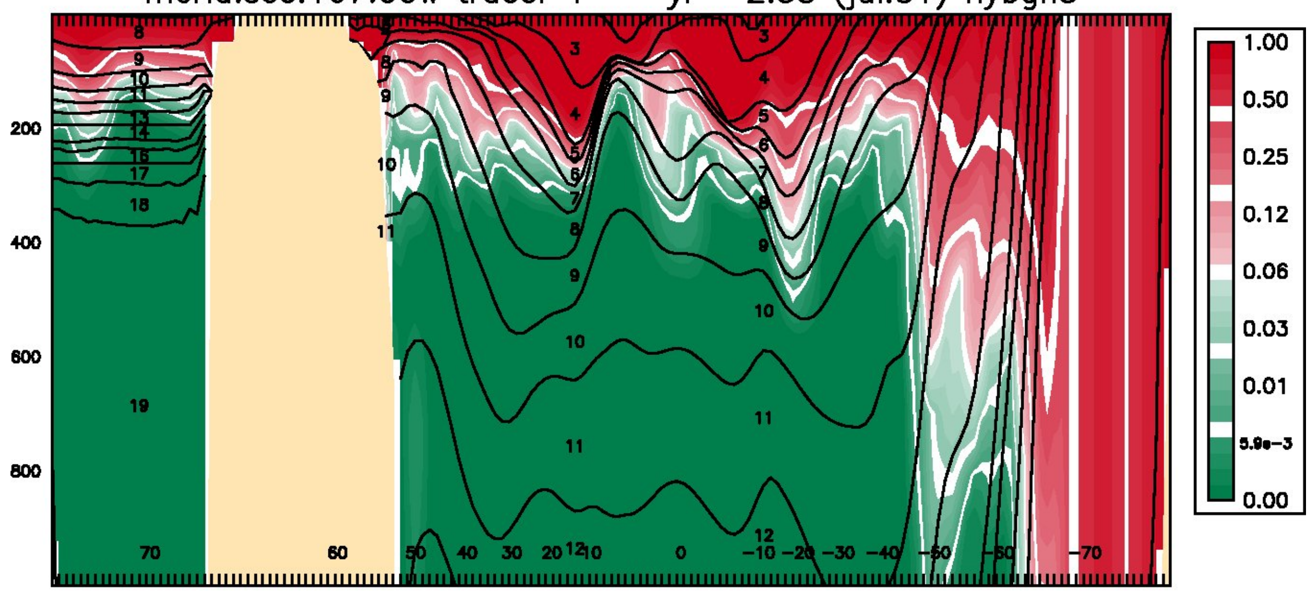
merid.sec.167.00w tracer 1 yr 2.58 (jul.31) hybgn3



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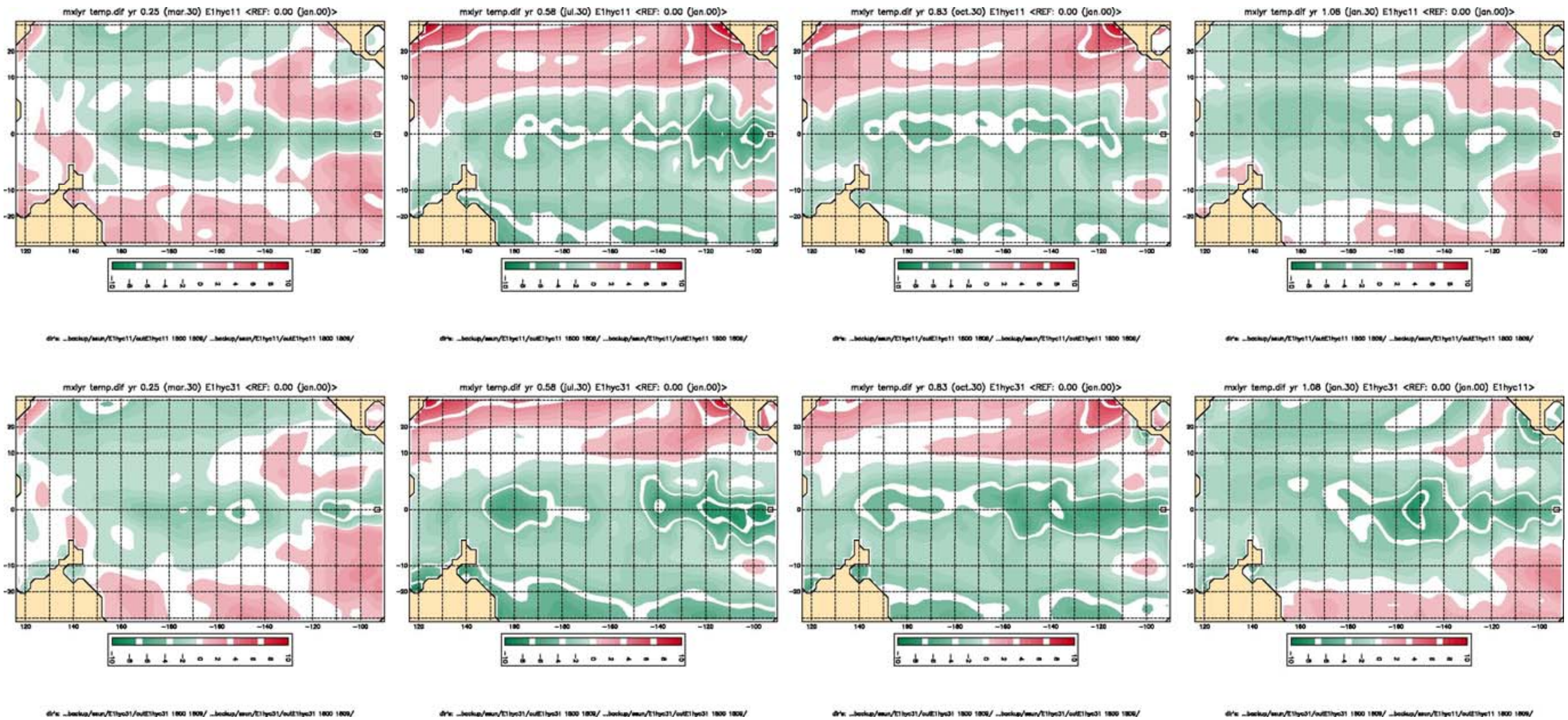
Comparison of 2 grid generators in coupled model

Top row: hybgn1

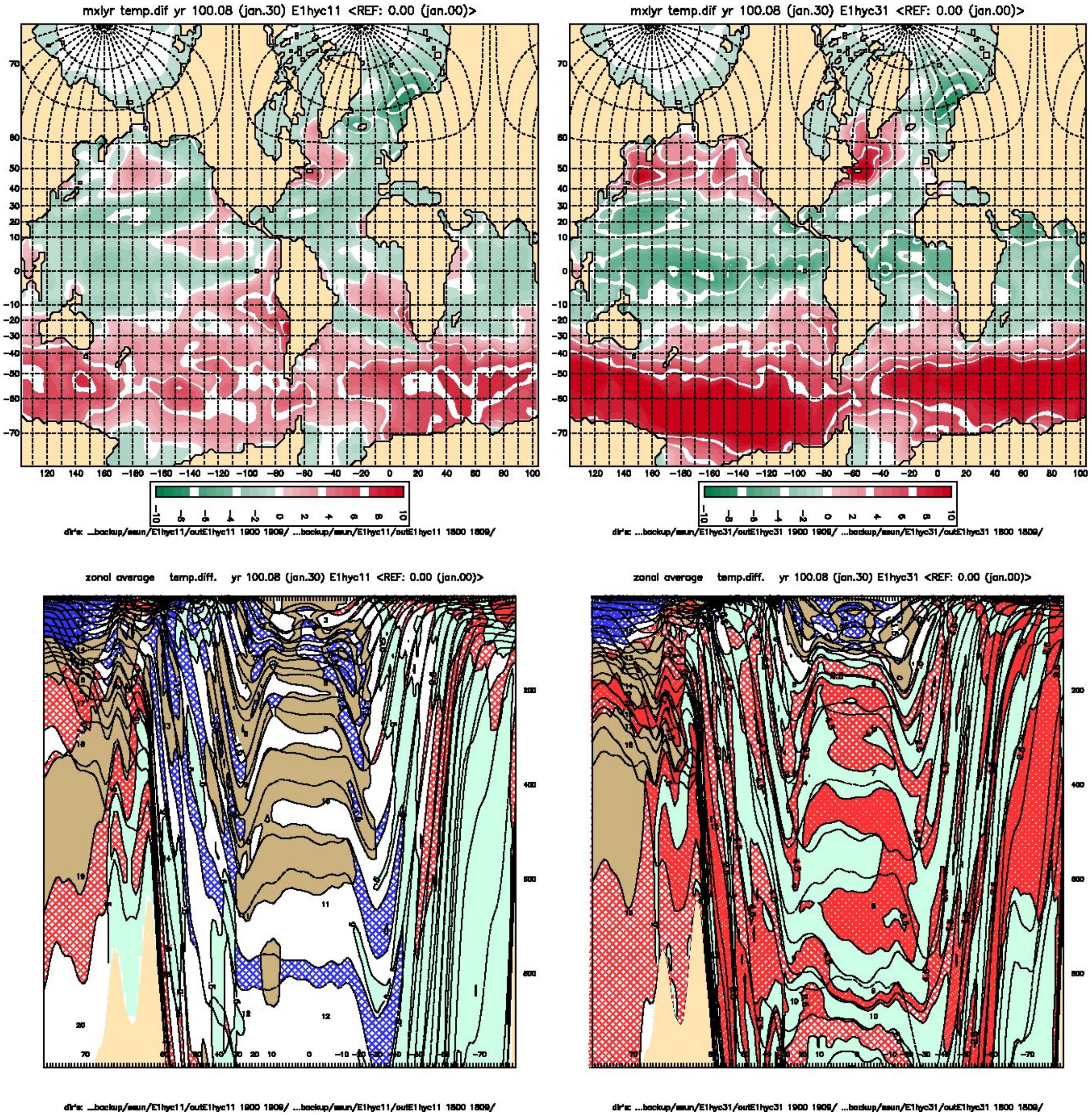
Bottom row: hybgn3

SST mnth 3 – mnth 0 | SST mnth 6 – mnth 0 | SST mnth 9 – mnth 0 | SST mnth 12 – mnth 0

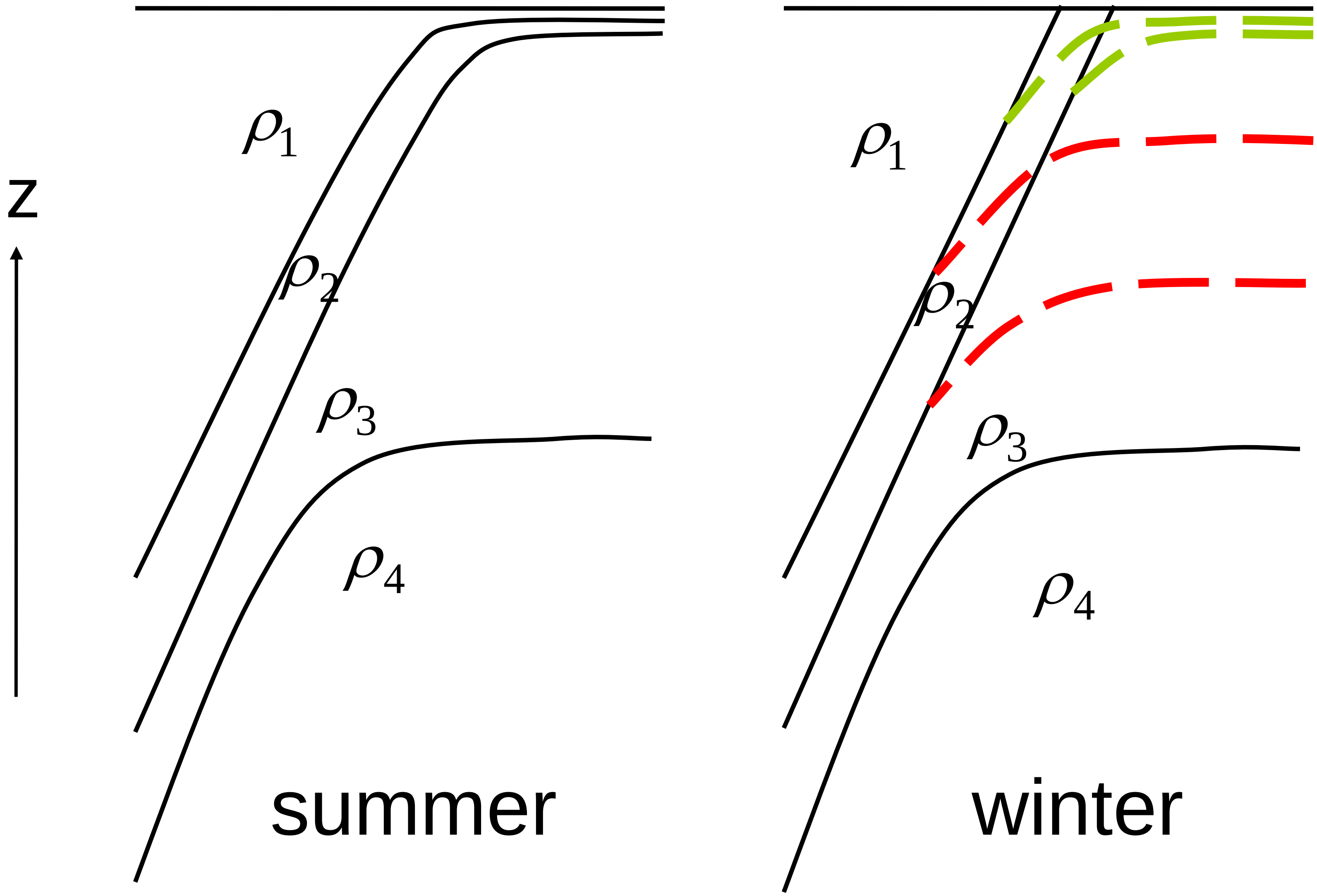


Comparison of 2 grid generators in coupled model: year 100

left: hybgn1
right: hybgn3



green: warming; brown: cooling; red hatching: isopycnals descending; blue hatching: isopycnals ascending



z
ρ_1
ρ_2
ρ_3
ρ_4
summer
ρ_1
ρ_2
ρ_3
ρ_4
winter

Certain things are easier in *z* coordinate models.

u,*v* are <u>always</u> the Cartesian velocity components, regardless of your model's vertical coordinate.

Hence, the pressure gradient term must <u>always</u> mimic the gradient of *p* at constant *z*, $\nabla_z p$

General recipe: $$\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z = \frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_s + g\left(\frac{\partial z}{\partial x}\right)_s$$

The 2-term expression on the right reduces to a single term if one of the following conditions is met:

(a) **s = z** $$\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z = \frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z$$

(b) **s = p** $$\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z = g\left(\frac{\partial z}{\partial x}\right)_p$$

(c) $\left(\frac{\partial p}{\rho}\right)_s = (\partial\Omega)_s$ $$\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z = \frac{\partial}{\partial x}(\Omega + gz)_s$$

(i.e. $(\partial p / \rho)_s$ is an exact differential on s surface)

(c) $\left(\frac{\partial p}{\rho}\right)_s = (\partial\Omega)_s$ $\qquad$ $\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z = \frac{\partial}{\partial x}\left(\Omega + gz\right)_s$

Examples:

(c-1) **s = s(ρ)** $\qquad \Omega = \frac{p}{\rho}$

(c-2) ideal gas with **s = θ** $\qquad \Omega = c_p T$

Counterexamples:

(c-3) **s = z/z_{bot}** (“sigma” coordinate)

(c-3) **s = ρ_{pot}** (isopycnic ocean models)

Finite volume approach:

$$\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z = \left(\frac{\partial p}{\partial s}\right)^{-1} \partial(\phi, p)/\partial(x, s)$$

$$\int \partial(\phi, p)/\partial(x, s)\, dx\, ds = \oint \phi\, dp - \oint p\, d\phi$$

Thermobaricity is numerically problematic because it spawns a second term in the horizontal PGF expression.

It is mainly an issue in abyssal flows.

Despite the efforts of Sun et al. (1999) and Hallberg (2005), the problem has not been solved to everyone’s satisfaction.

Oceanographic Application

Isentropic charts lose in distinctness somewhat when applied to the ocean. In the first place, two samples of water of the same specific volume at one pressure do not in general have the same specific volume at another pressure….

In the second place, when two samples of the same specific volume but different temperature and salinity mix, the resulting mixture has a lesser specific volume than the original samples.

R. B. Montgomery: A suggested Method for Representing Gradient Flow in Isentropic Surfaces. Bull. Amer. Meteor. Soc., 1937.

Thermobaricity: temperature dependence of the (adiabatic) compressibility coefficient $\rho^{-1}\partial\rho/\partial p$

$$\boxed{\frac{\partial^2\rho}{\partial p\partial T}\neq 0}$$

Example of a non-thermobaric fluid: **ideal gas**

(1st Law =>) $$c_v\frac{dp}{p}-c_p\frac{d\rho}{\rho}=0 \quad\Rightarrow\quad \frac{1}{\rho}\frac{\partial\rho}{\partial p}=\frac{c_v}{c_p}\frac{1}{p}$$

i.e., atmospheric compressibility depends <u>only</u> on *p*

Summary of recurring issues

- Vertical diffusion near z/ρ coord. interface
- Thermobaricity
- Subgridscale eddy mixing (Gent-McWilliams)
- z-coordinate-centric mixing schemes
- Lack of ideas on grid generator development

Lesson 4

Model development will continue indefinitely

Man May Work from Sun to Sun But Woman’s Work is Never Done

The T/S/ρ conundrum in isopycnic-coordinate models

T, S, ρ_{pot} are materially conserved in adiabatic flow.

The three variables are related: $\rho_{pot} = \rho_{pot}(T,S)$ and T,S are of similar importance.

Due to numerical errors and nonlinearities in the equation of state, T,S advection is unlikely to conserve ρ_{pot}.

This is extremely inconvenient in models using ρ_{pot} as independent variable.