Adiabatic density surface
the only
truly neutrally buoyant surface

Rui Xin Huang

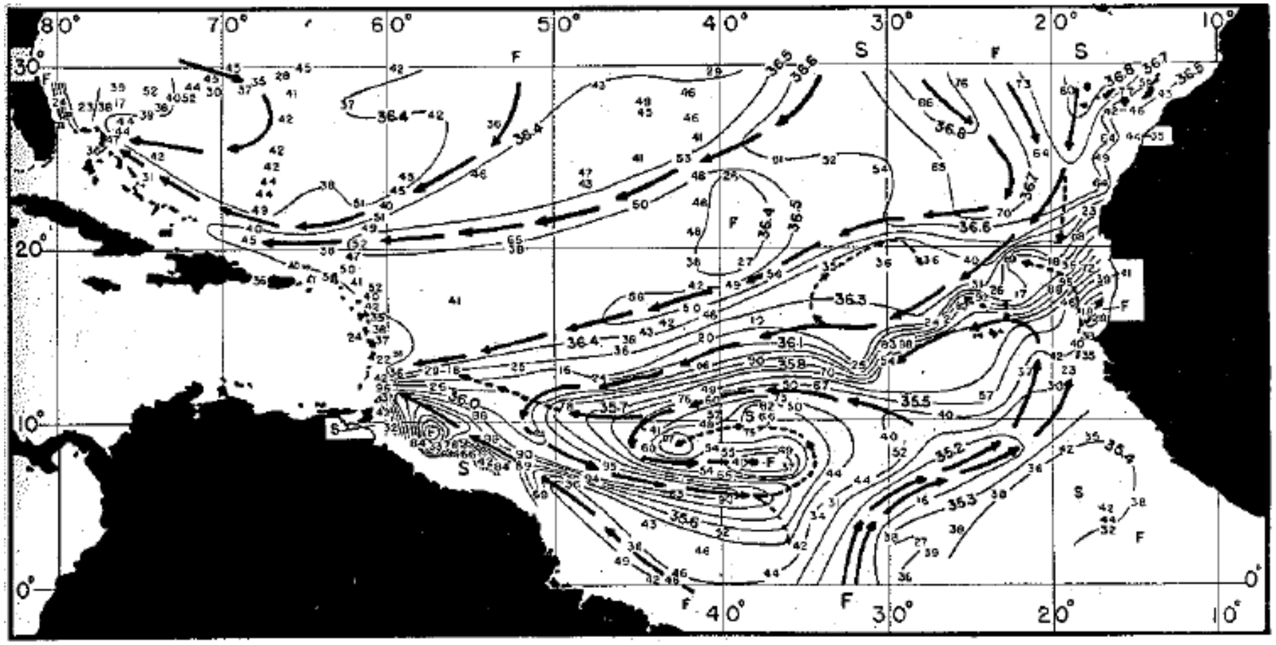
Woods Hole Oceanographic Institution, Woods Hole, USA

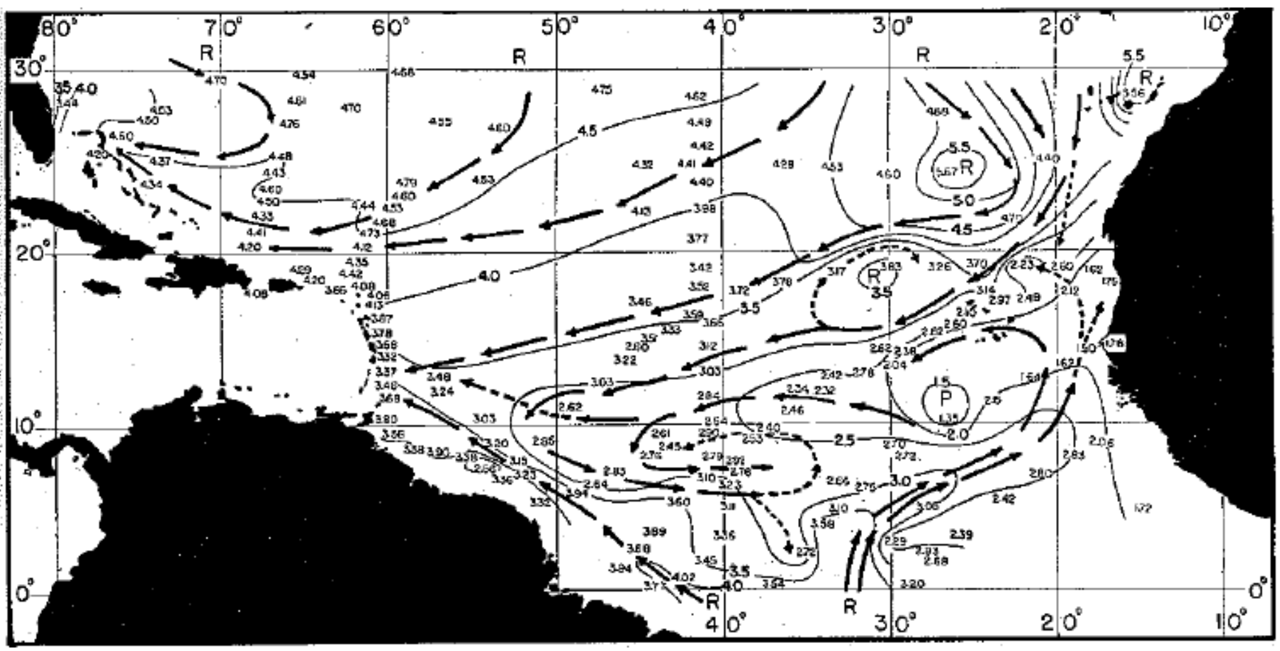
What is isopycnal/isentropic analysis

- Isentropic analysis (Montgomery, 1938)

  specific entropy of sea water is a function of potential temperature only (to a very good approximation!)
- Isopycnal analysis
- Potential density analysis
- Neutral density analysis

- Is neutral surface really neutrally buoyant?





Isentropic analysis by Montgomery (1938) salinity Oxygen on $\sigma_t = 26.5$

A close look at isentropic analysis

by Montgomery (1938)

- Specific entropy of sea water is a function of potential temperature only (to a very good approximation! New subroutines can be used to calculate specific entropy conveniently).
- Motions in the ocean do not conserve entropy →using constant-entropy surface is no good
- In-situ density is not conserved along trajectory
- →Using potential density instead
- →Neutral density --- is it really neutral ??

Isopycnal/neutral movement a free lunch?

- “Neutral surfaces are defined so that small isentropic and adiabatic displacements of a fluid parcel in a neutral surface do not produce a buoyancy restoring force on the parcel.” (McDougall, 1987);
- “A neutral trajectory is a three-dimensional path in the ocean, and is defined such that no buoyancy forces act on a water parcel when it is moved a small distance along this path.” (Eden and Willebrand, 1999)

(McDougall and Jackett 1988). Continuous density surfaces can be associated with a density variable, such as potential density or neutral density, which is constant on the surface. These surfaces are mathematically well defined (i.e., continuous), but water parcels moving along these continuous density surfaces encounter buoyant restoring forces.

KLOCKER & McDougall (2010) may change this statement slightly

Using the reference frame of continuous density surfaces to describe water-mass transformation, an additional diapycnal advection process has to be taken into account. This additional diapycnal advection process occurs even in the absence of the dissipation of mechanical energy and is due to the ill-defined nature of neutral trajectories, causing flow through any continuous density surface.

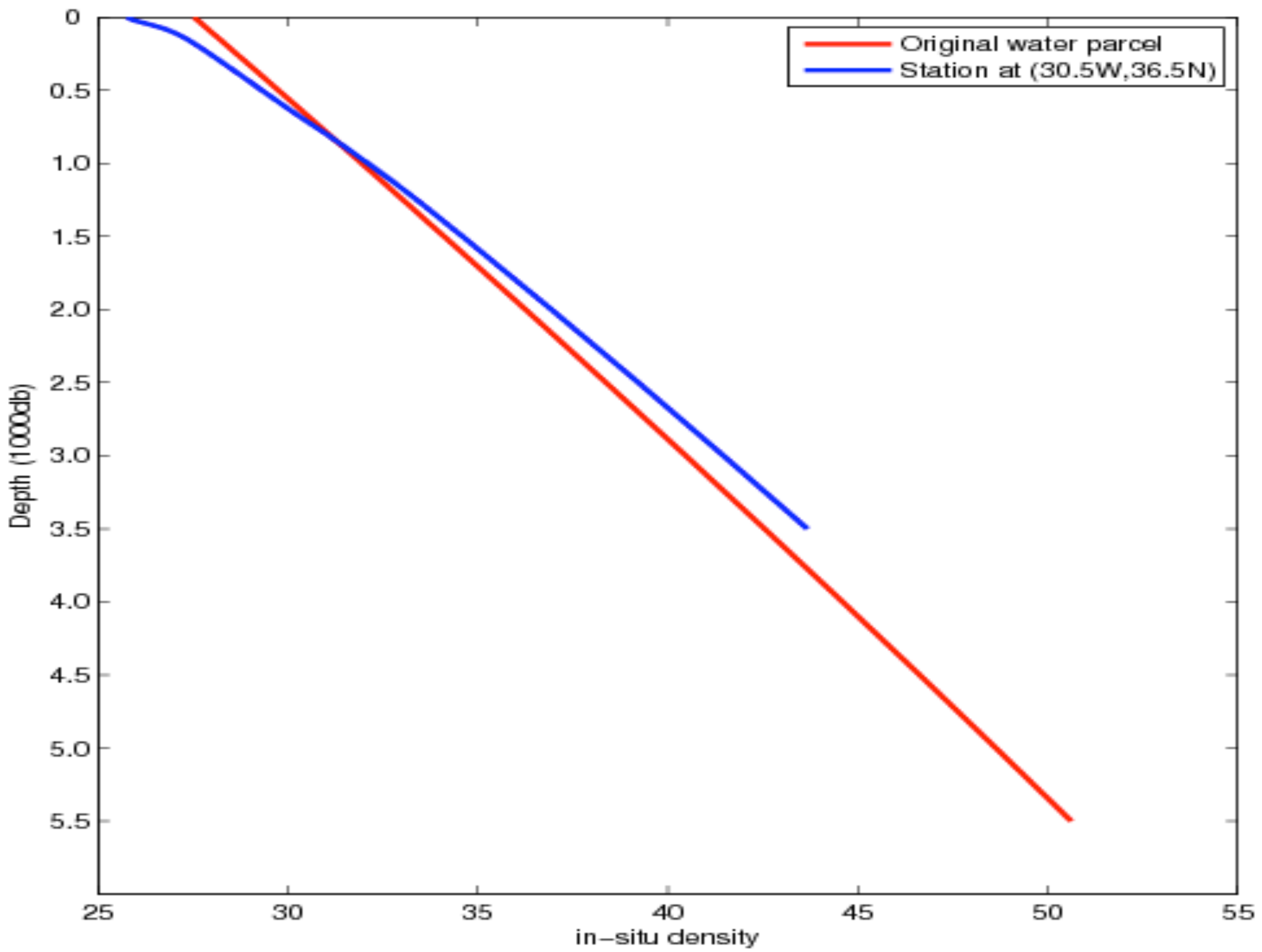
How to define an adiabatic density surface?

- Pick up a water parcel at given latitude, longitude and depth
- Plot the functional relation $\rho_{adia} = \rho_{adia}(p)$ for the in-situ density of this water parcel during adiabatic motions
- Pick up any station in the world ocean, and plot the functional relation of in-situ density and pressure at this station $\rho_{station} = \rho_{station}(p)$
- Find the intersection of these two curve

$$\rho_{adia}(p_{adia}) = \rho_{station}(p_{adia})$$

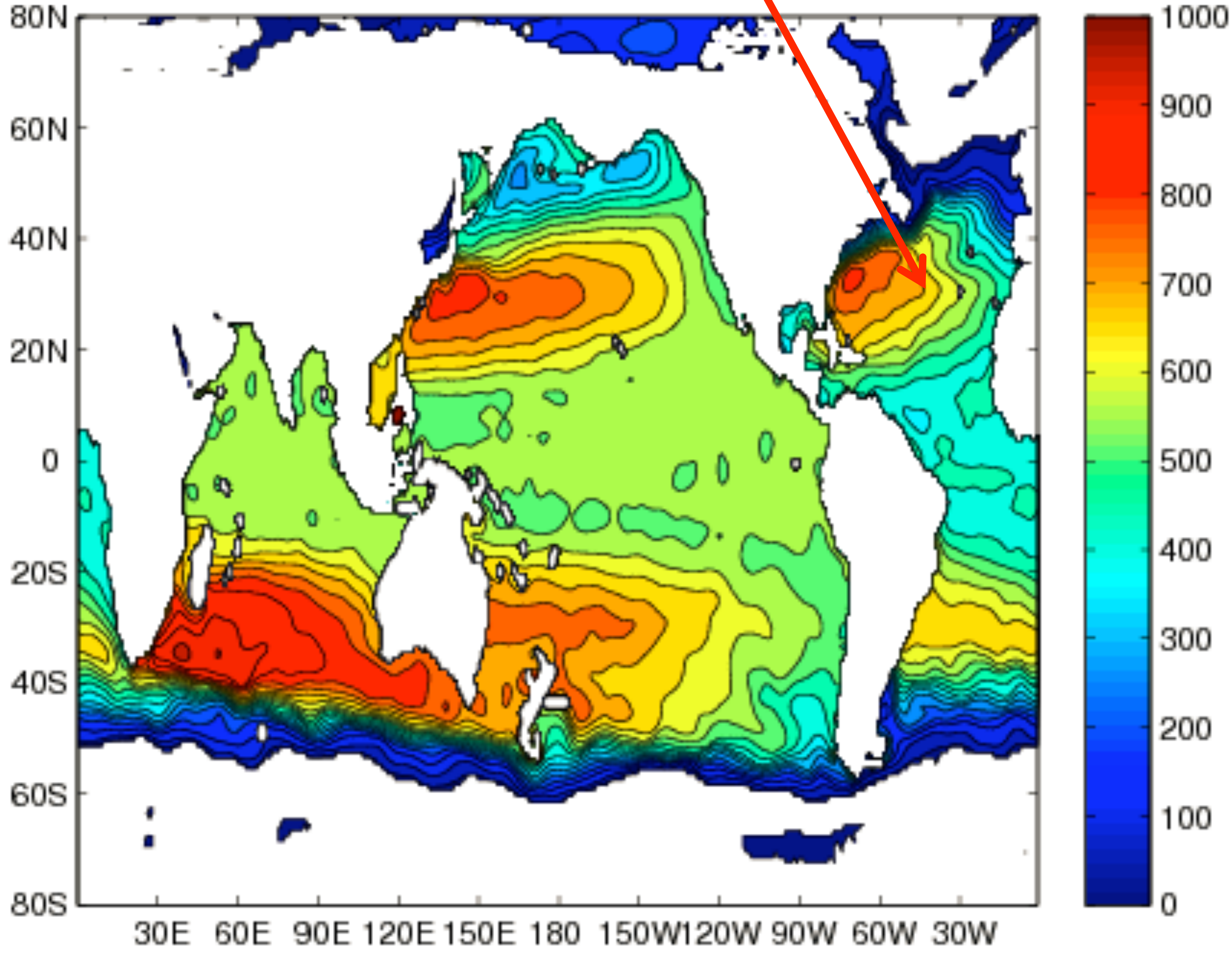
- Plot a maps, using these depth

Adiabatic density surface and how to define it ?

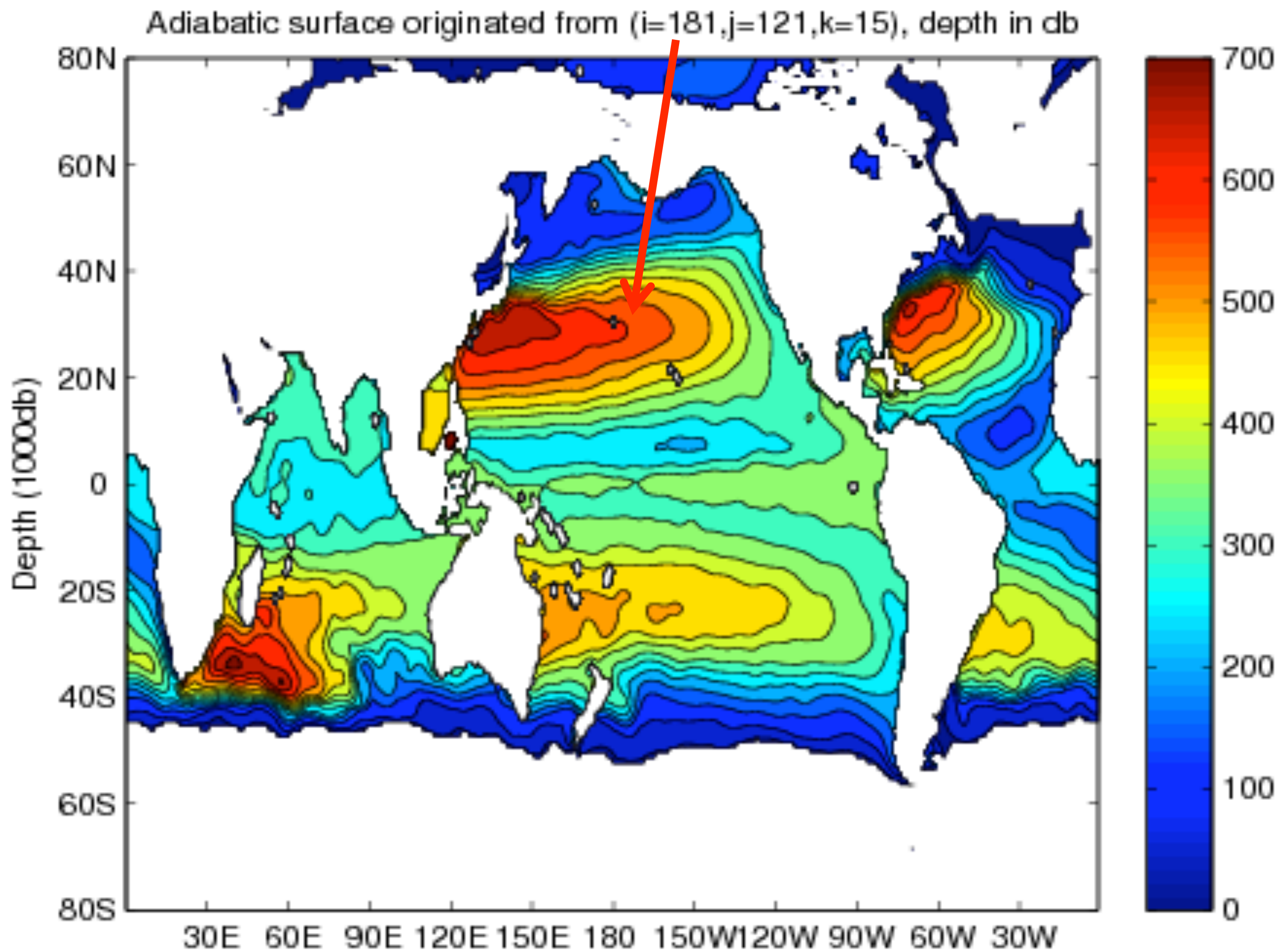


An adiabatic density surface

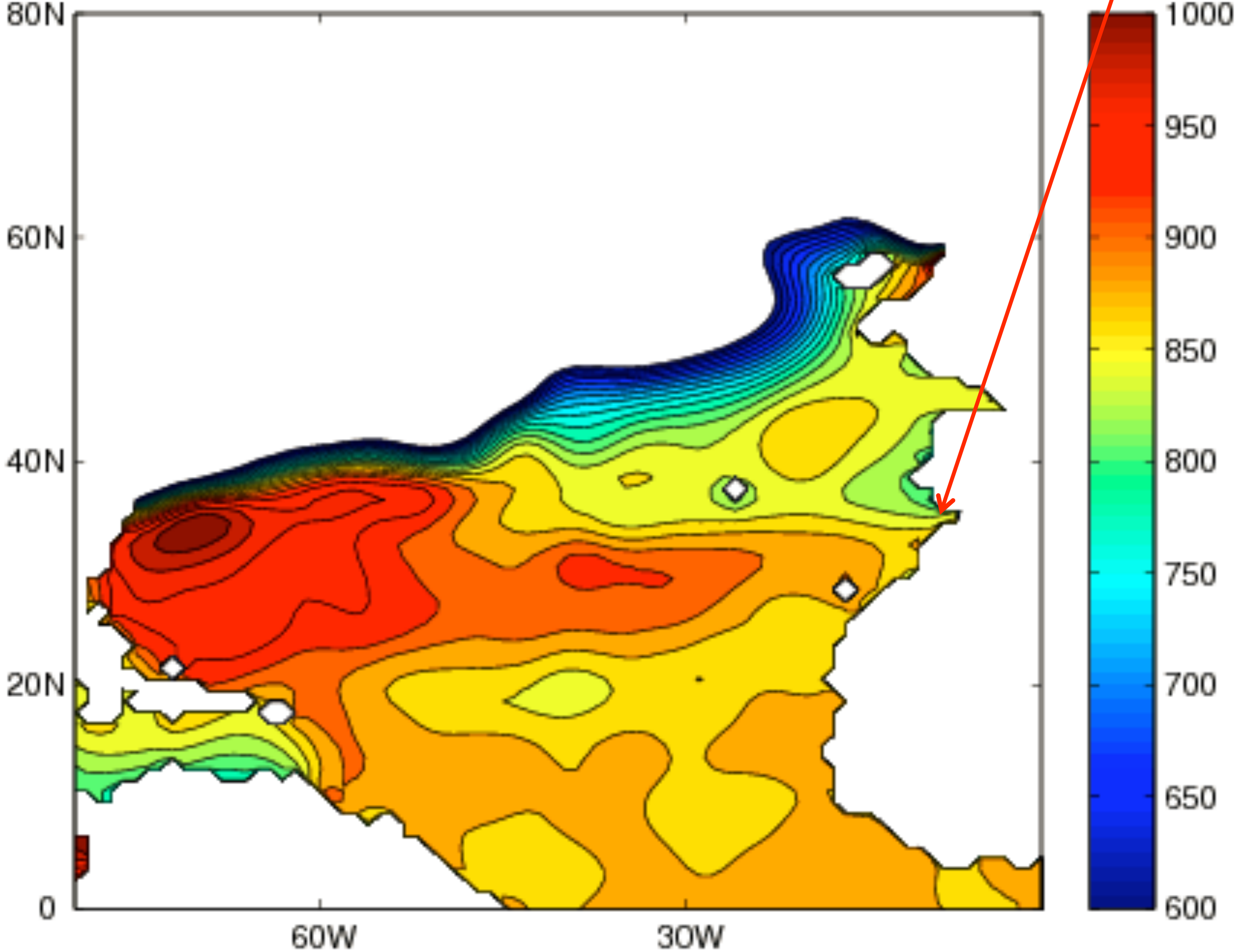
taken from the North Atlantic at (30.5W, 30.0N) and at a depth of 600db.



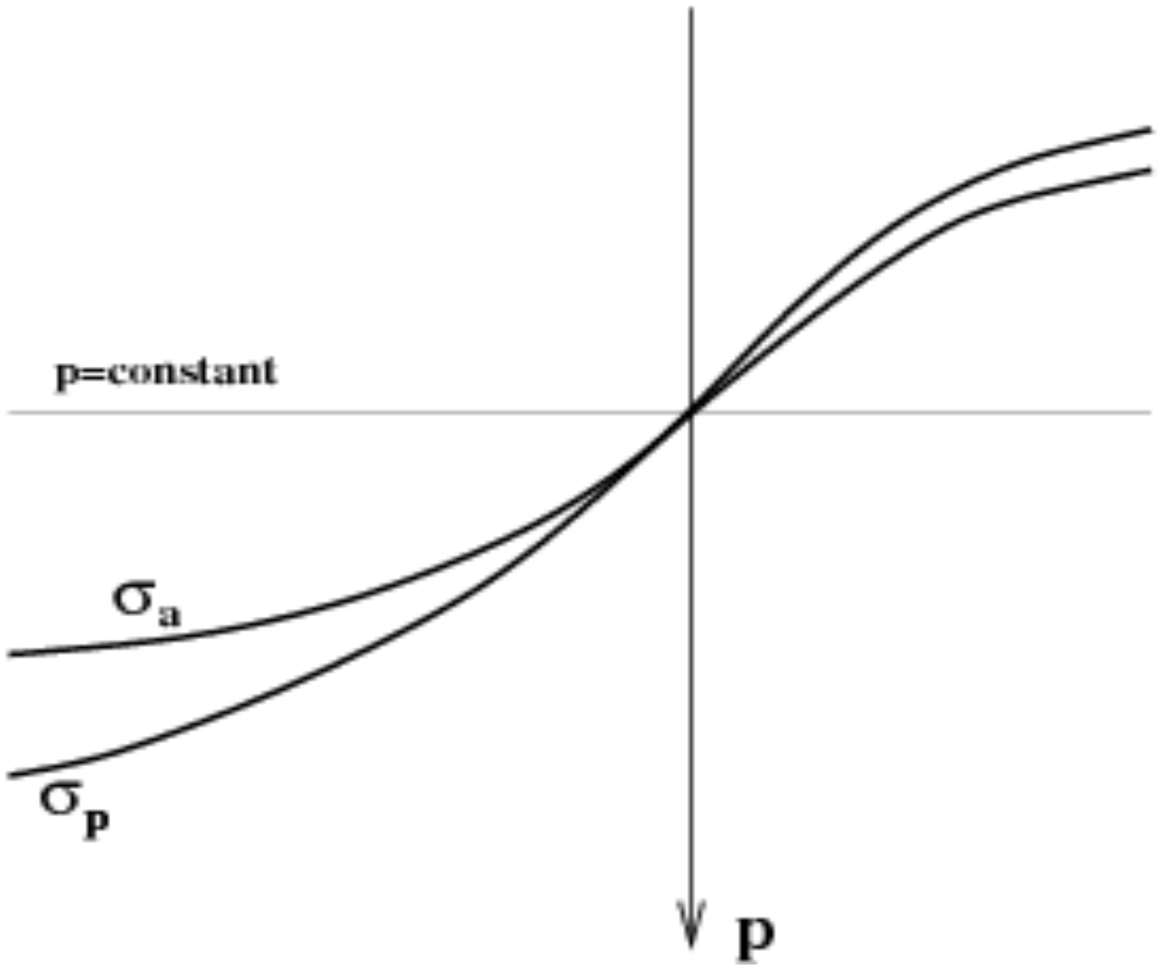
An adiabatic density surface for Pacific



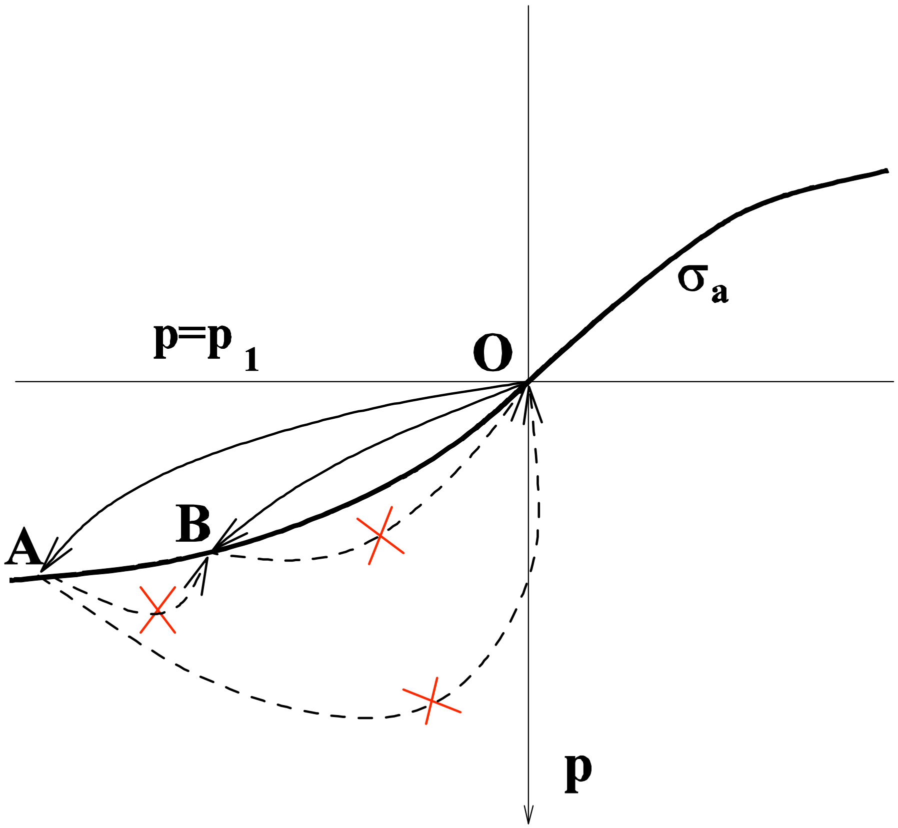
An adiabatic density surface for Mediterranean outflow originated from (10.5W, 38.5N, 800db).



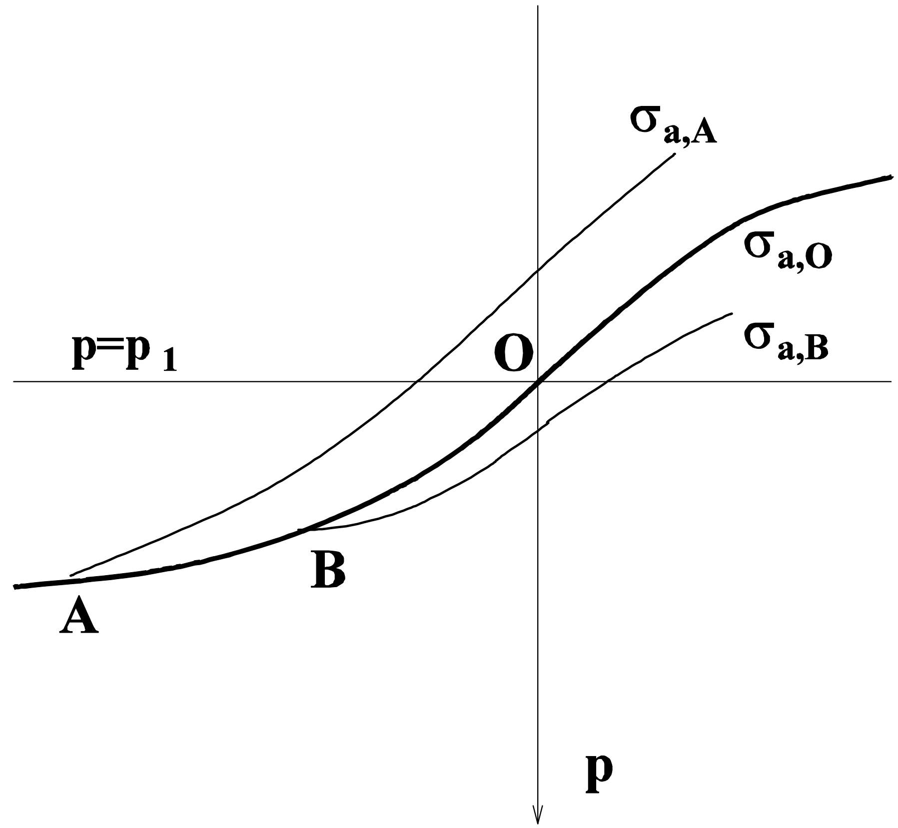
Adiabatic density surface and potential density surface



Adiabatic density surface:
permeable & non-permutable moves ?

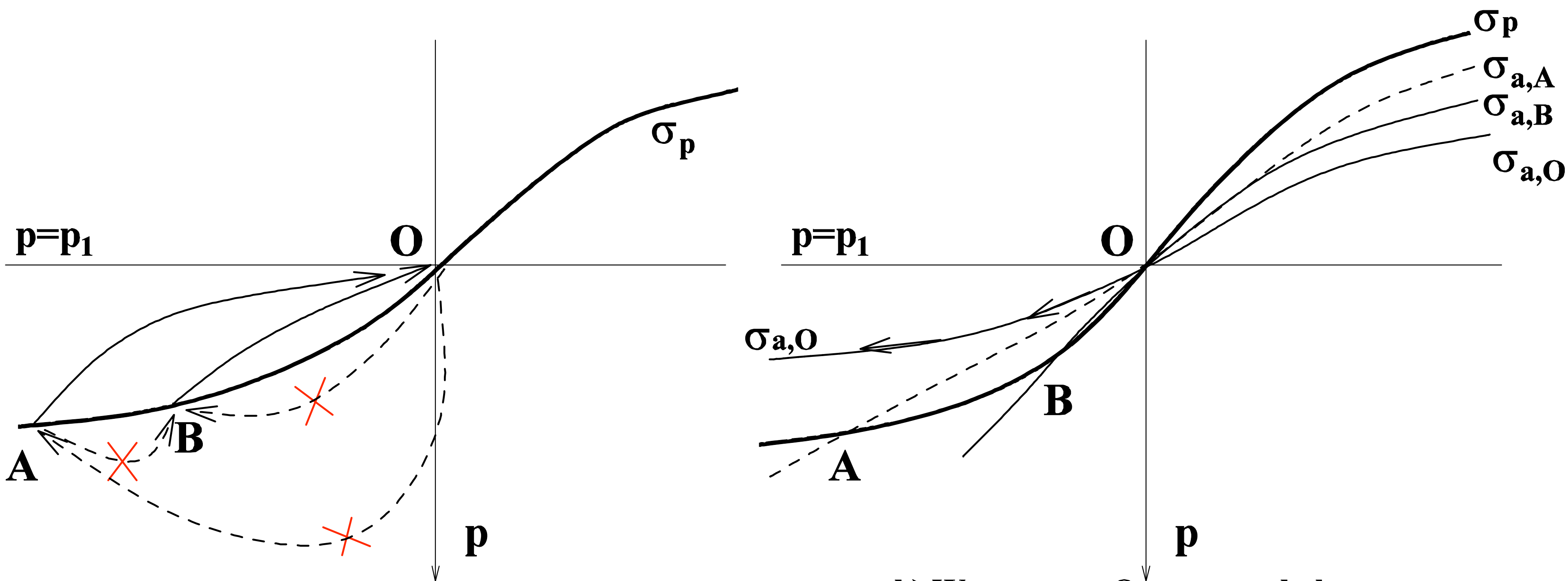


a) Water mass O can travel to any point on its adiabatic density surface



b) Water masses A & B on this adiabatic density surface cannot travel to point O

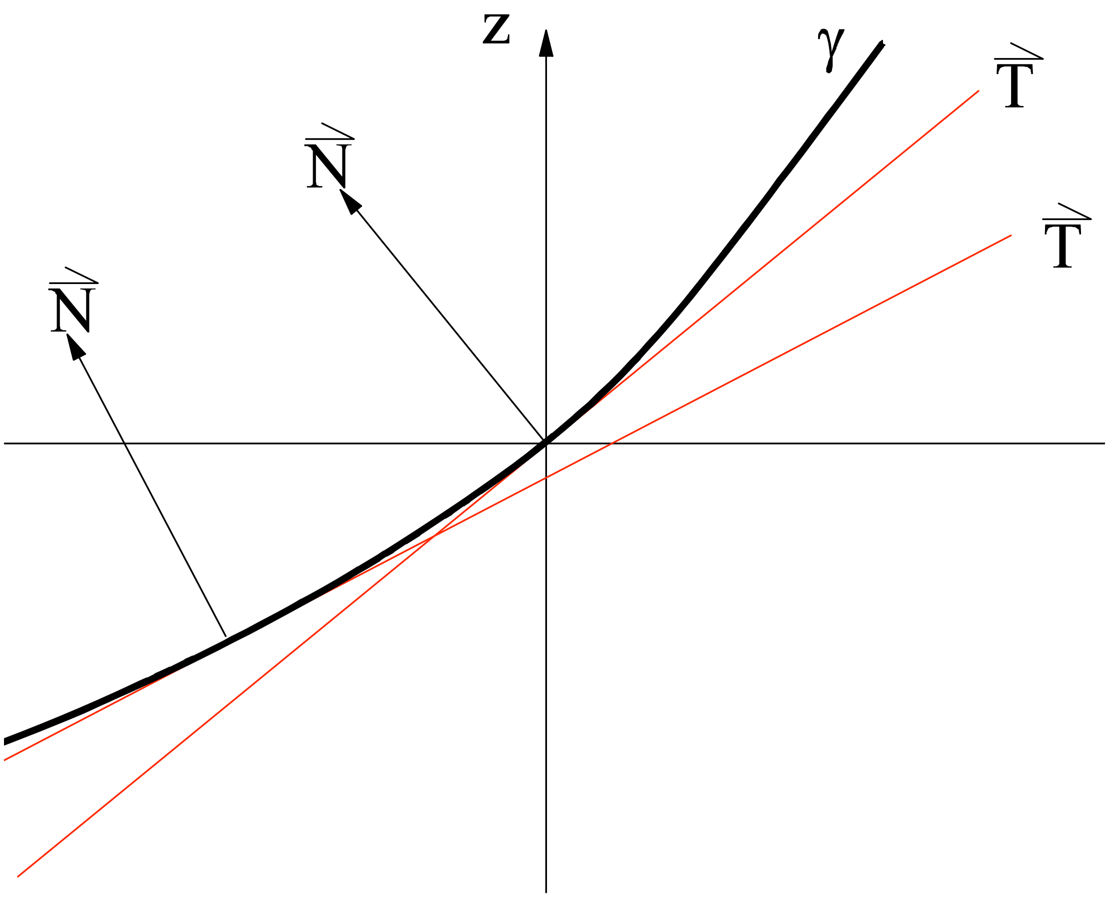
Potential density surface:
permeable and non-permeable moves ?



a) Water masses A & B on a potential potential surface can travel to point O

b) Water mass O can travel along its adiabatic density surface only

Neutral surface is not locally defined

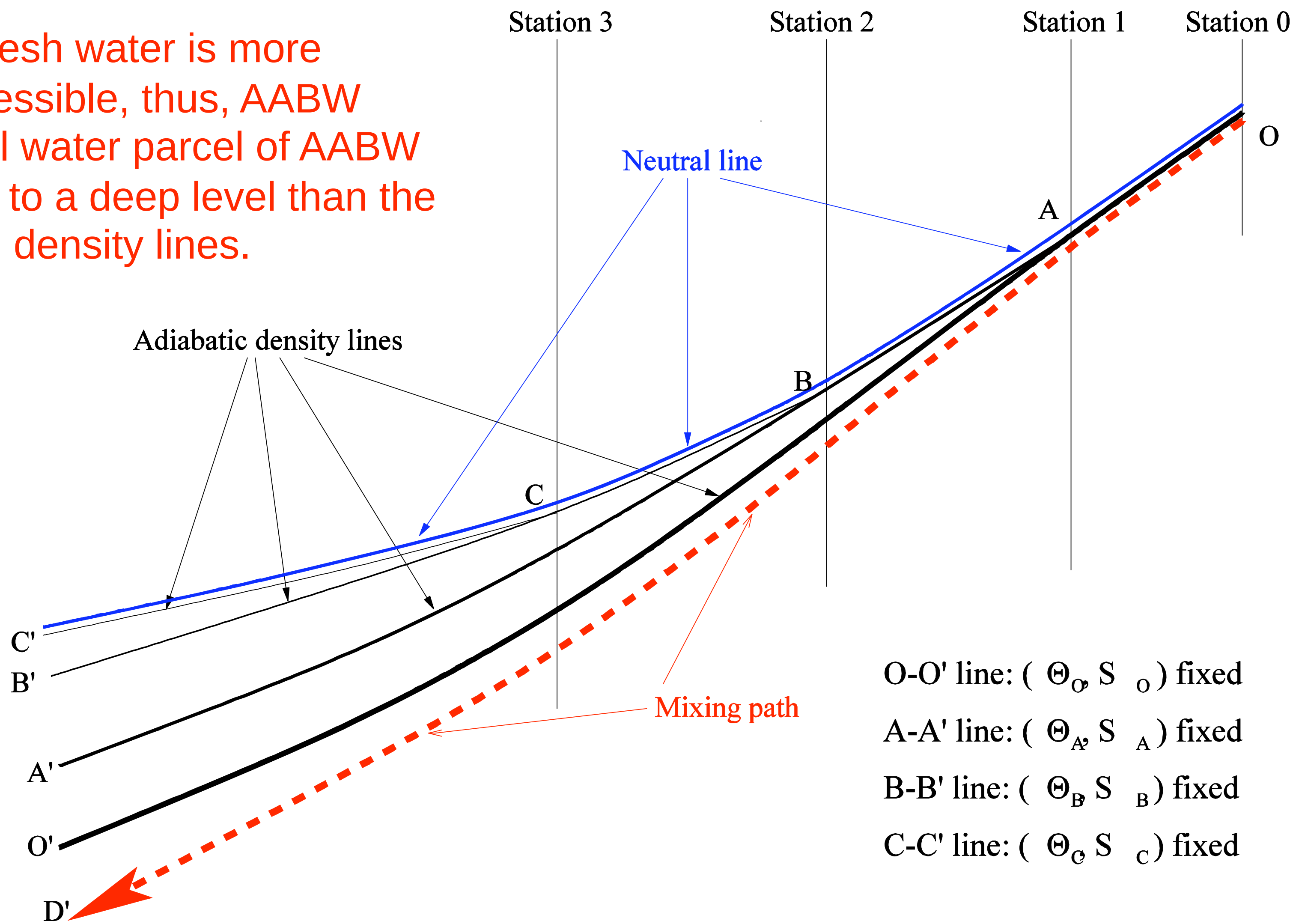


Adiabatic and potential density surfaces are locally defined

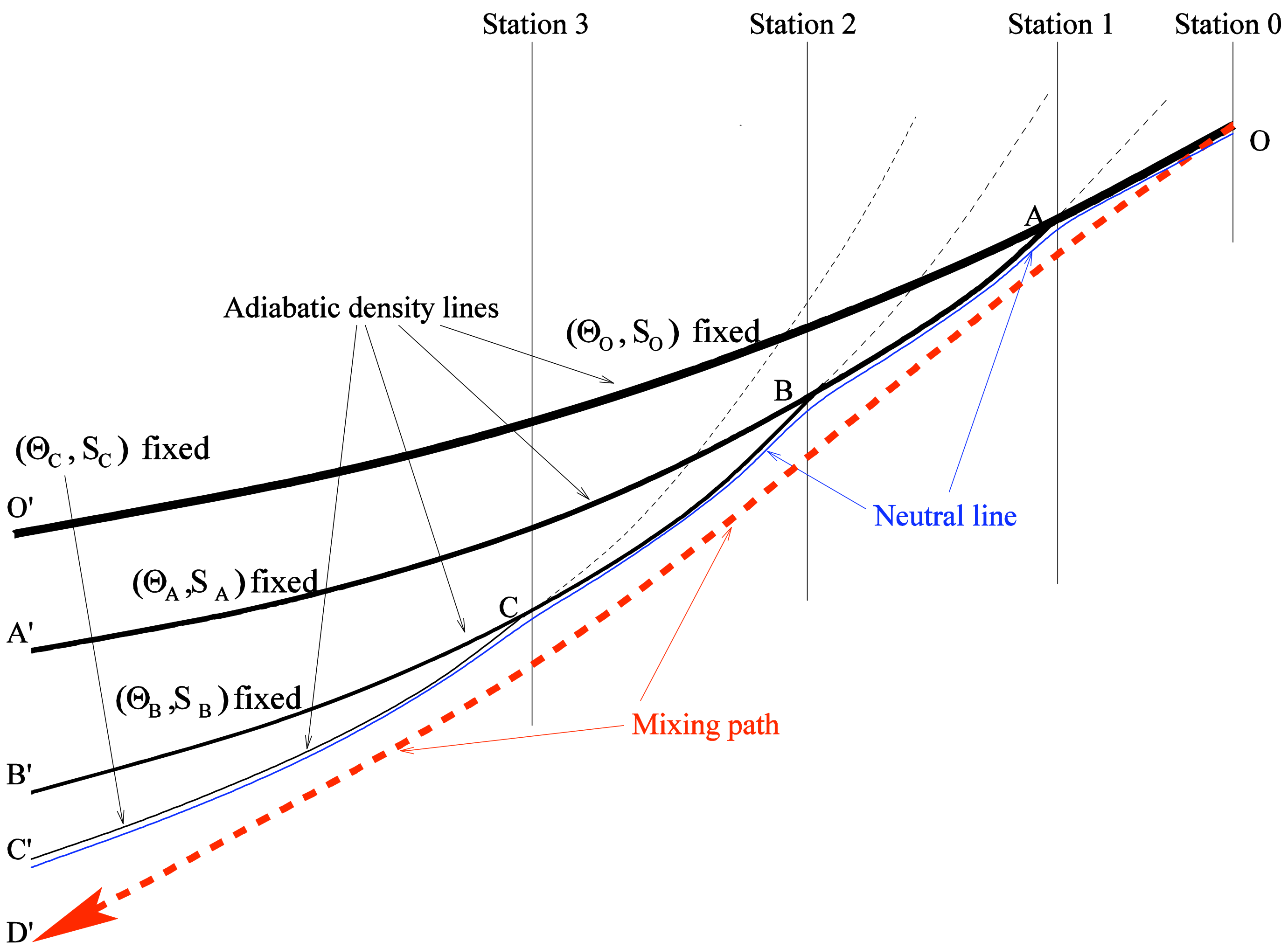
Adiabatic density line may be more close to the mixing path

Revisiting Forster and Carmack (1976): Frontal zone mixing and Antarctic bottom water …

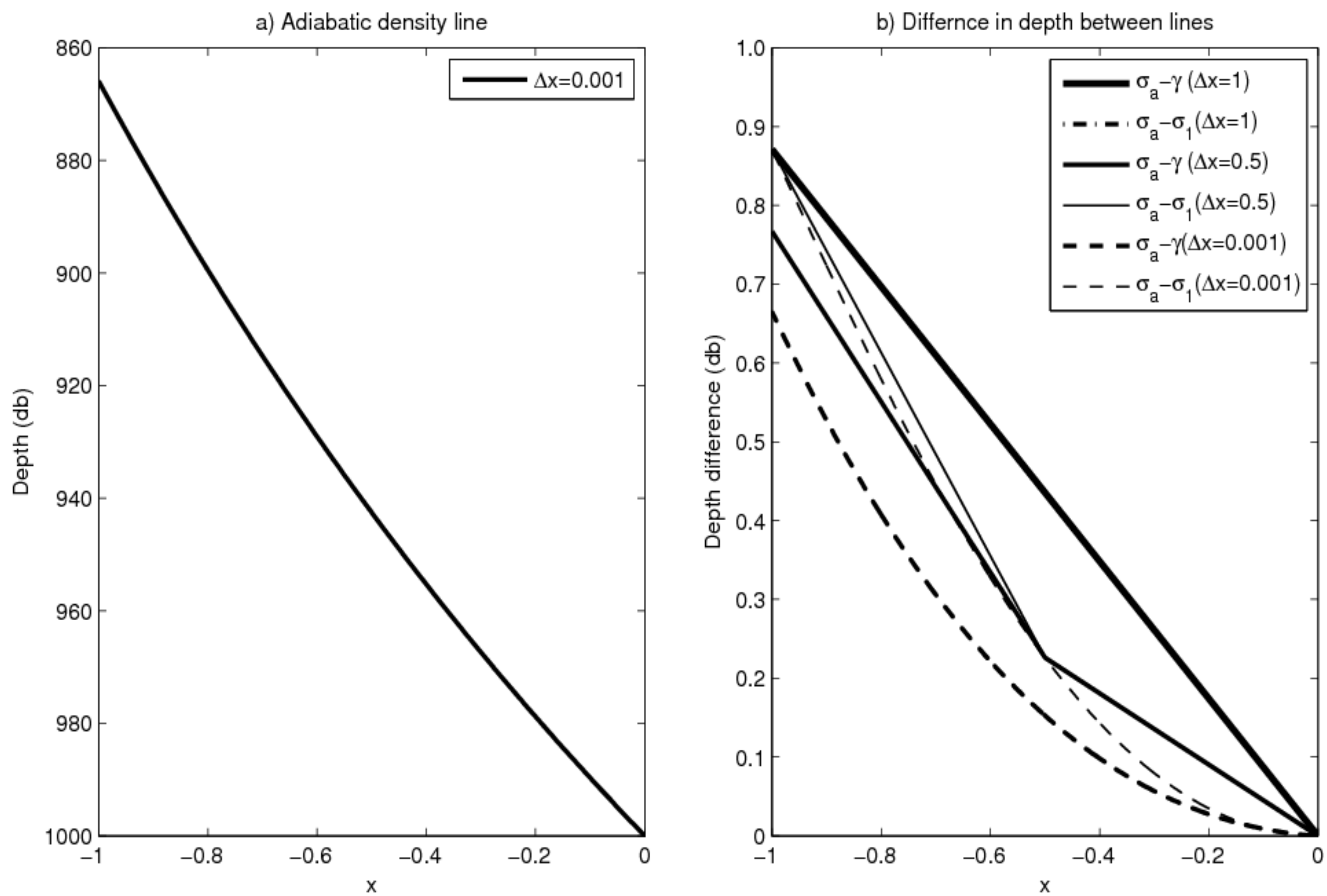
Cold/fresh water is more compressible, thus, AABW original water parcel of AABW moves to a deep level than the neutral density lines.



Warm/salty water is less compressible, thus original water from MOW may spread to a shallower level than the mixing path



Adiabatic density surface,
neutral density and potential density



Adiabatic density surface,
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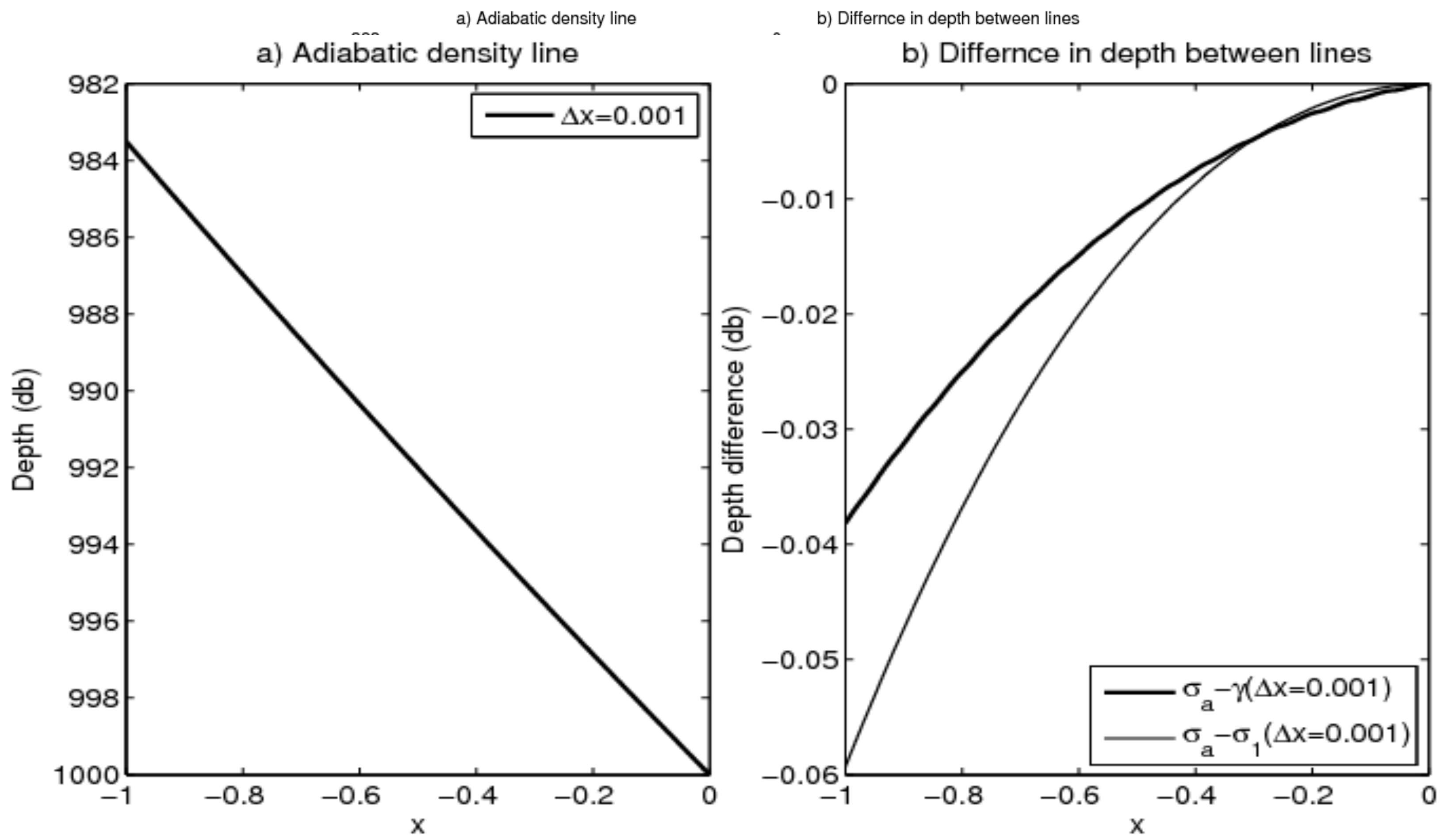
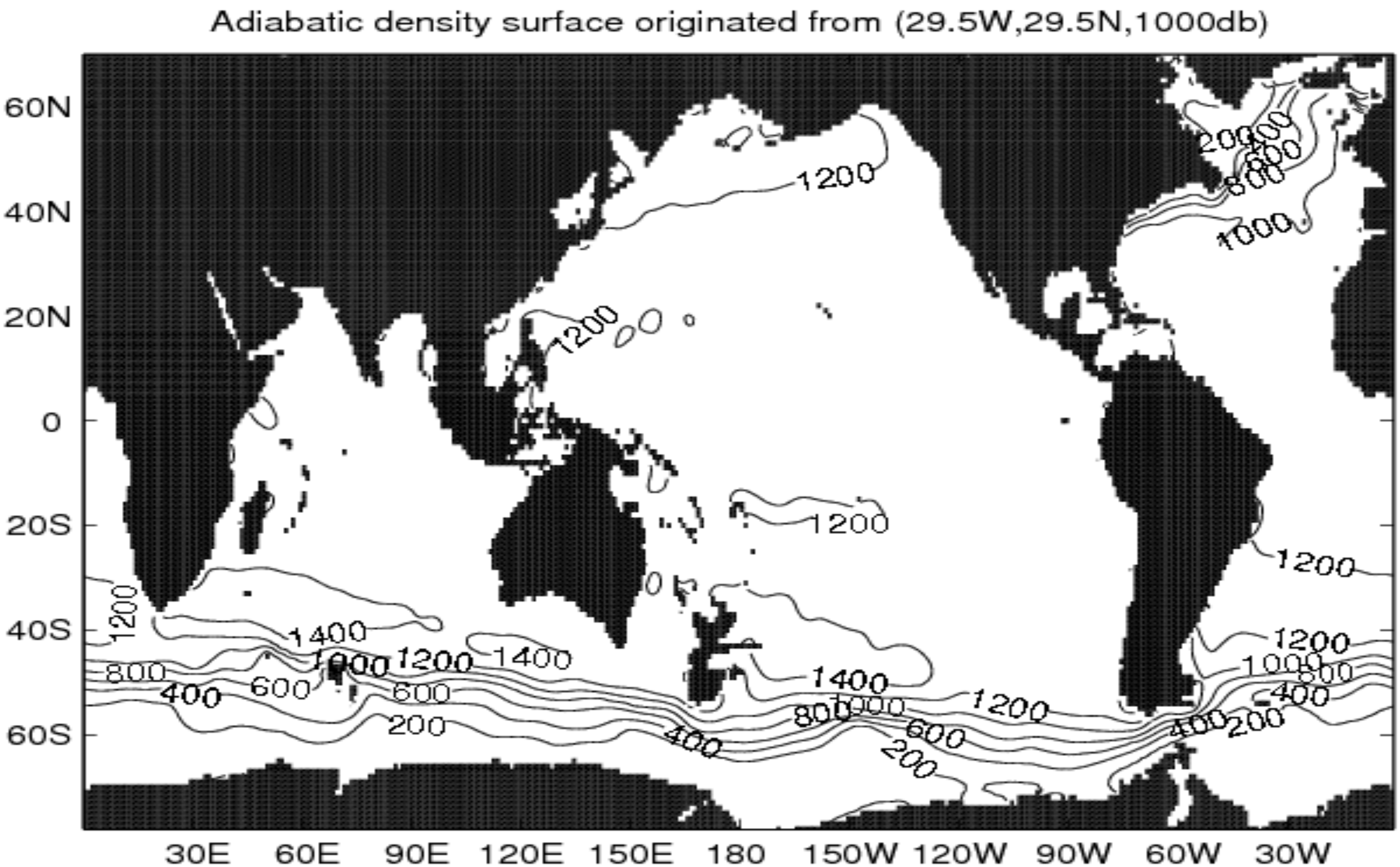
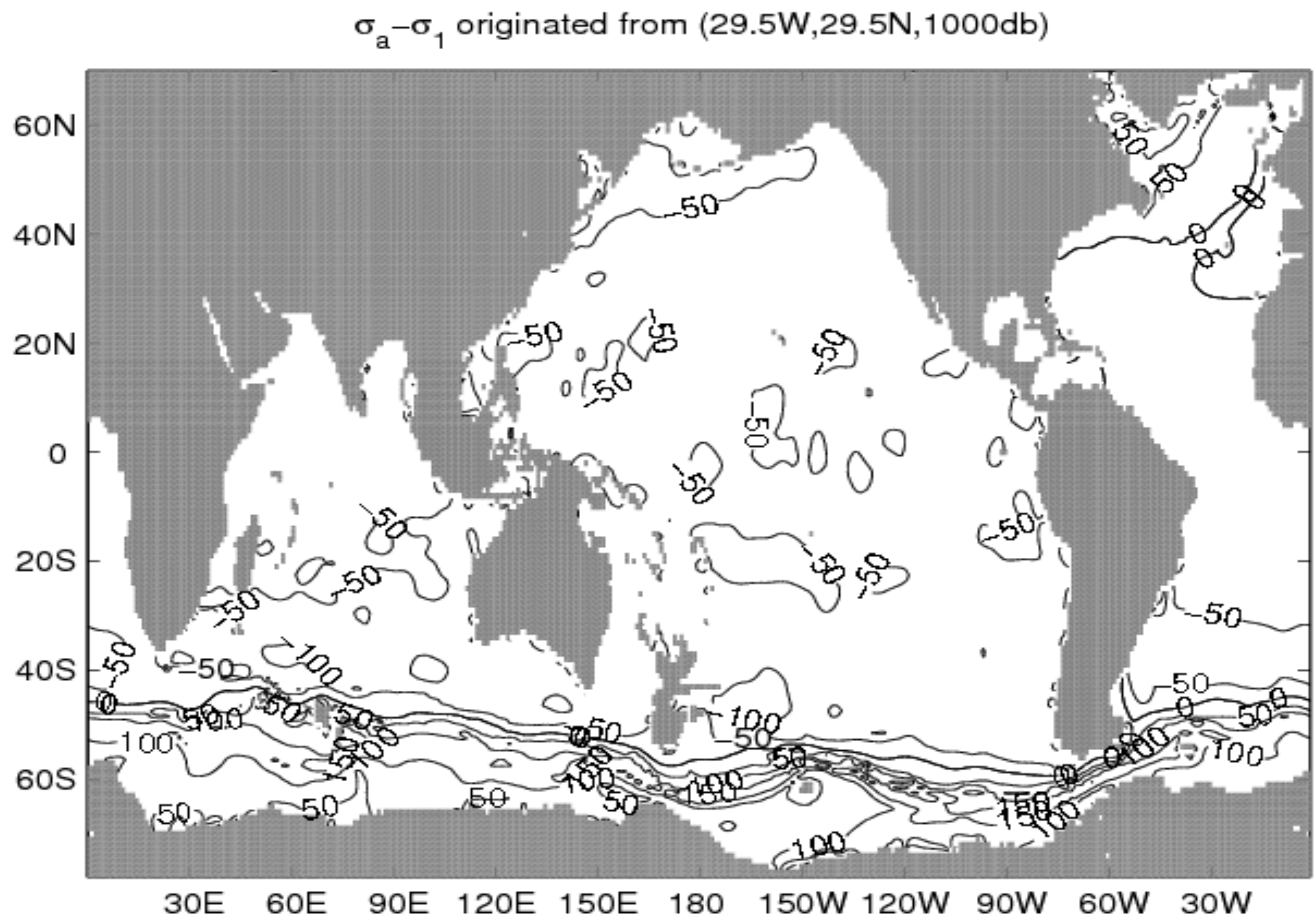


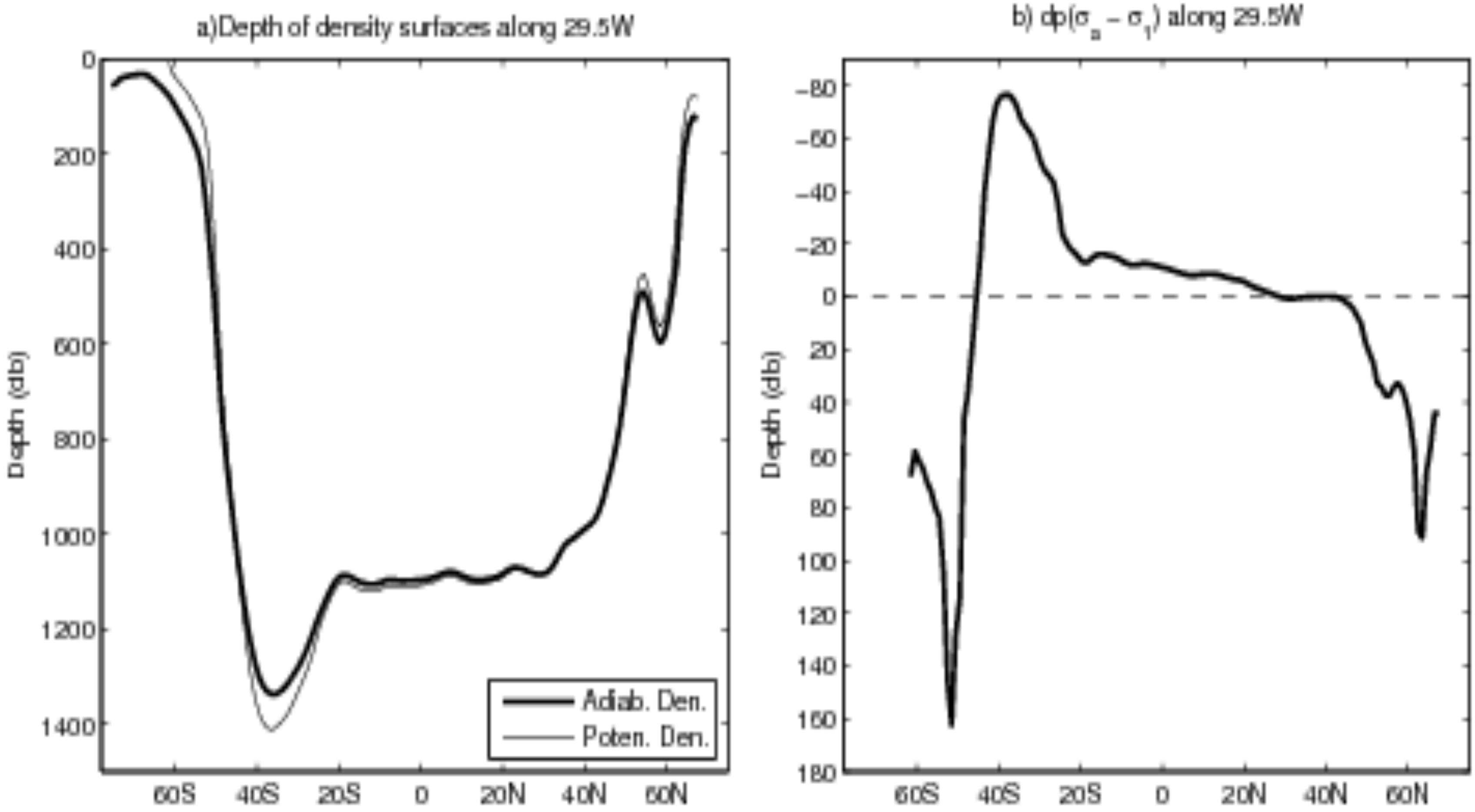
Adiabatic density surface (1000db)



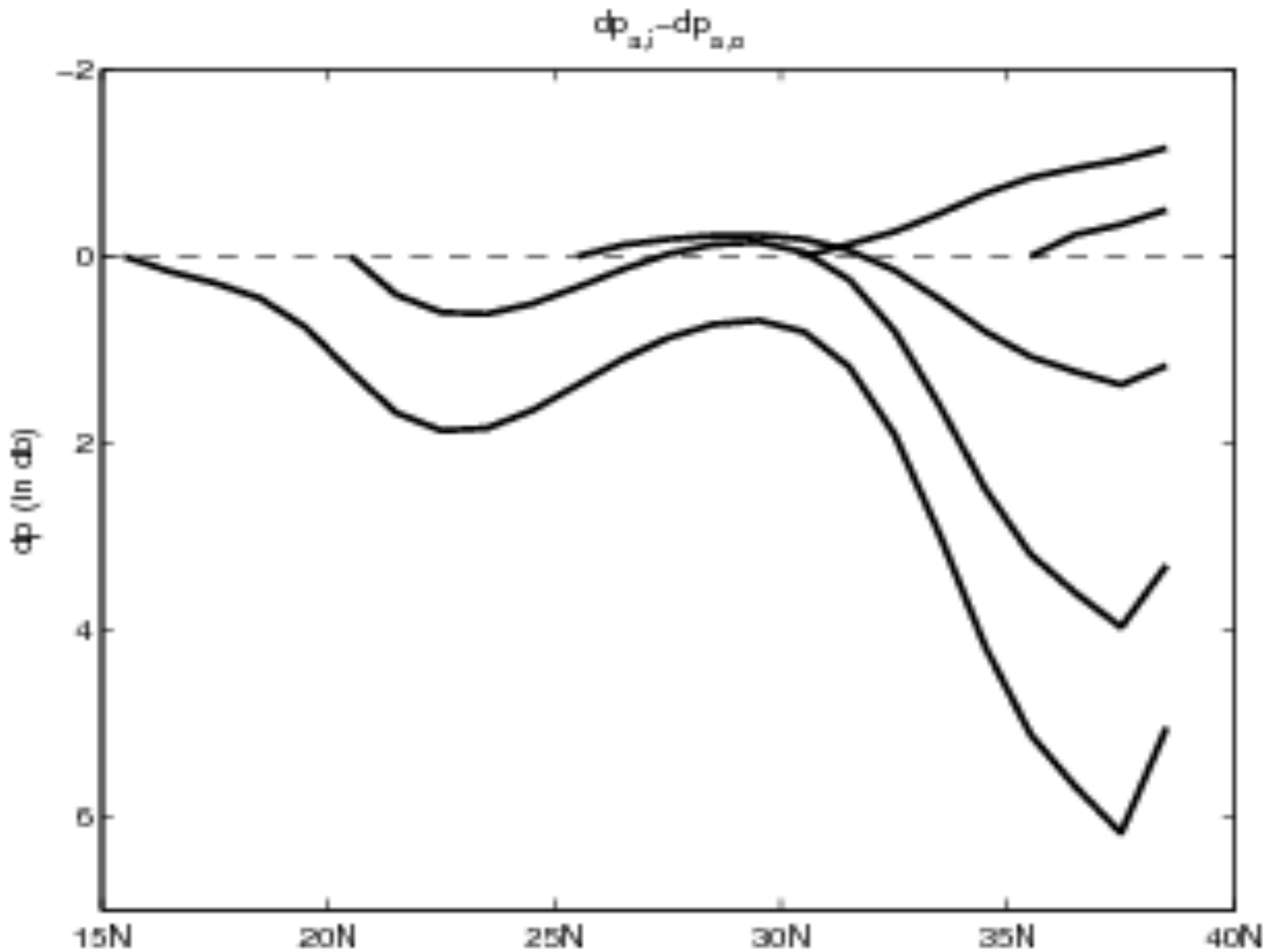
Depth difference



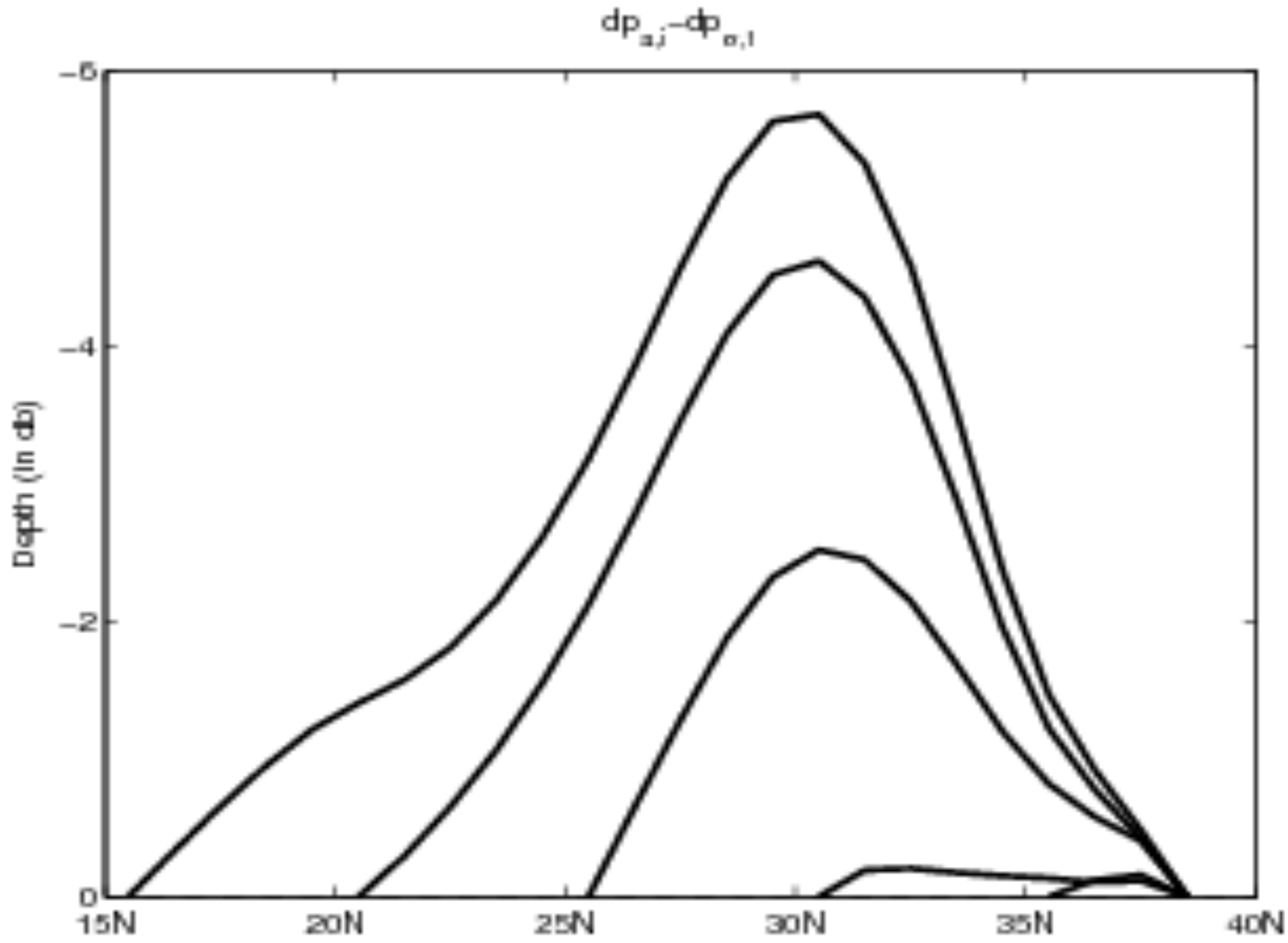
Depth difference along 29.5W



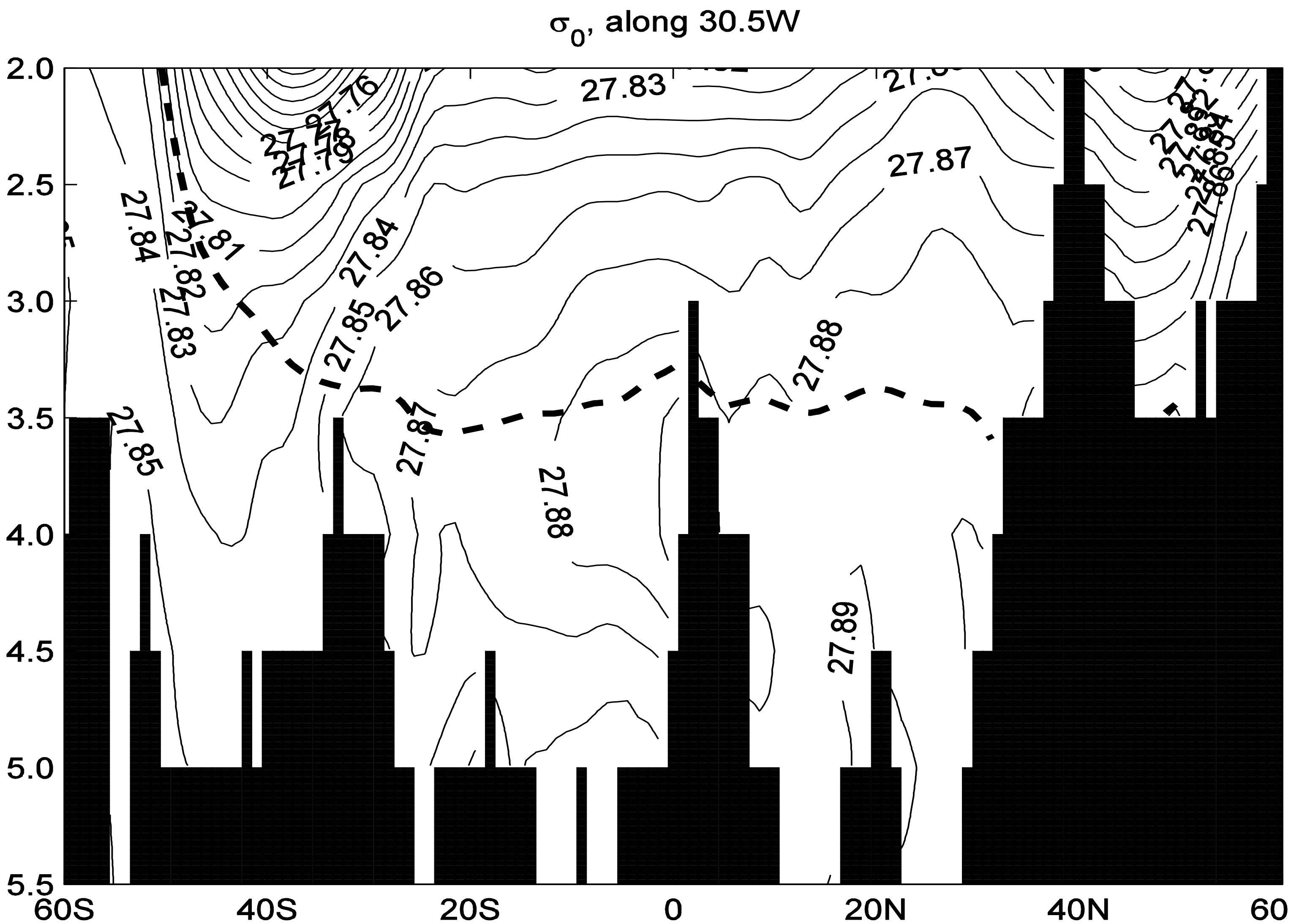
Departure of the ADS



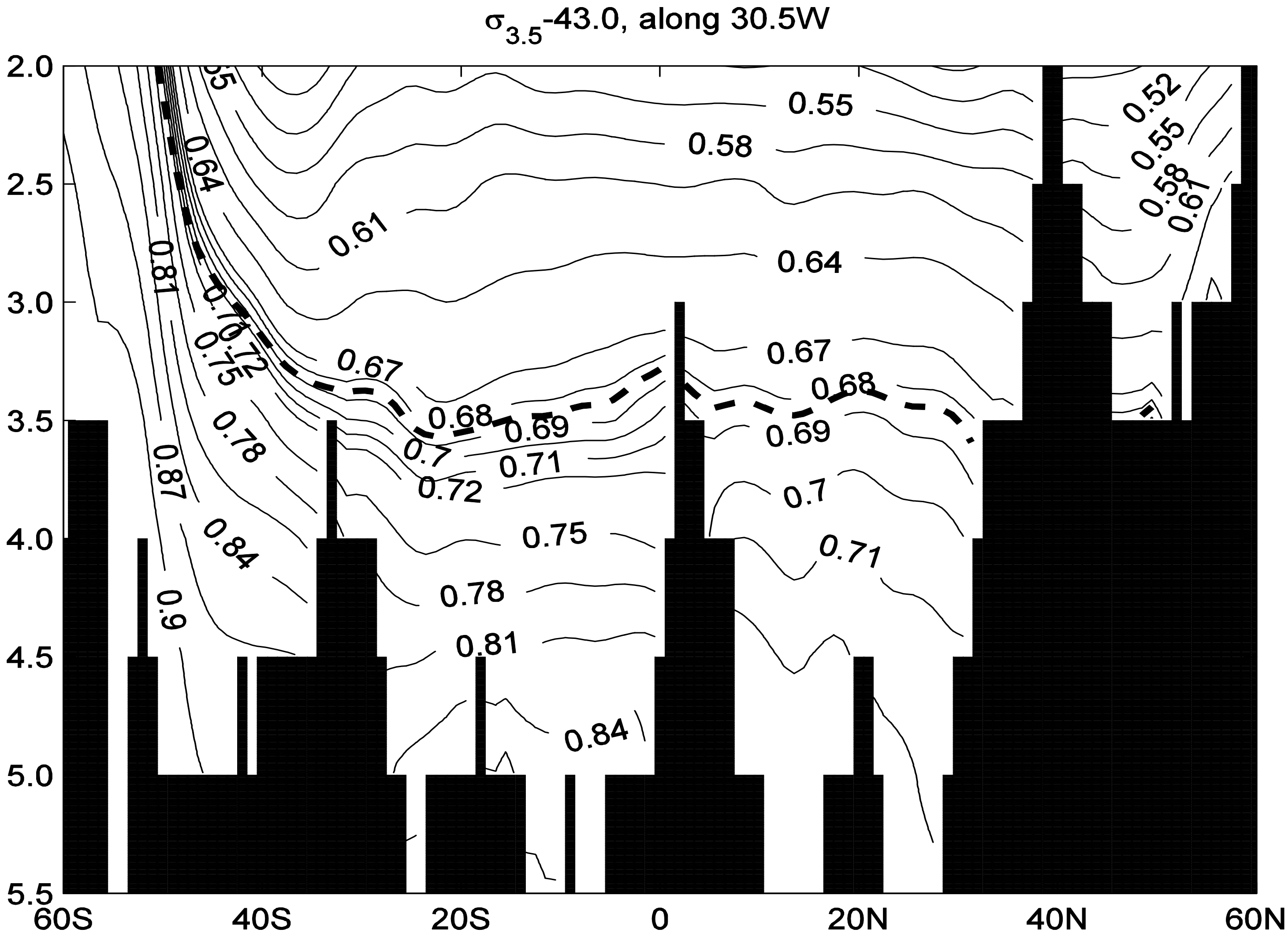
Departure of ADS from the PDS



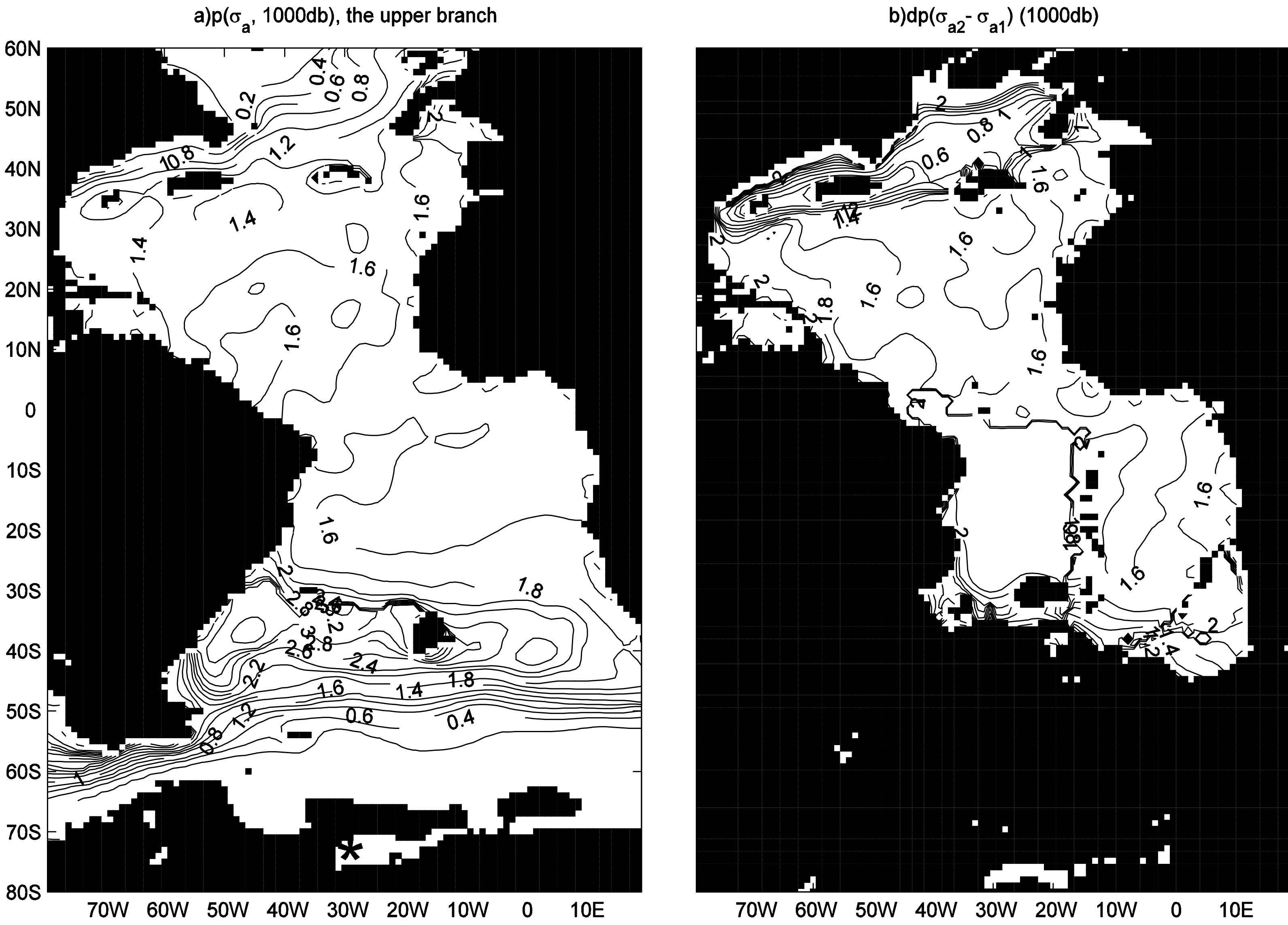
Sigma_0 along 30.5W



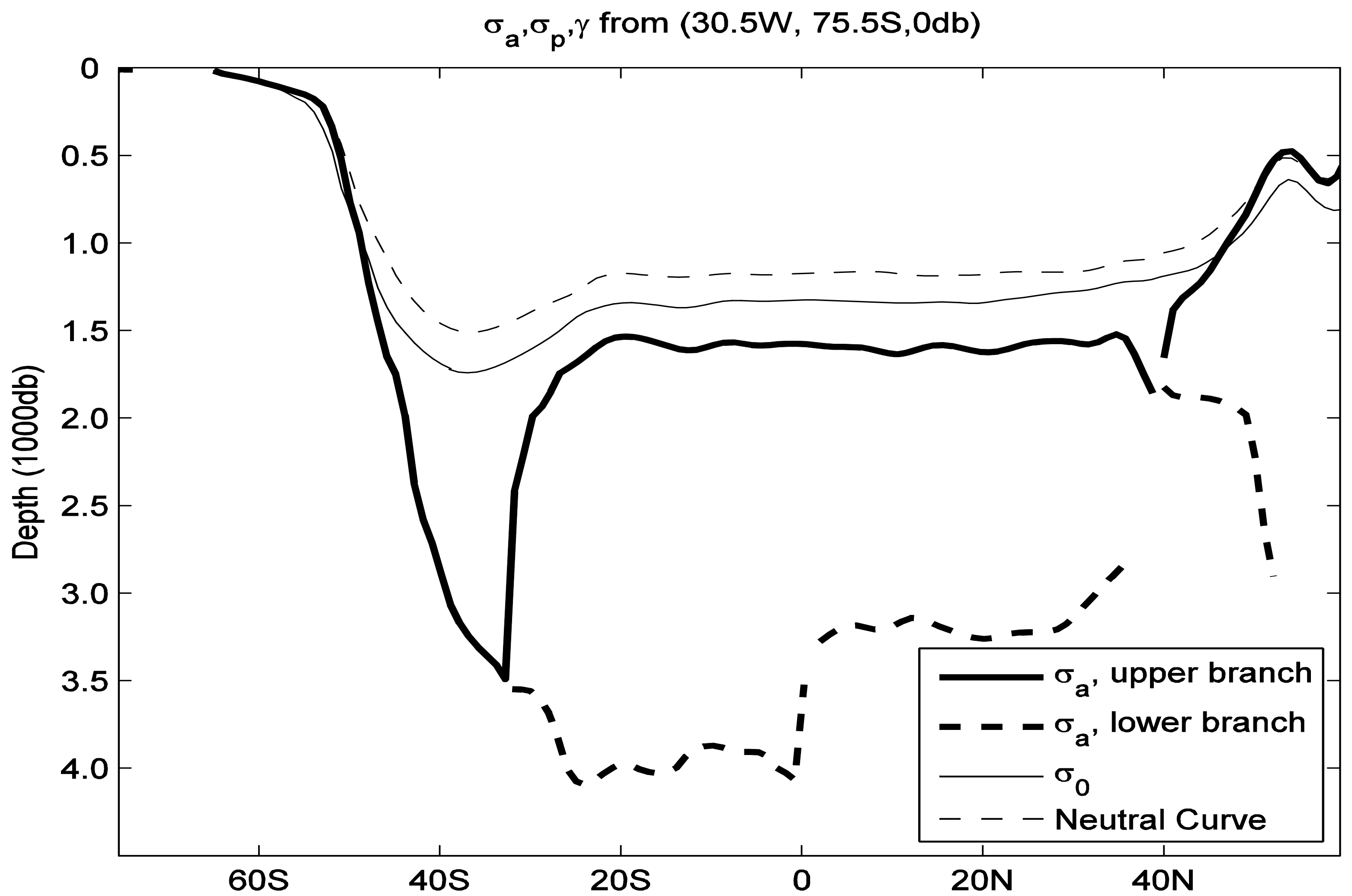
Potential density along 30.5W



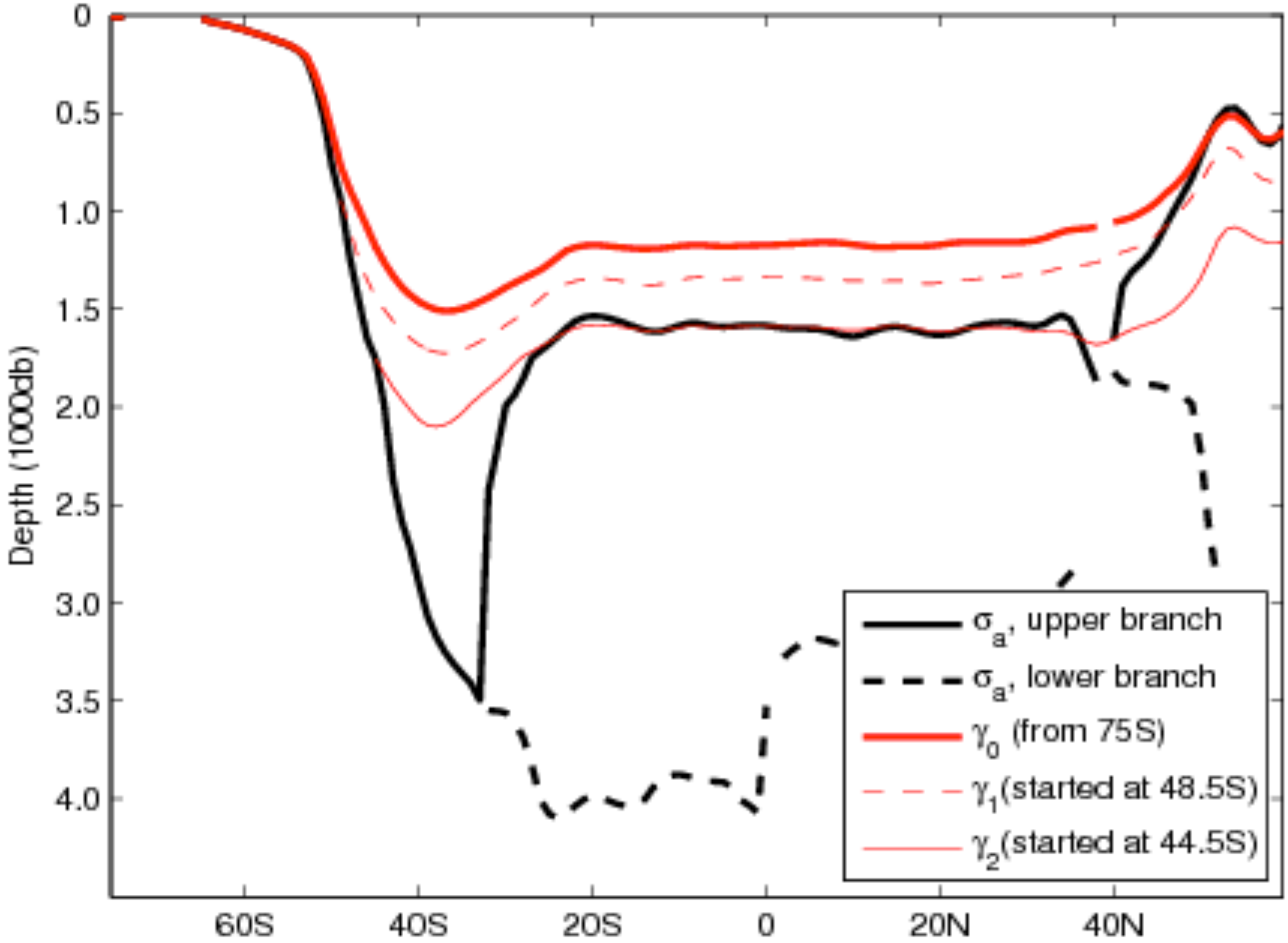
Two sheets of ADS



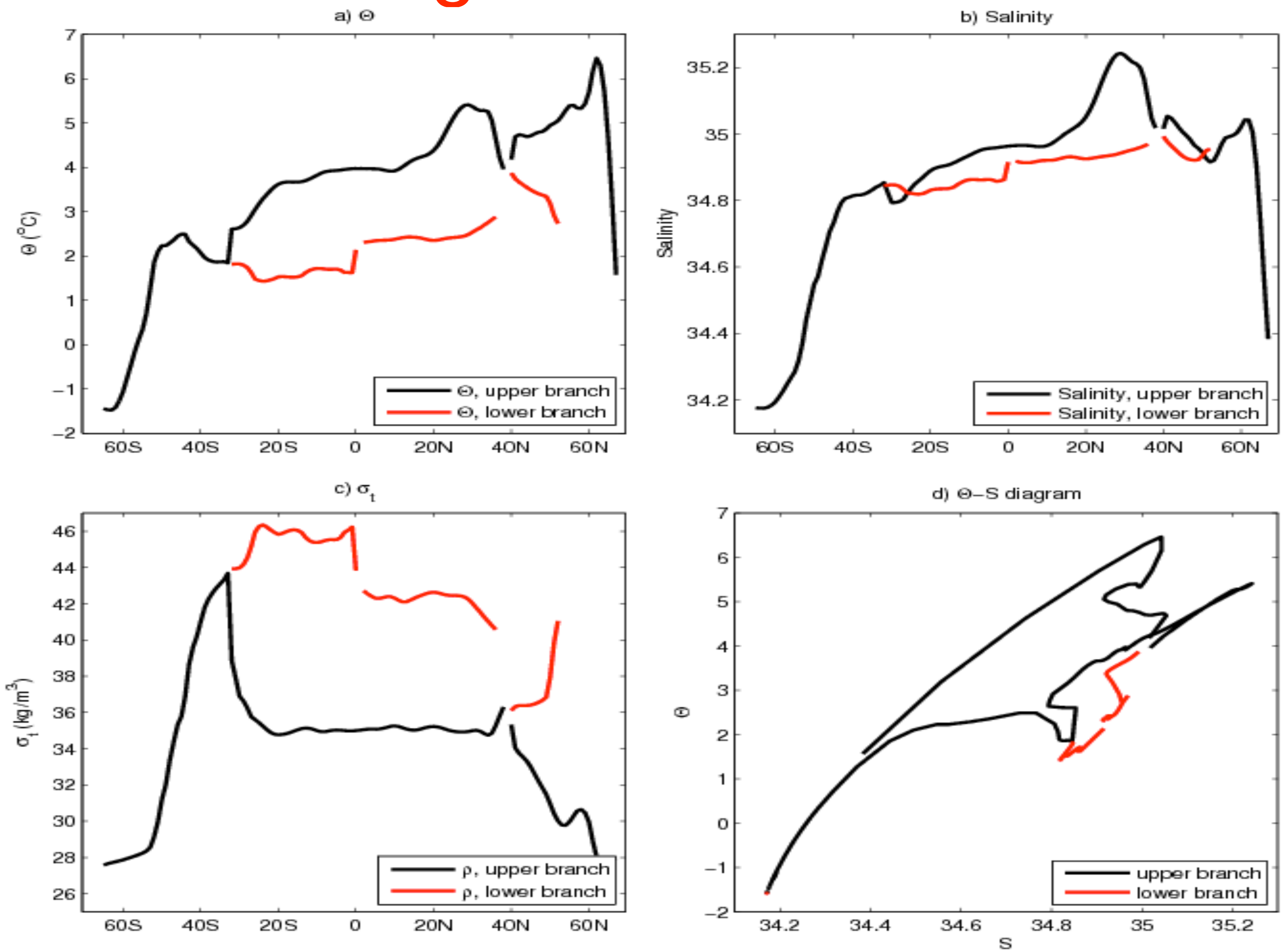
ADS (with bifurcation), PDS and NDS



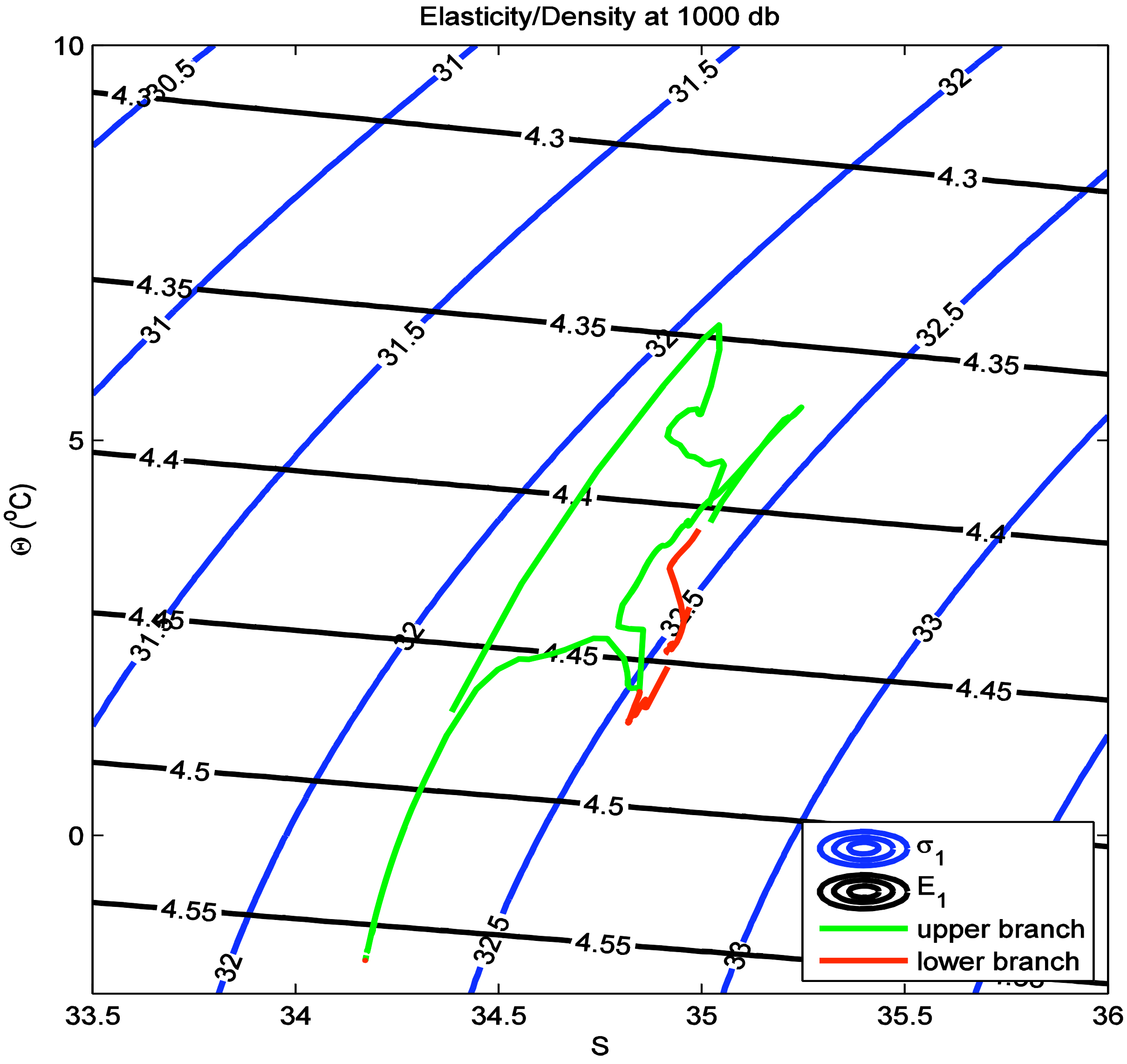
The adiabatic surface may be closer to the mixing path?



Water properties change along adiabatic curves



Cold & fresh water is more compressible



Adiabatic density line may be more close to the mixing path

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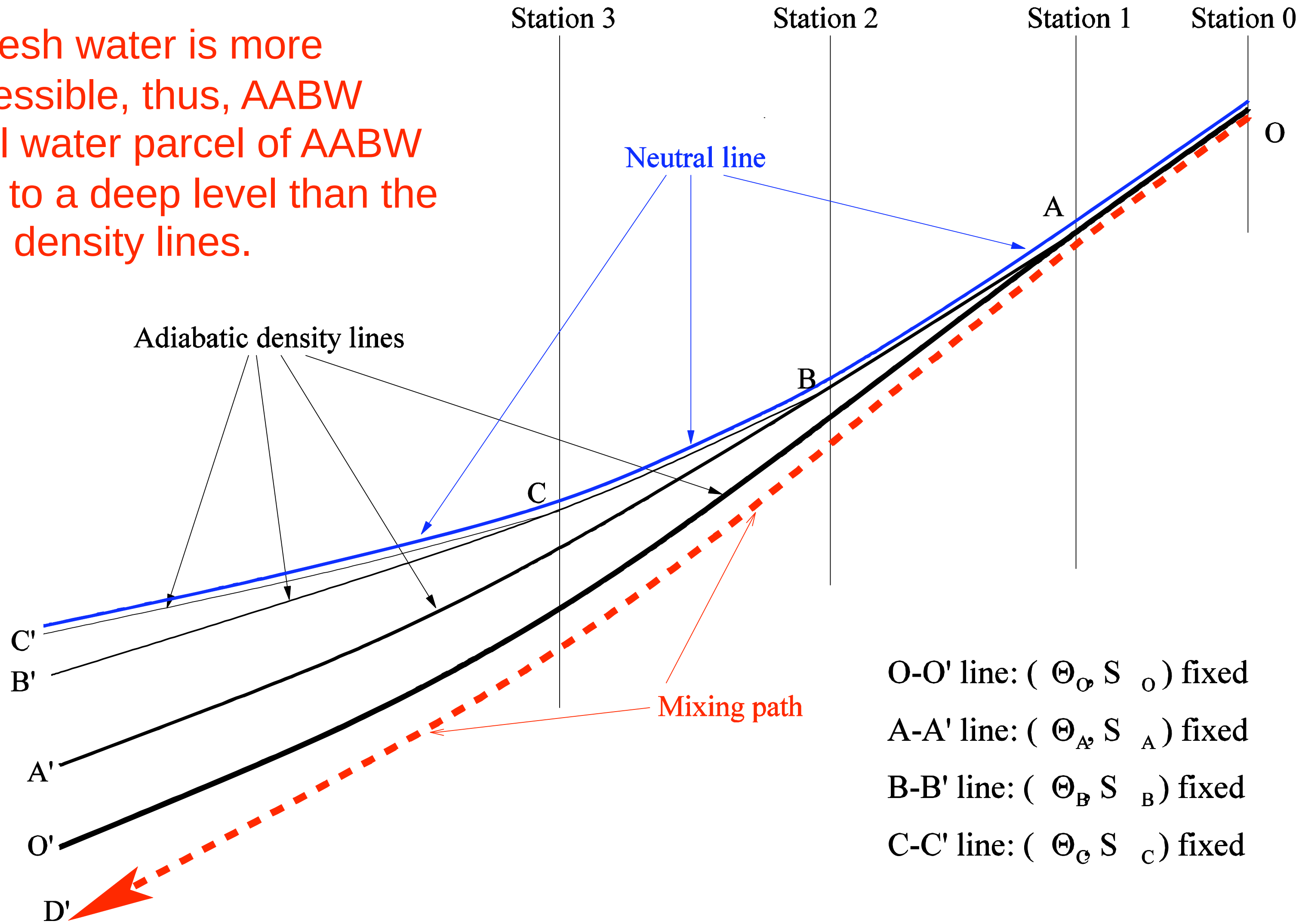
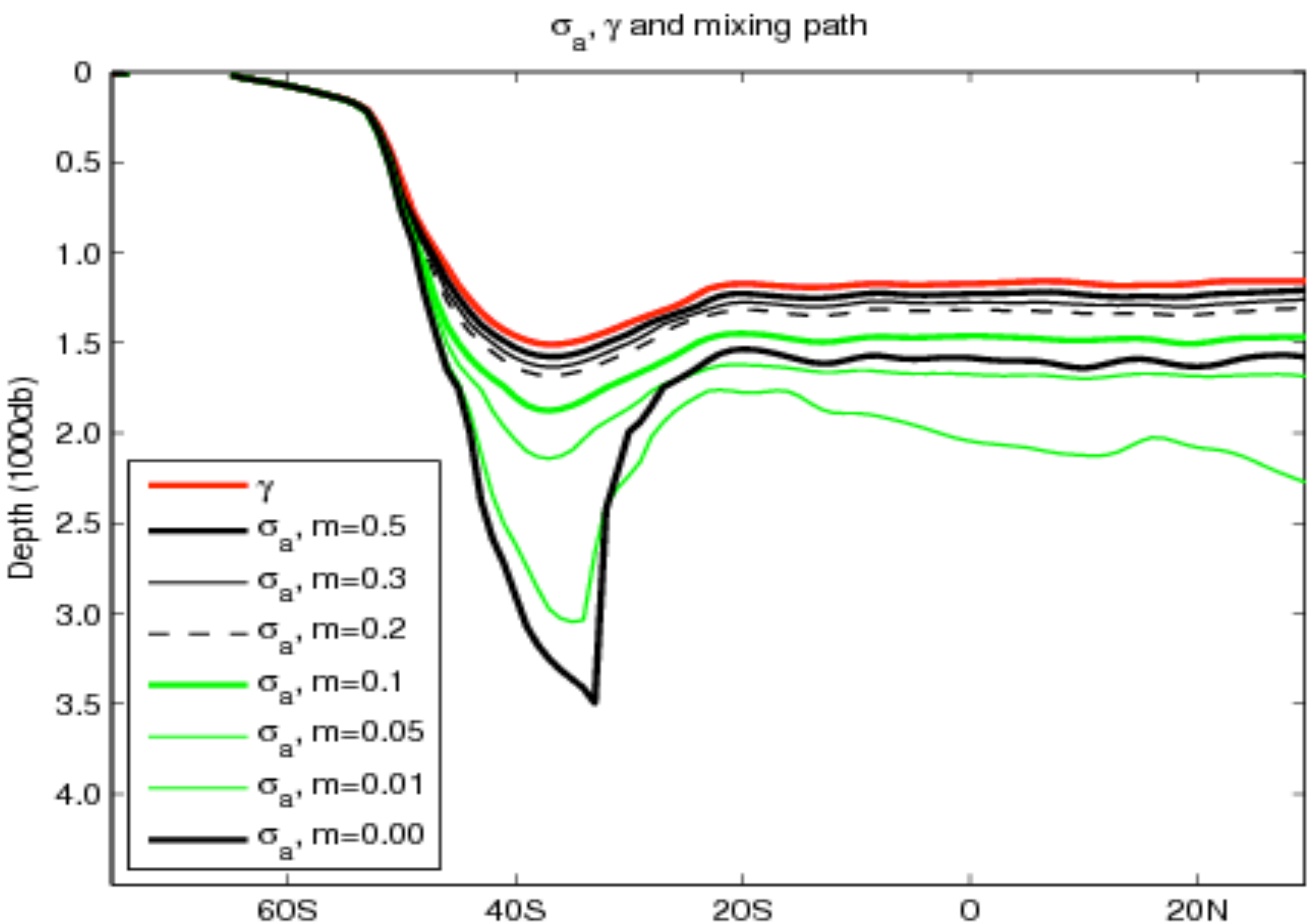
Cold/fresh water is more compressible, thus, AABW original water parcel of AABW moves to a deep level than the neutral density lines.

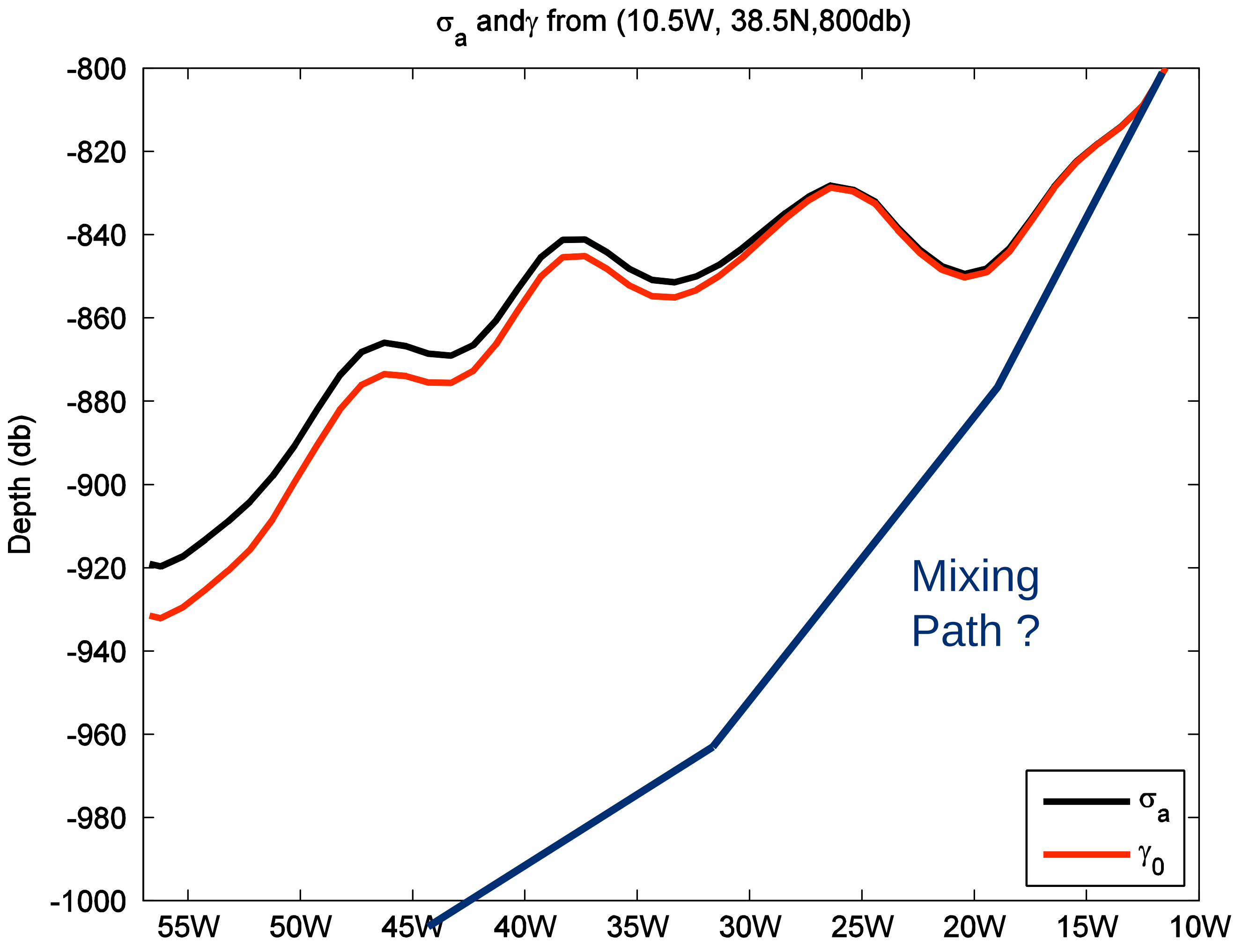


Mixing paths

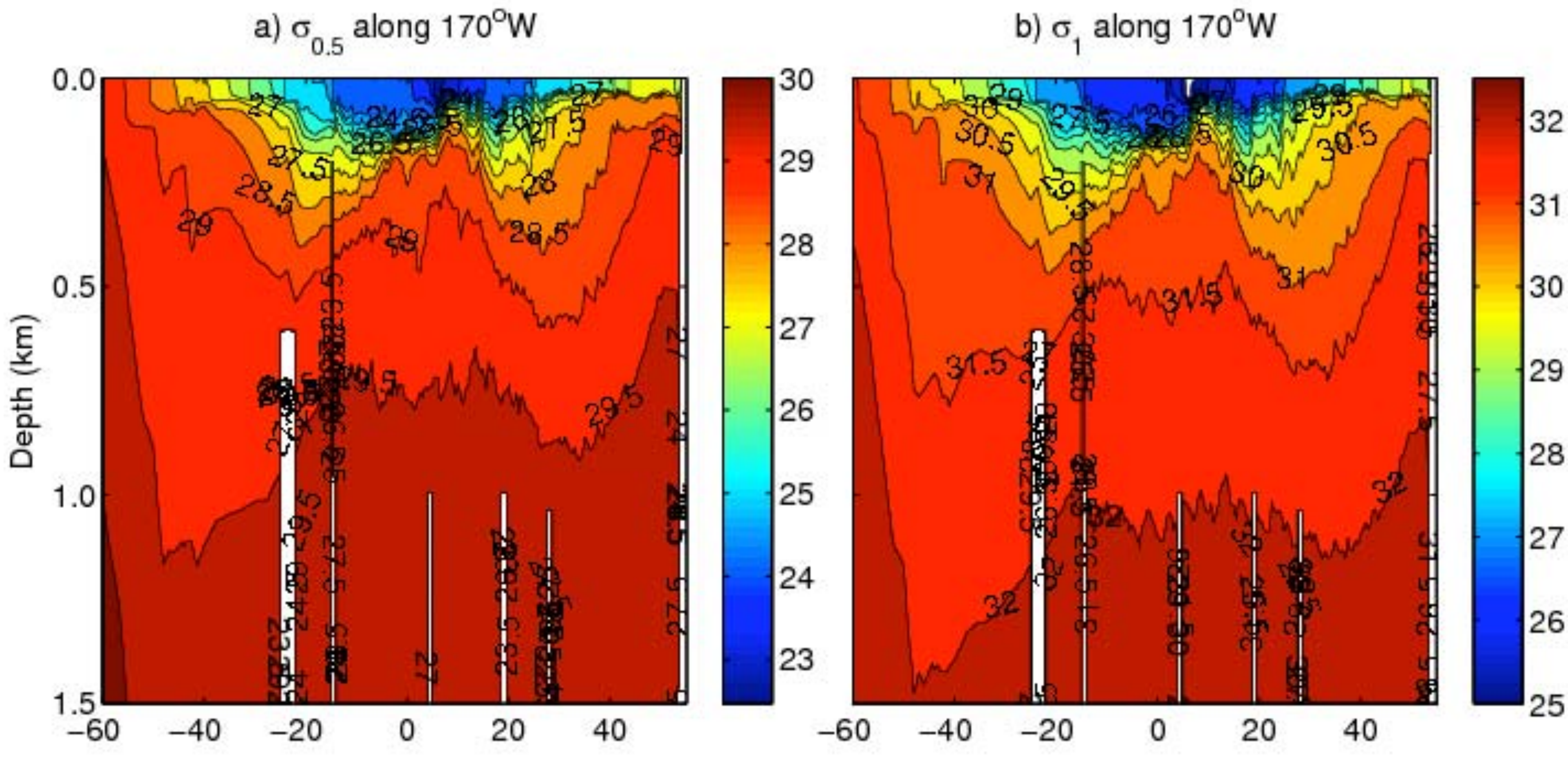


MOW is warm/salty, it is less compressible than the environmental water

γ is lower than σ_a



WOCE P15 Section



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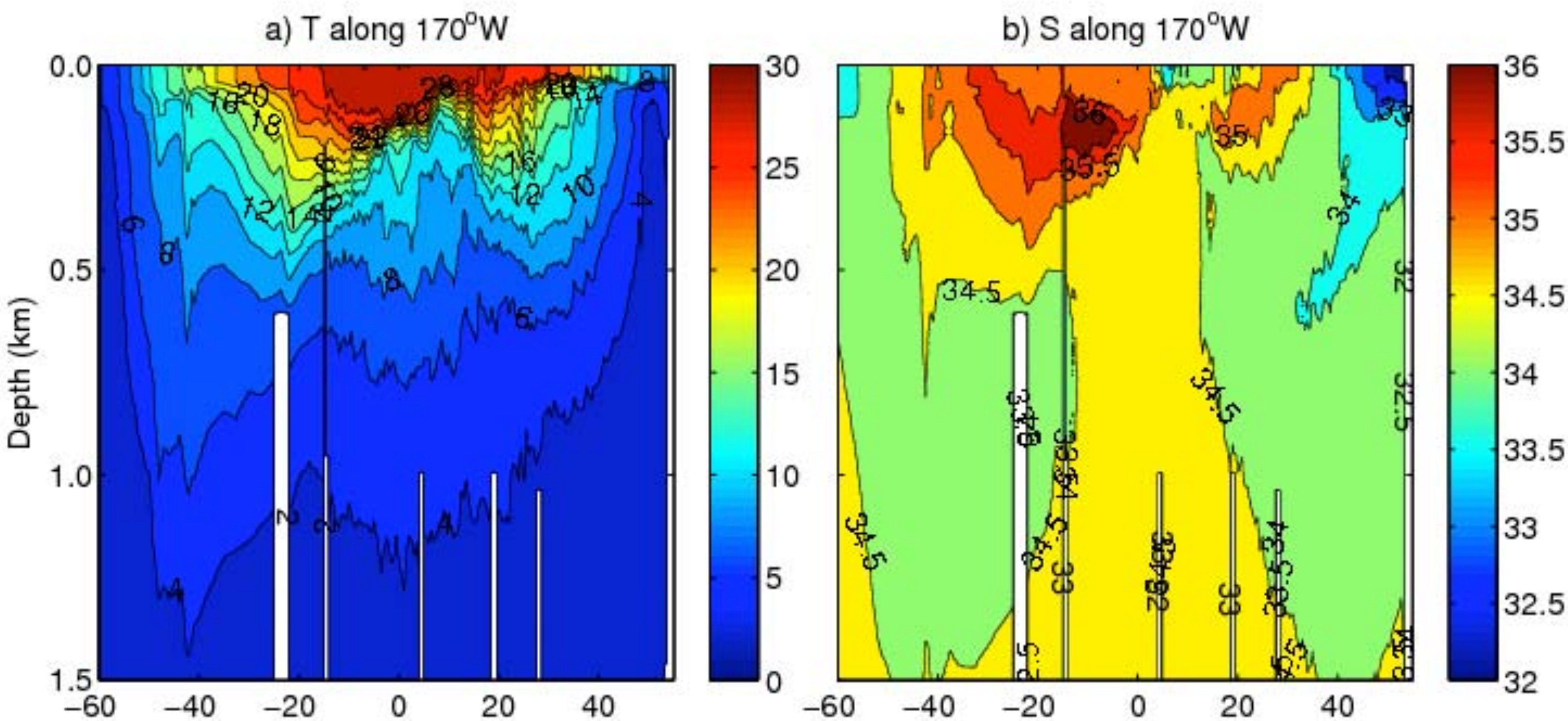
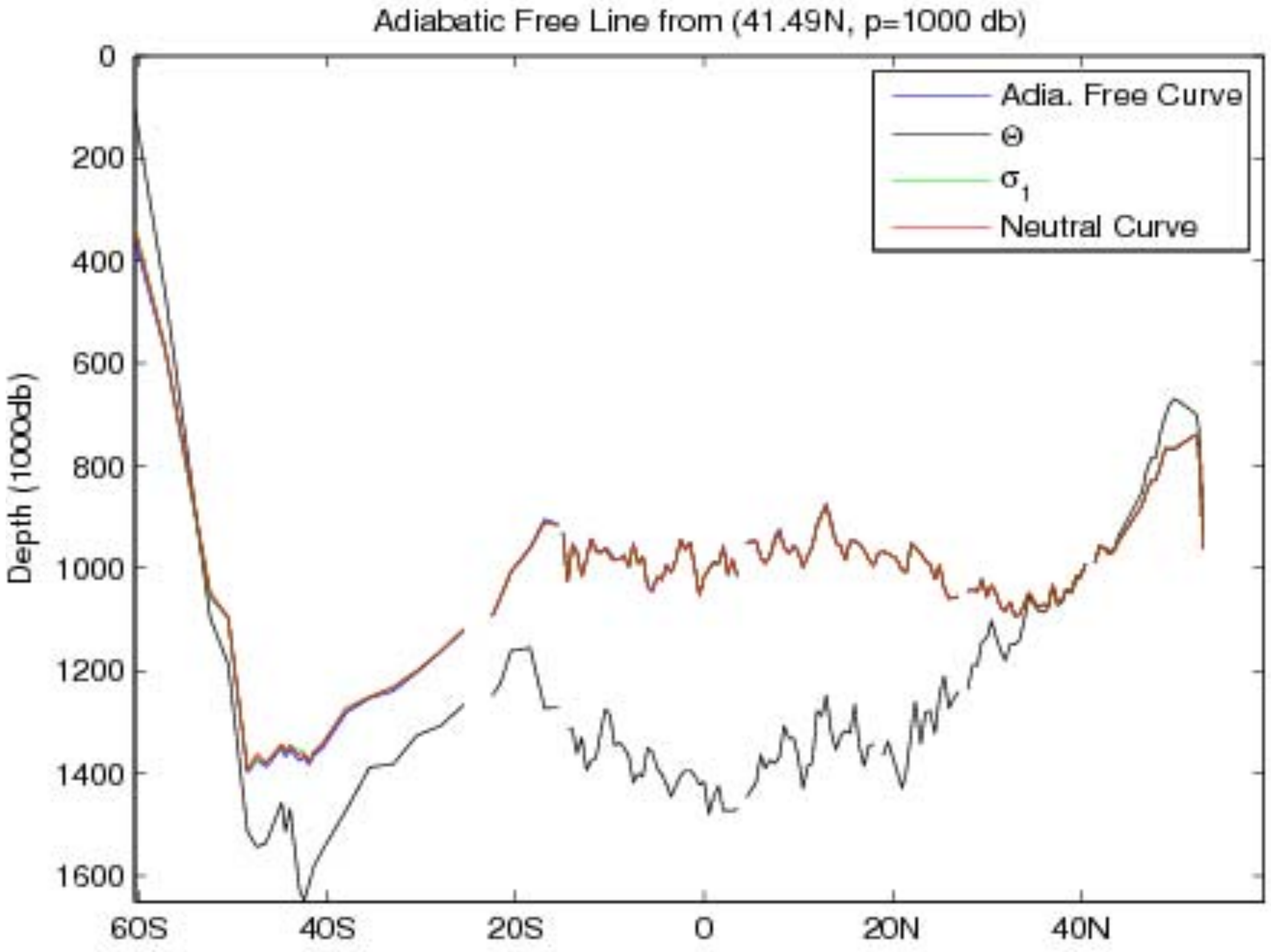
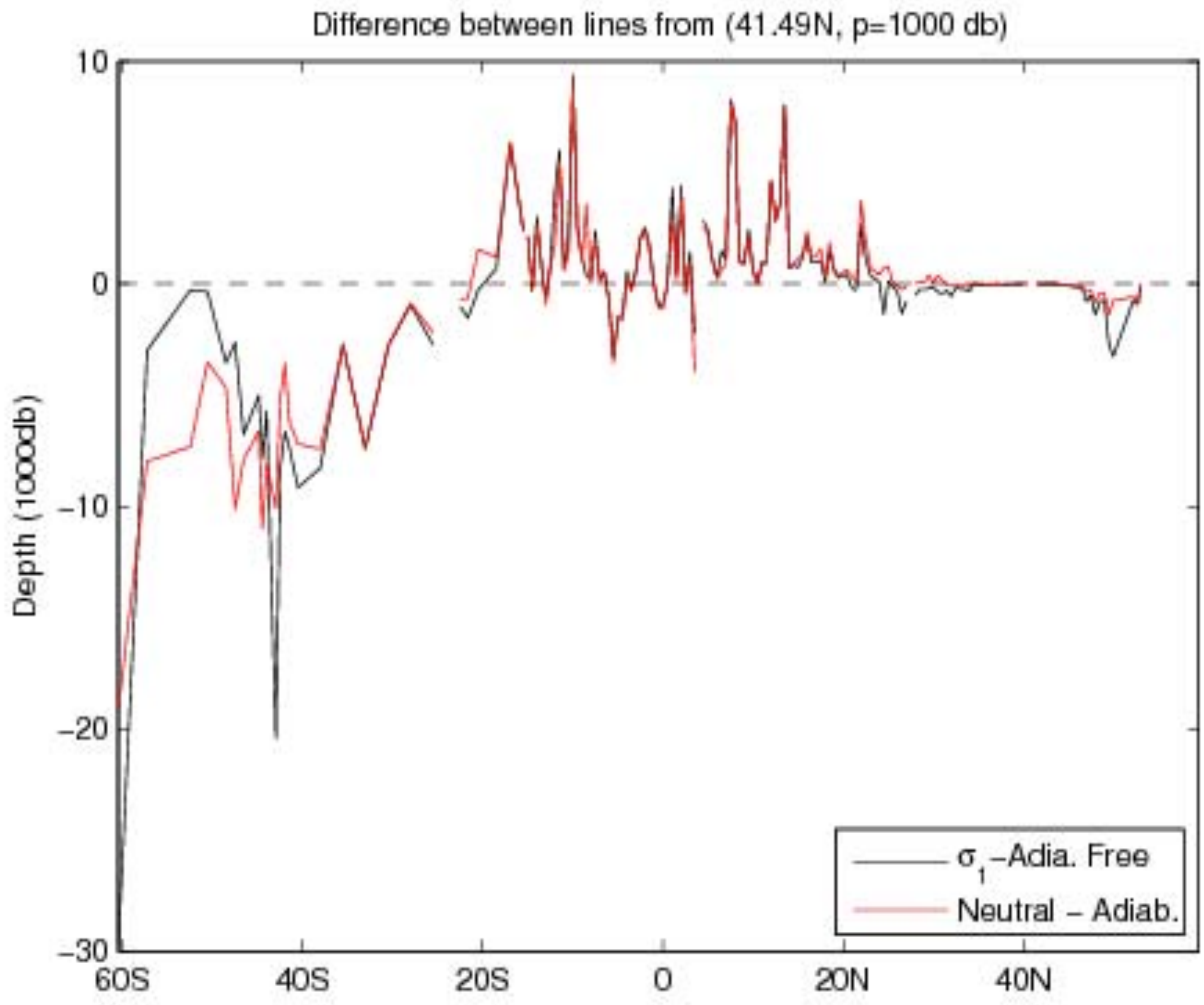


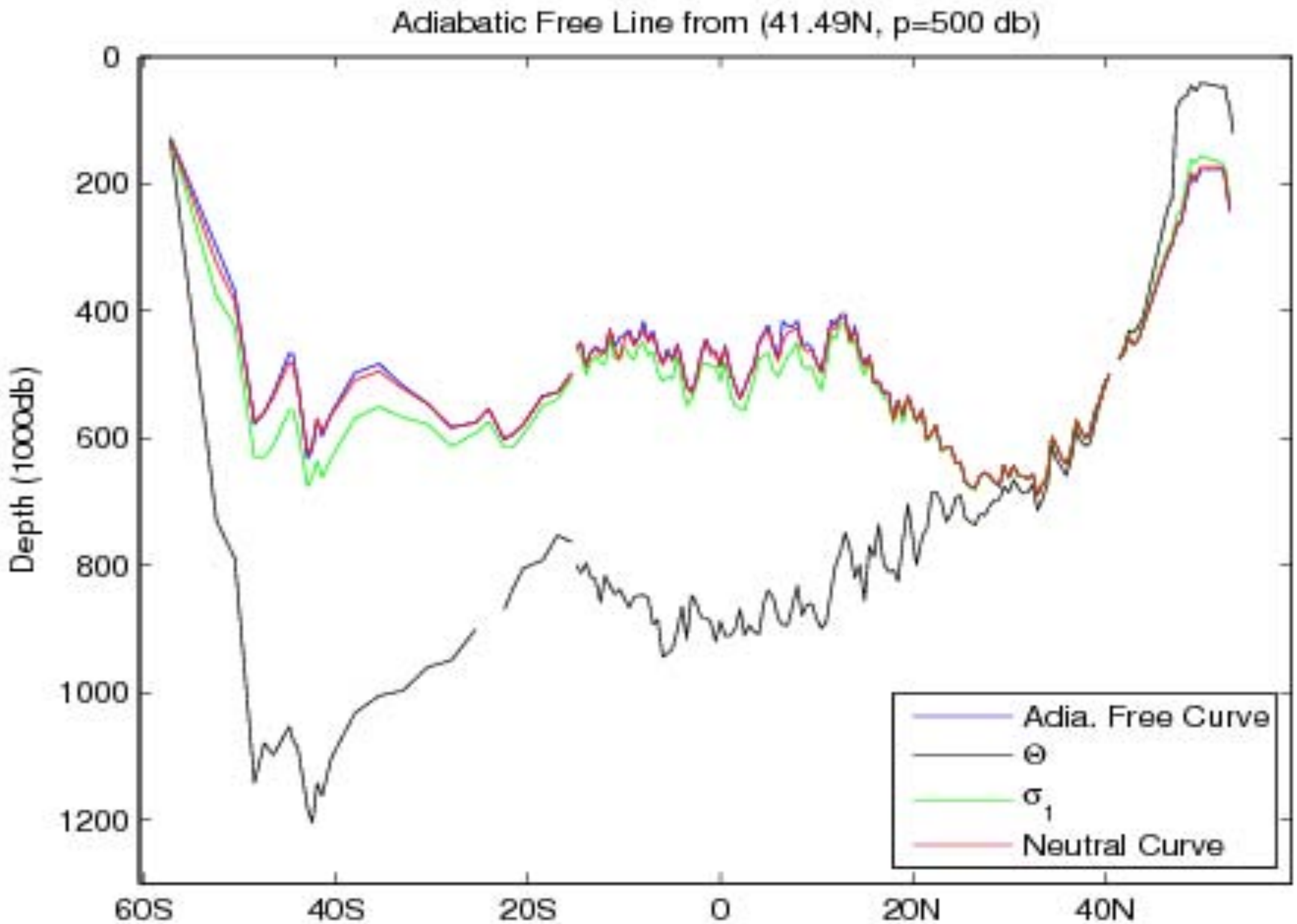
Adiabatic density & other curves



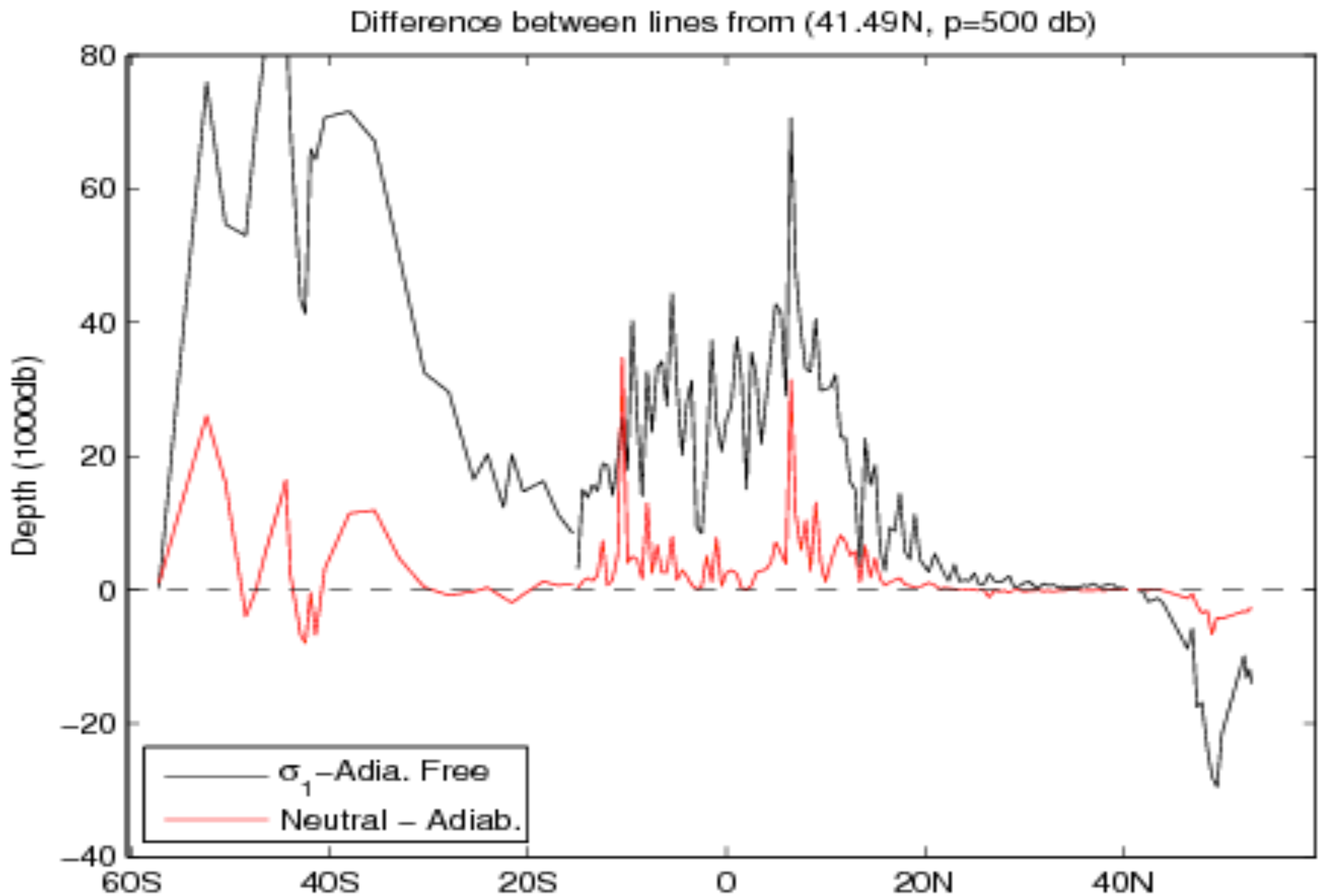
Diff. between different curves



Adiabatic density & other curves



Difference between different curves



Energetics of quasi-horizontal stirring and mixing

Rui Xin Huang

Woods Hole Oceanographic Institution, Woods Hole, USA

Major points

- What really matters is eddy diffusion, not molecular diffusion
- Eddy diffusion consists of Stirring and Mixing
- Isopycnal exchange of watermass is no free lunch
- Cabbeling is always a sink of GPE
- A new type of meso-scale instability

  this instability is supported by energy released in connection with the non-constant elasticity of seawater.

Subgrid parameterization of diffusion

- Diapycnal mixing, eddy transport -GM90 scheme

  Types of lateral diffusion (mixing):

  horizontal, sigmas-surface, isopycnal

  2) isopycnal diffusion: the classical concept → isopycnal diffusion is isotropic ?

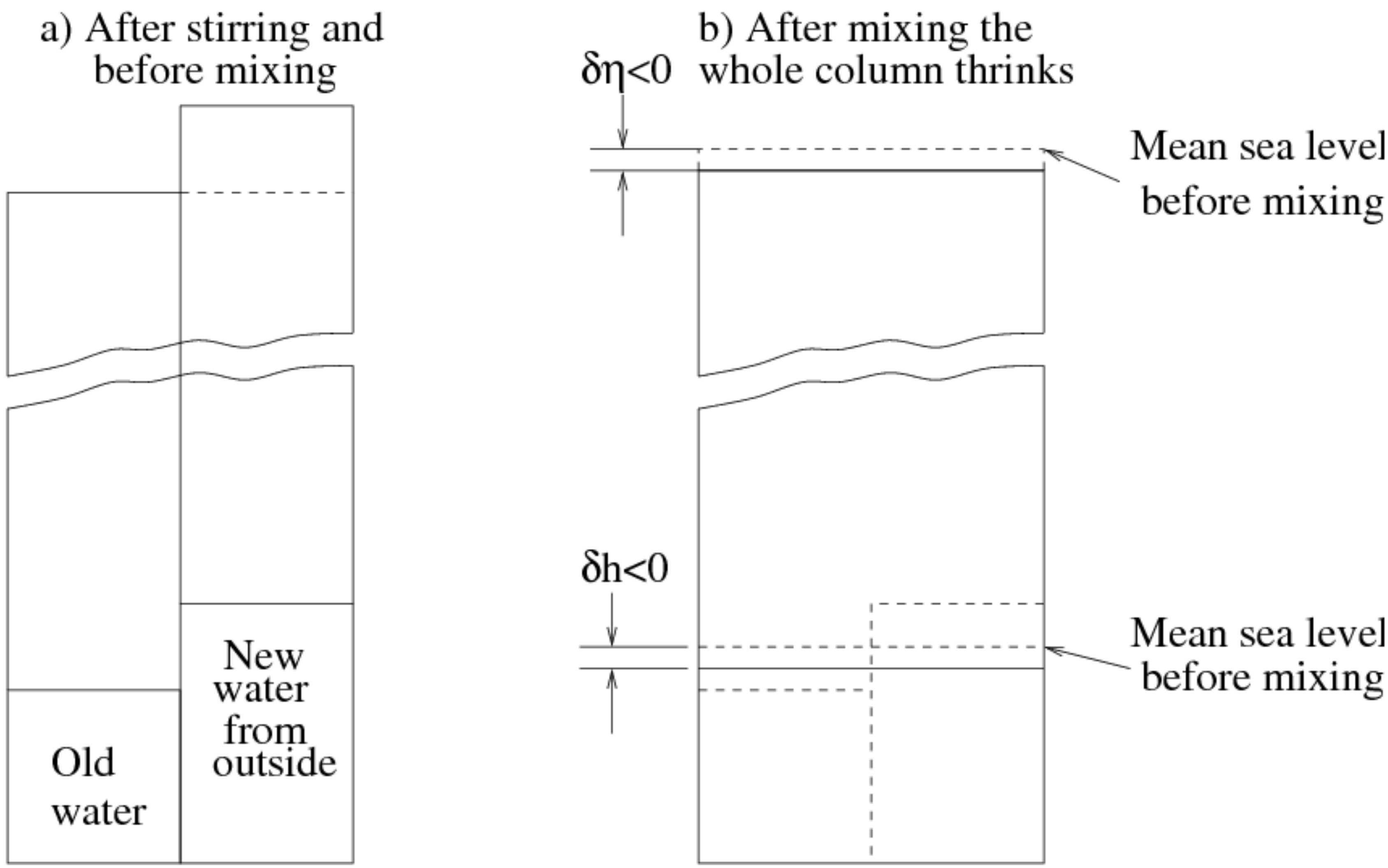
  isopycnal stirring is free of GPE changes ?

- Momentum:

  Difficult to measure momentum transfer/mixing.

  Remains a great grand challenge.

Eddy diffusion is separated into 2 steps: stirring & mixing

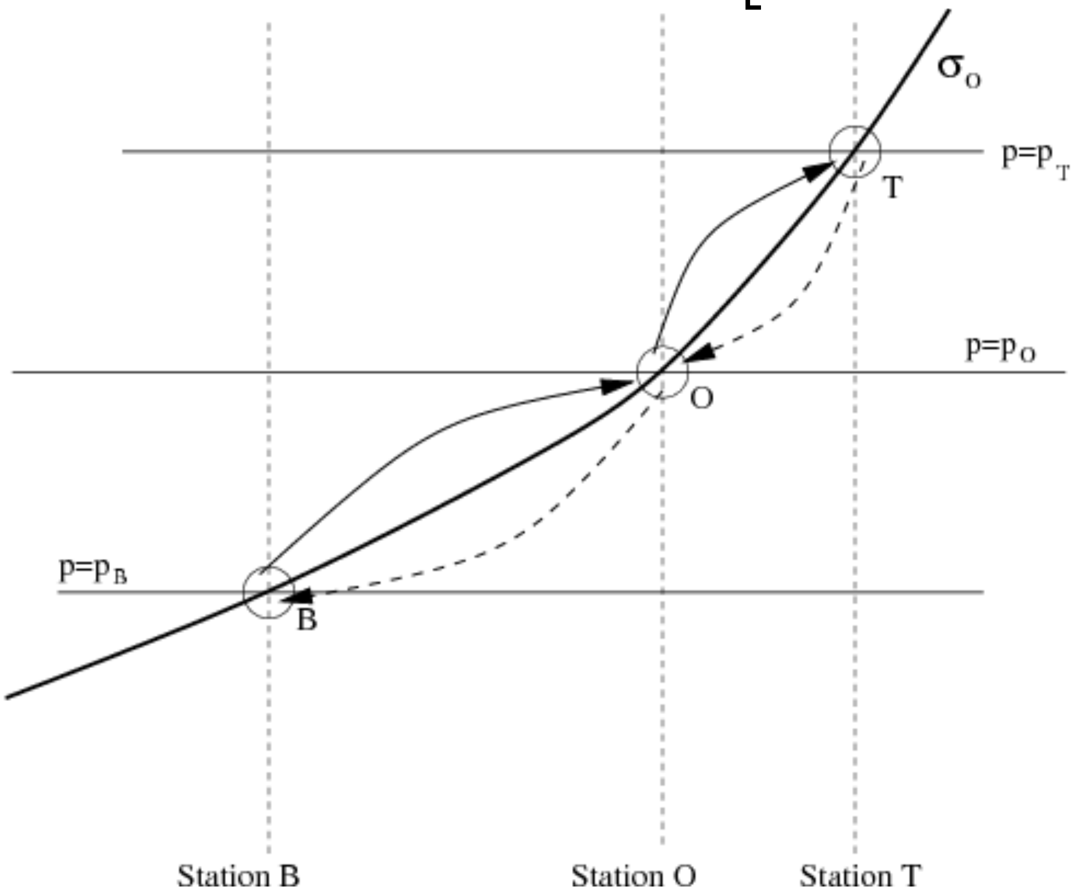


Stirring along isopycnal surface

induces density anomaly and GPE changes

$$\rho\left(B \rightarrow T\right) - \rho\left(T\right) = \left[E\left(T,O\right) - E\left(B,O\right)\right]\Delta p / 2000$$

$$\rho\left(T \rightarrow B\right) - \rho\left(B\right) = \left[E\left(T,O\right) - E\left(B,O\right)\right]\Delta p / 2000$$



Density anomaly at both stations is non-zero.

They have the same value

Cabbling: Mixing two boxes with 1 kg of sea water

$$T_1 = 0^o C, P_1 = 0$$
$$S_1 = 34$$
$$\rho_1 = 1027.30053\, kg / m^3$$

$$T_2 = 10^o C, P_2 = 0$$
$$S_2 = 35.44367$$
$$\rho_2 = 1027.30053\, kg / m^3$$

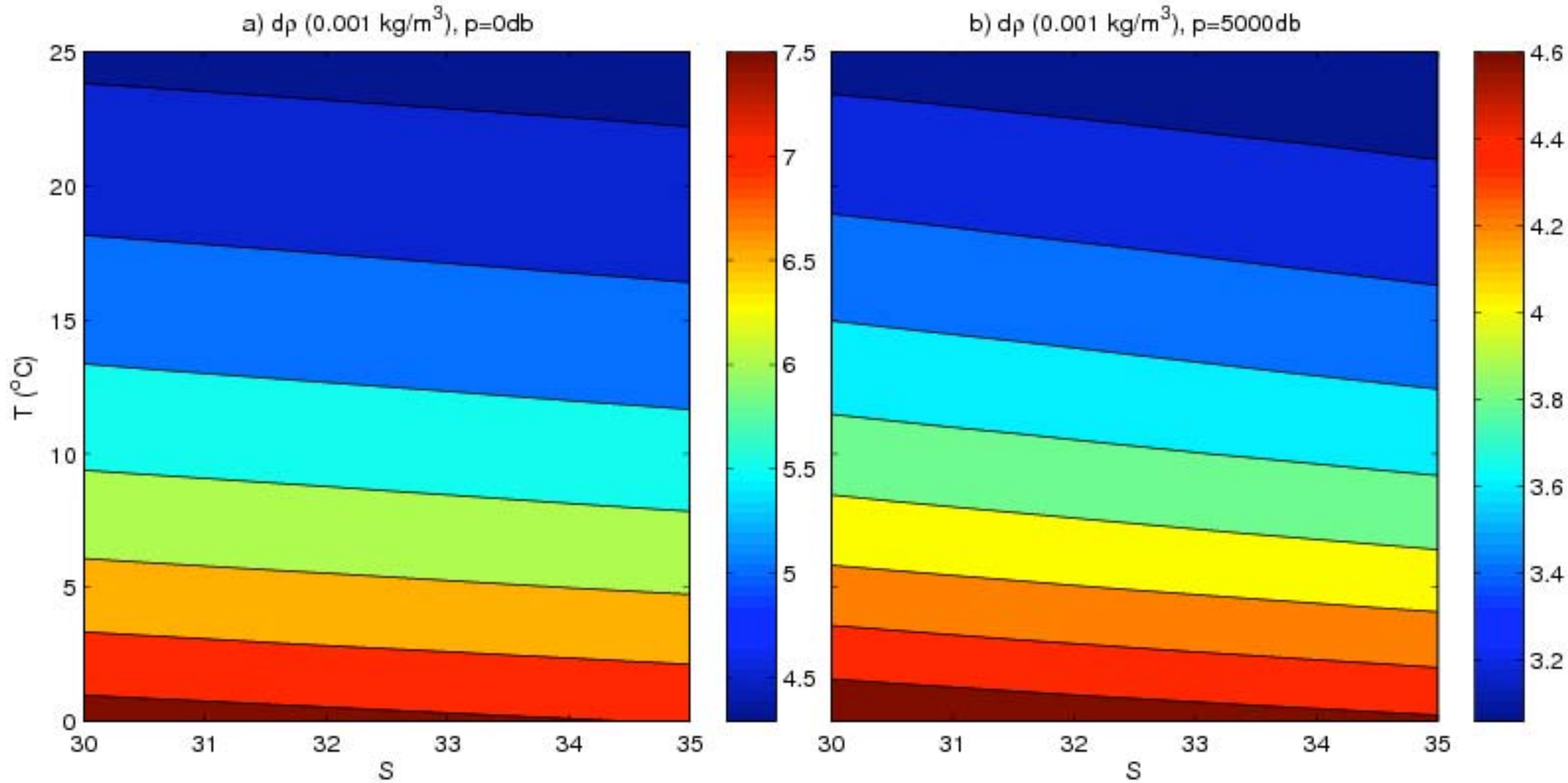
$$T_3 = 5^o C, P_3 = 0$$
$$S_3 = 34.721836$$
$$\rho_3 = 1027.4551105\, kg / m^3$$

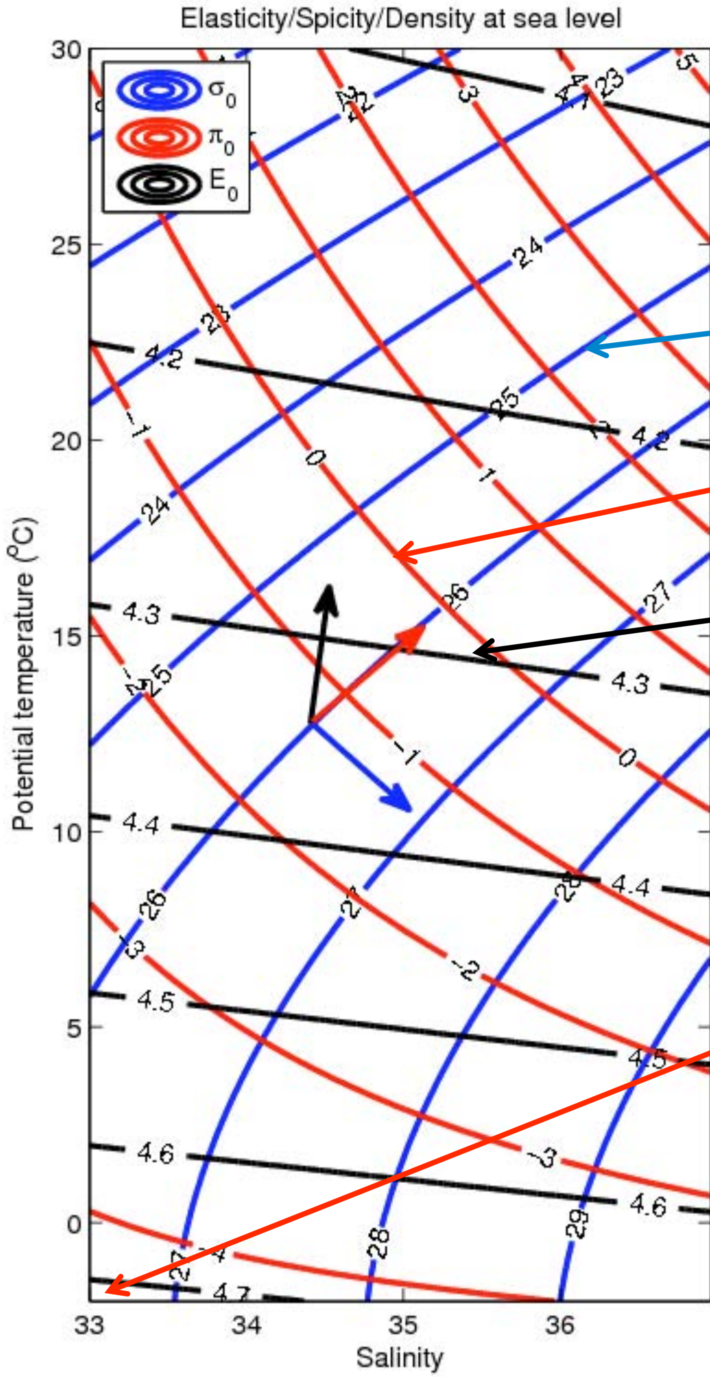
$$\delta\rho = 0.15458\, kg / m^3$$
$$\delta\rho / \rho = 0.015\%$$

Cabbling

Two parcels with different T &S, but same density mix can produce water with larger density which sinks to a deeper level

Density increase due to mixing of parcels with dT=0.1C,dS=0.1





Density

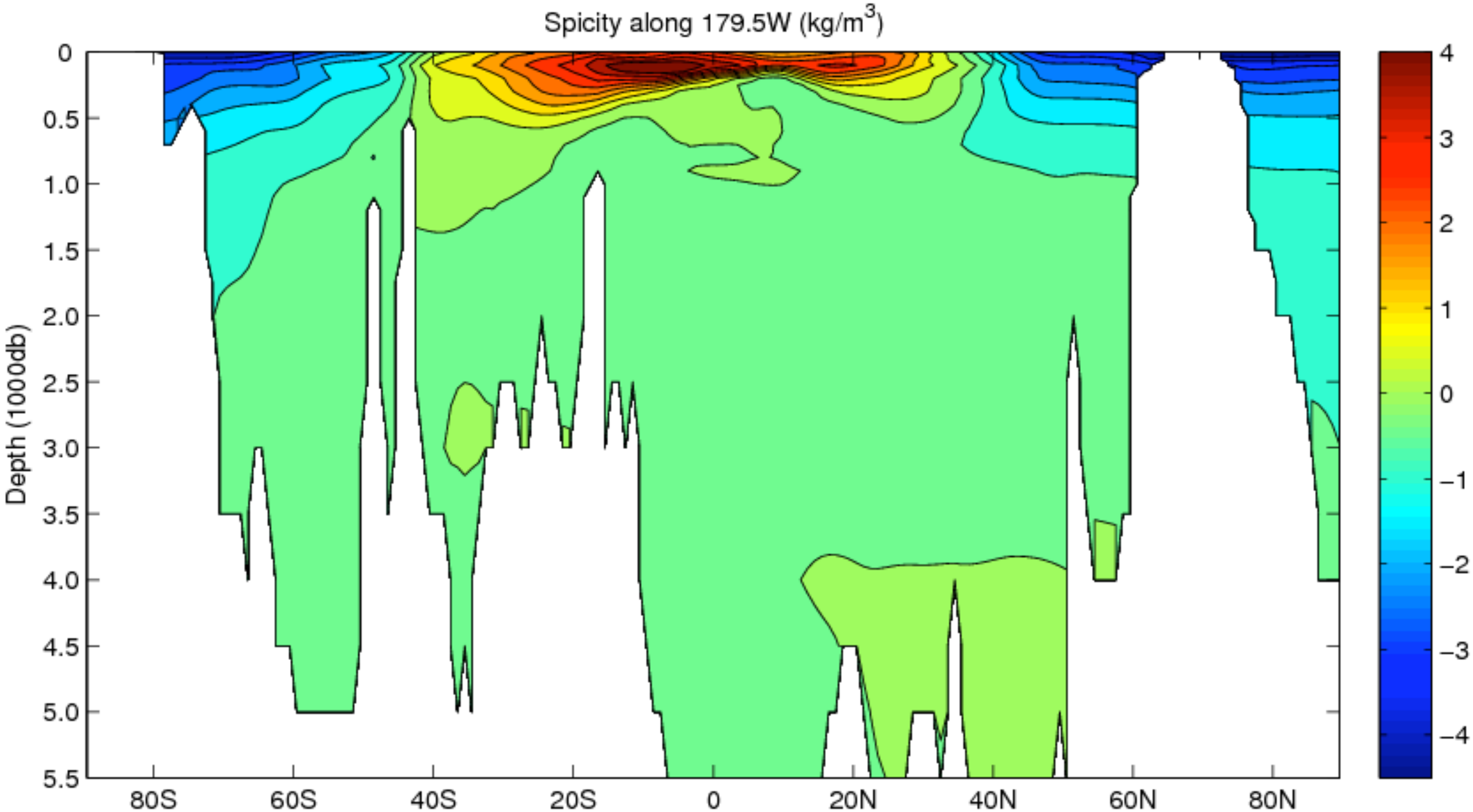
spicity

elasticity

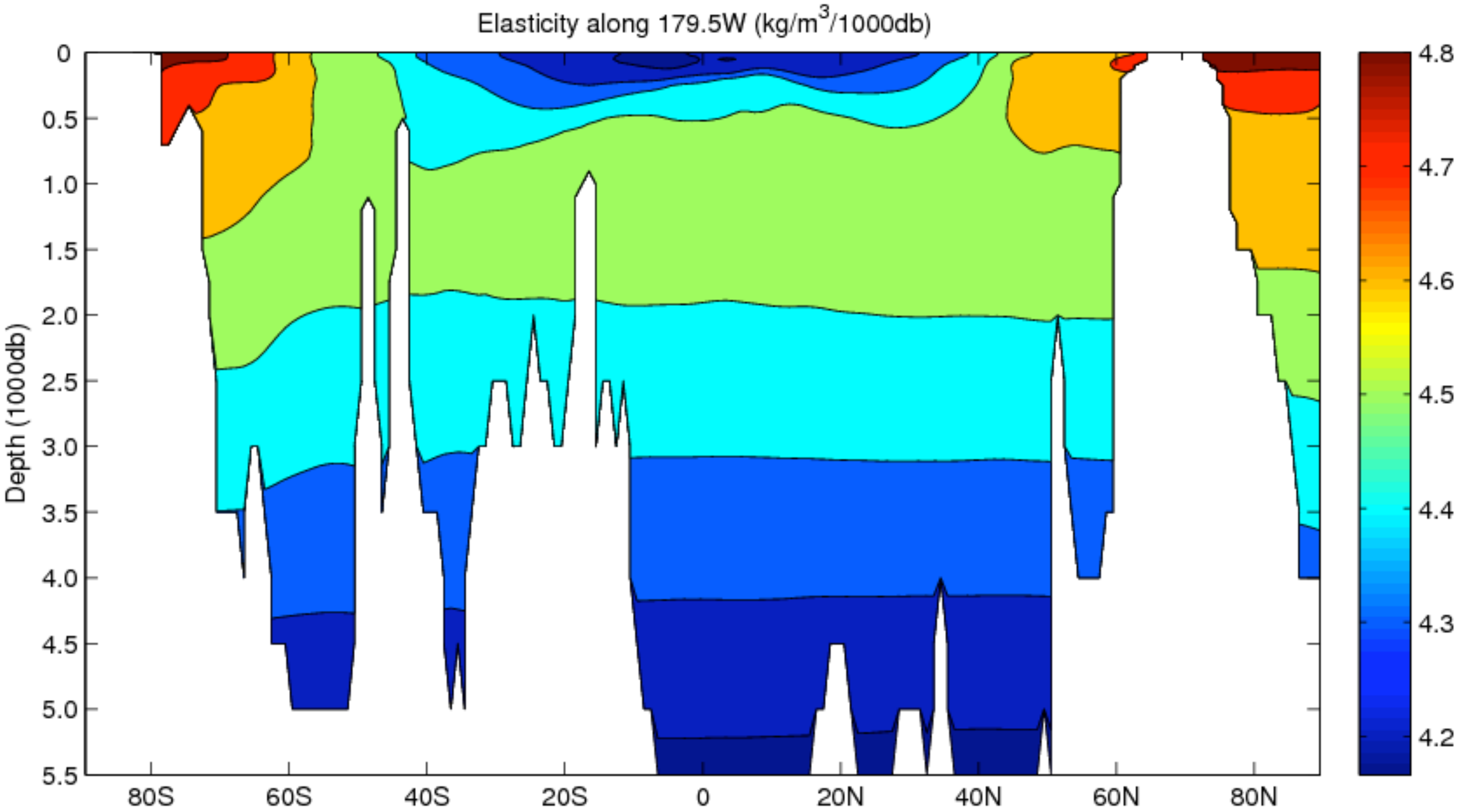
$$E = 1000db\rho K_\eta \left(kg/m^3\right)$$

Cold & fresh water is more compressible

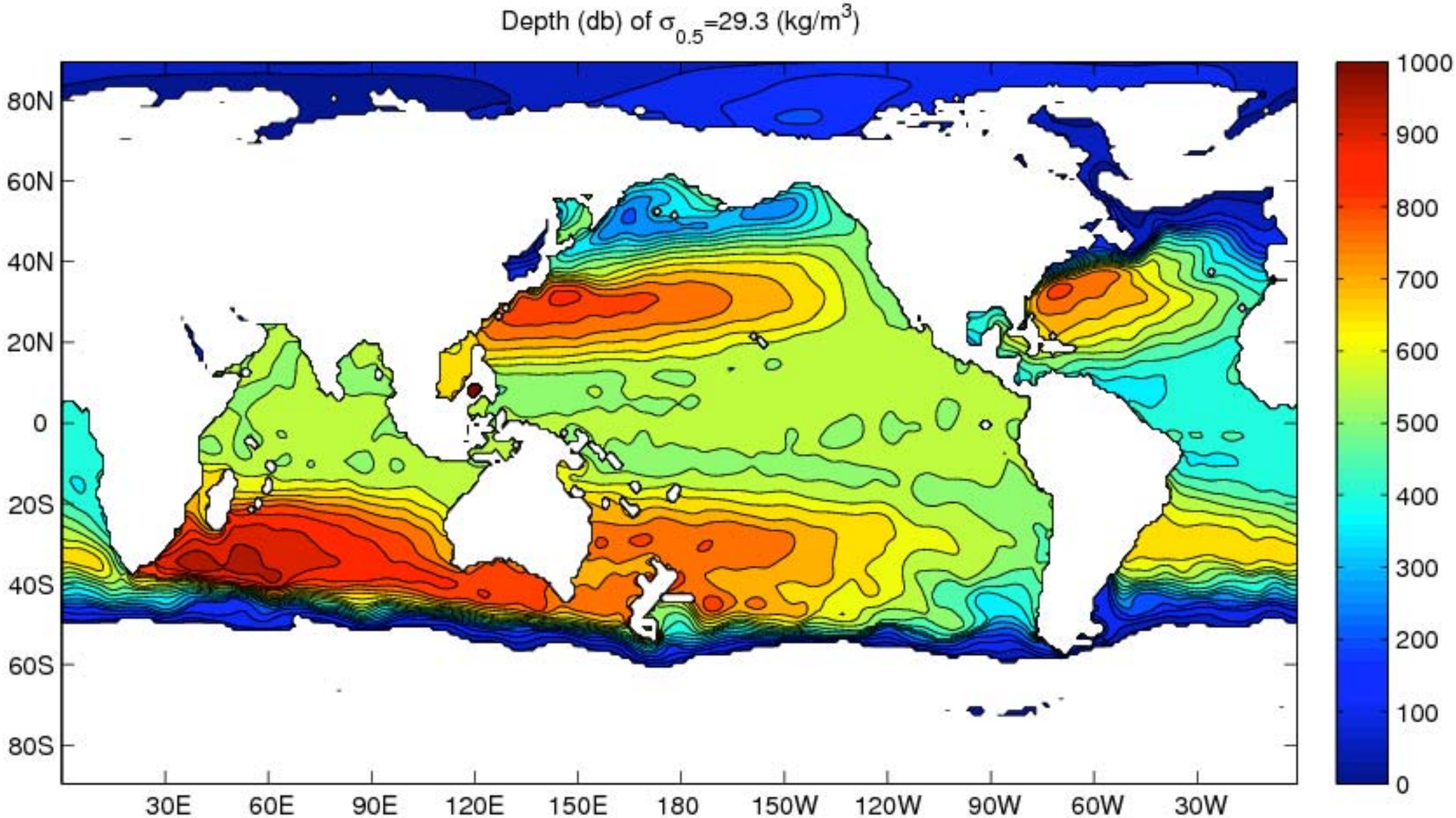
Spicity along 179.5W



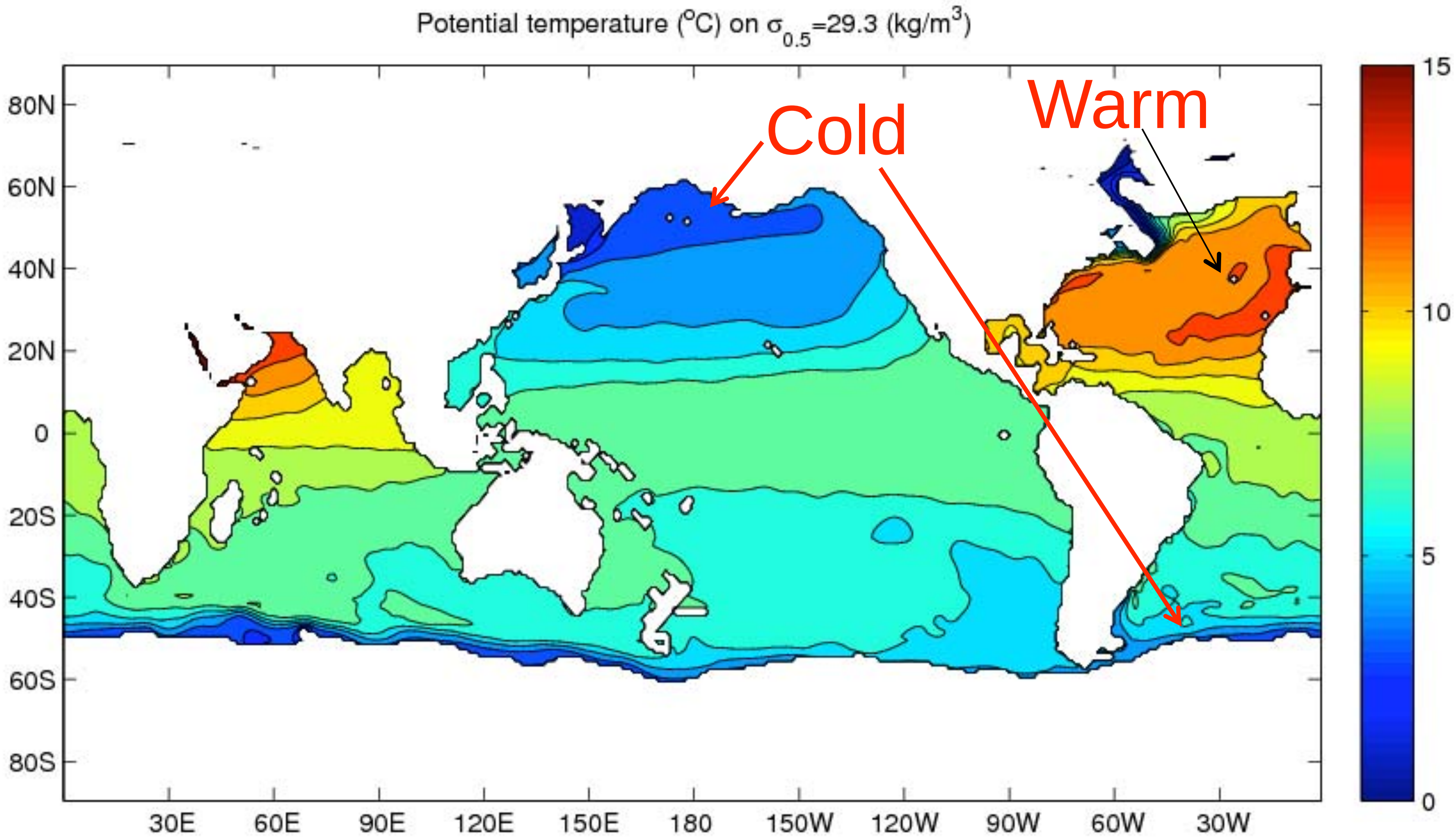
Elasticity along 179.5W



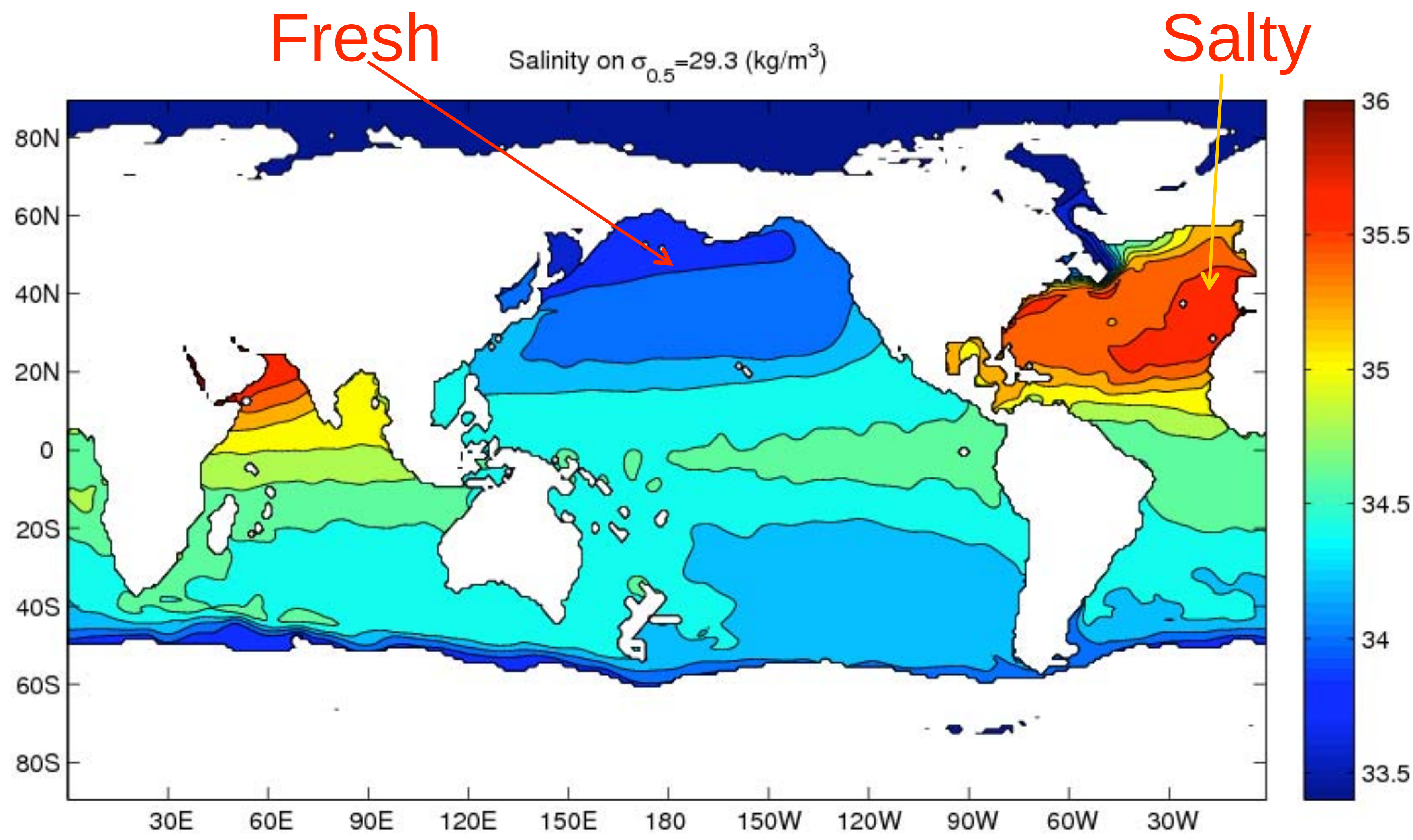
Depth of $\sigma_{0.5} = 29.3(kg/m^3)$



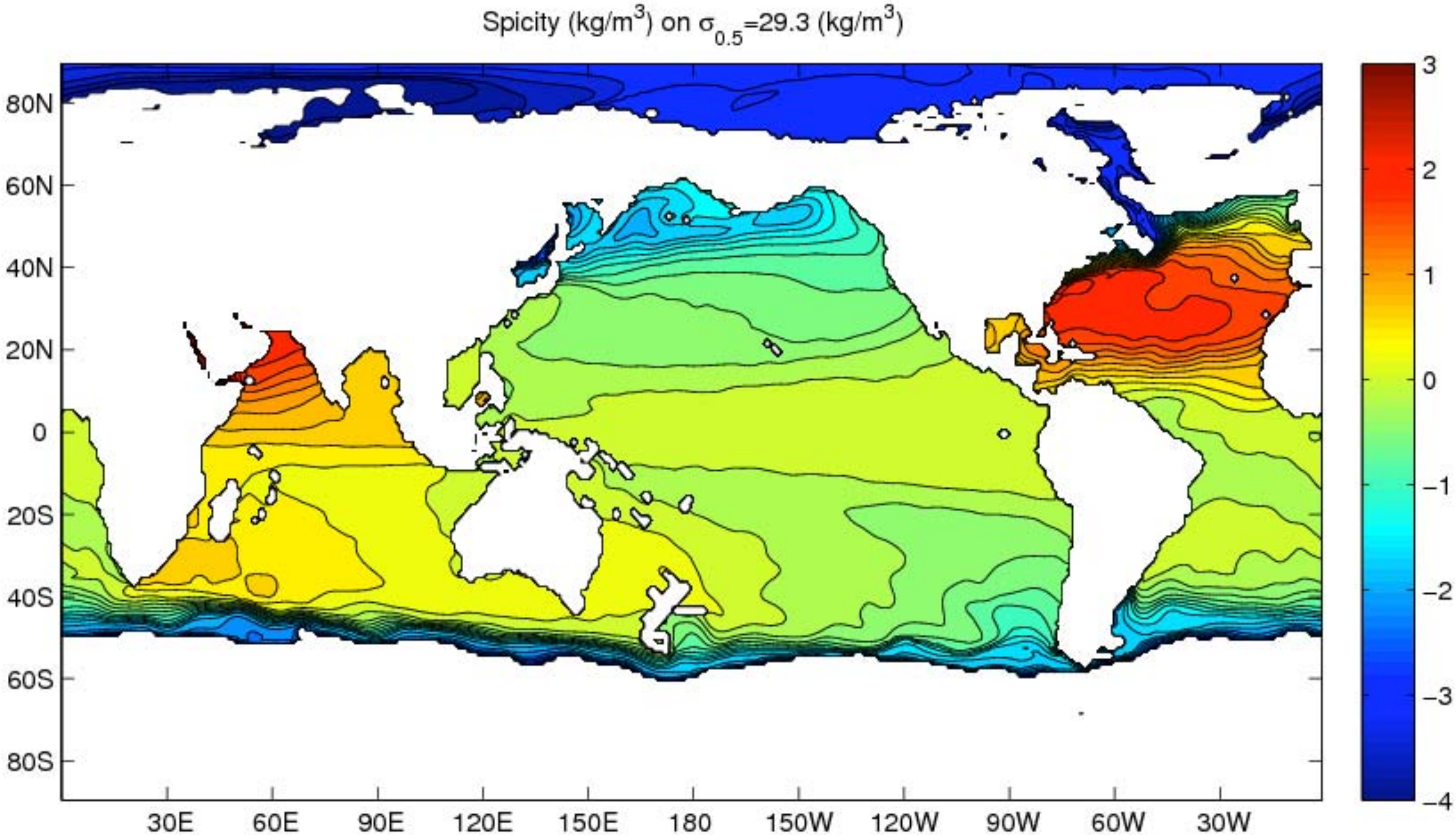
Potential temperature on $\sigma_{0.5} = 29.3(kg/m^3)$



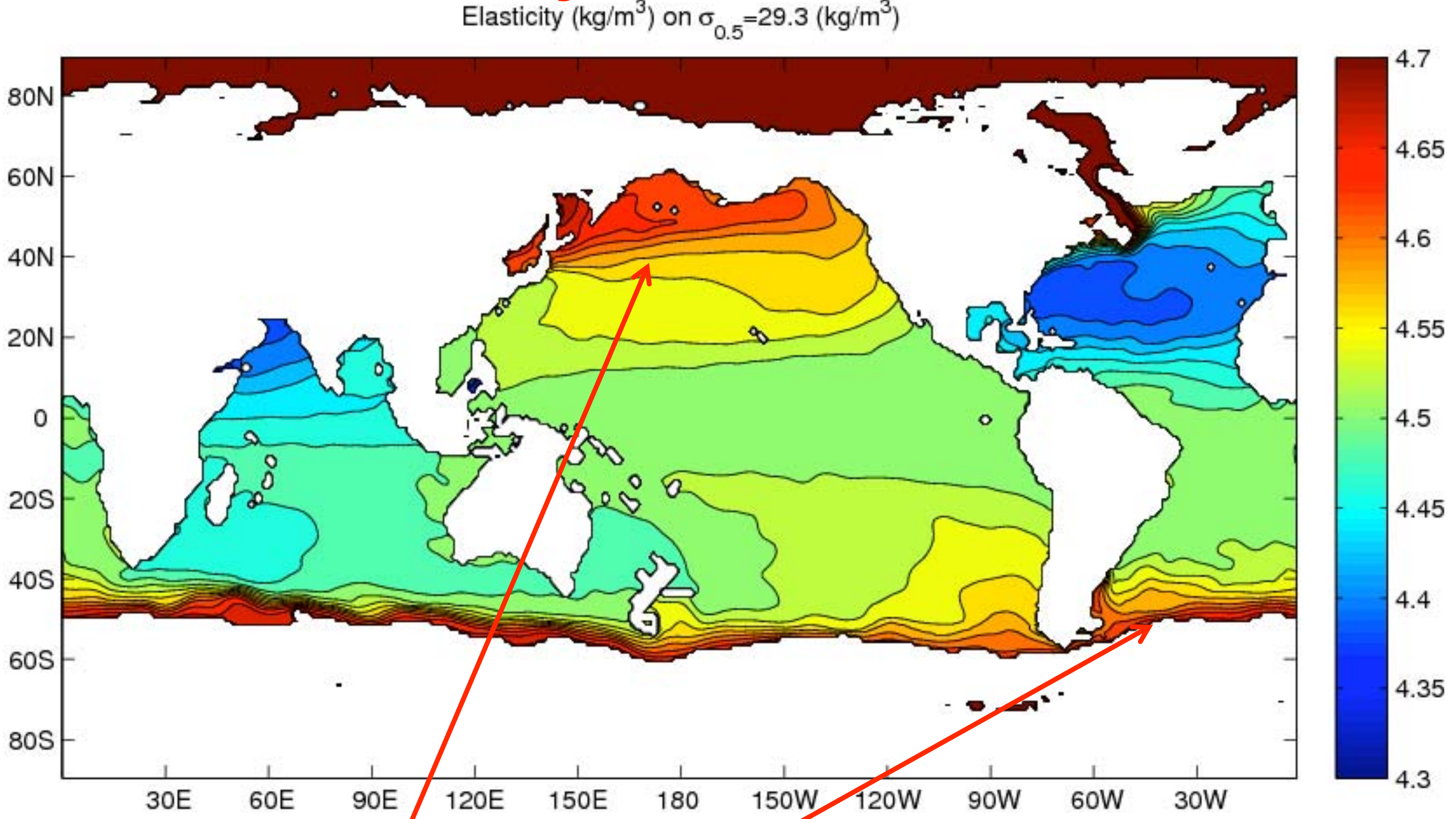
Salinity on $\sigma_{0.5} = 29.3(kg/m^3)$



Spicity on $\sigma_{0.5} = 29.3 (kg/m^3)$



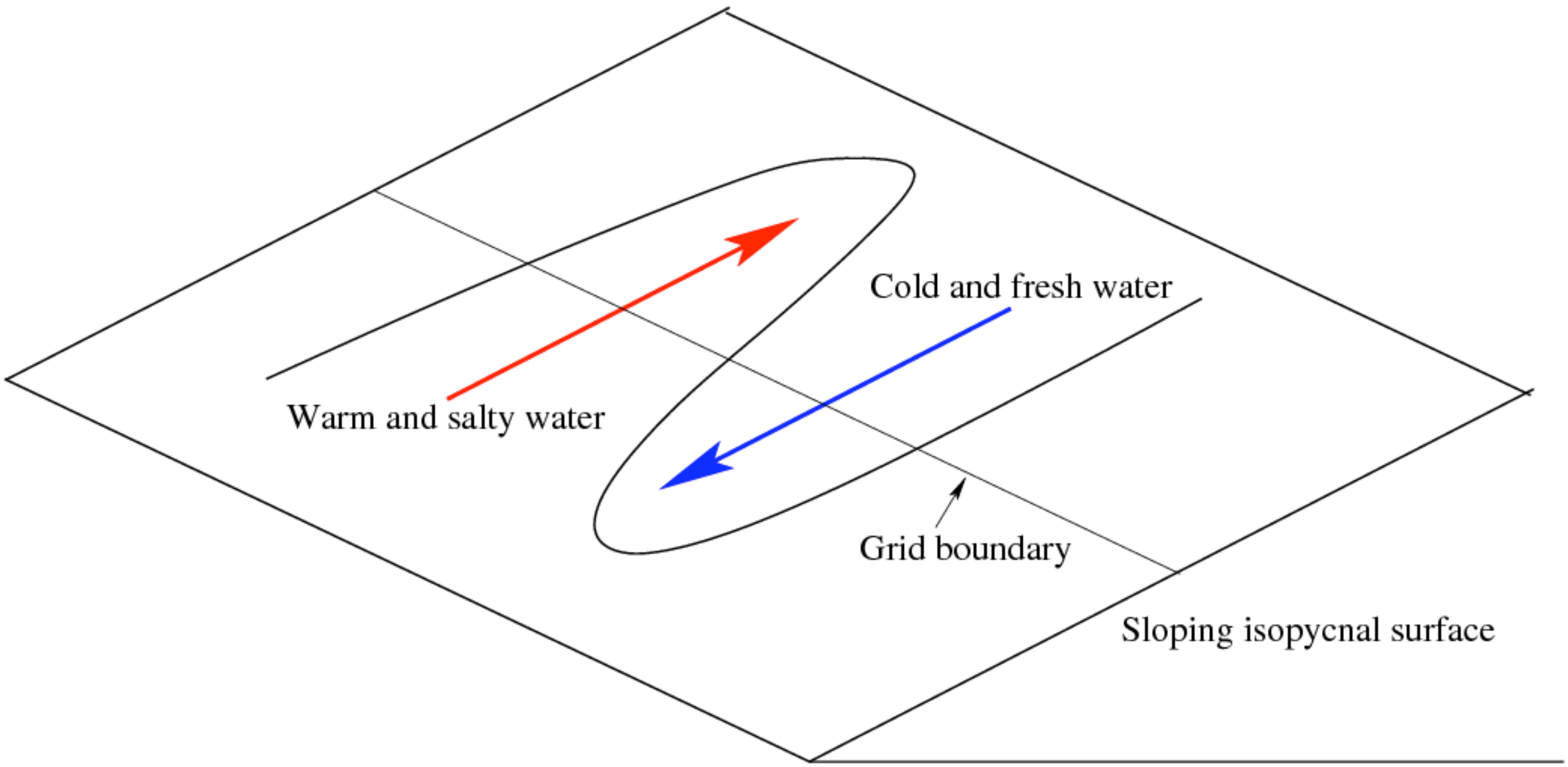
Elasticity on $\sigma_{0.5} = 29.3(kg/m^3)$



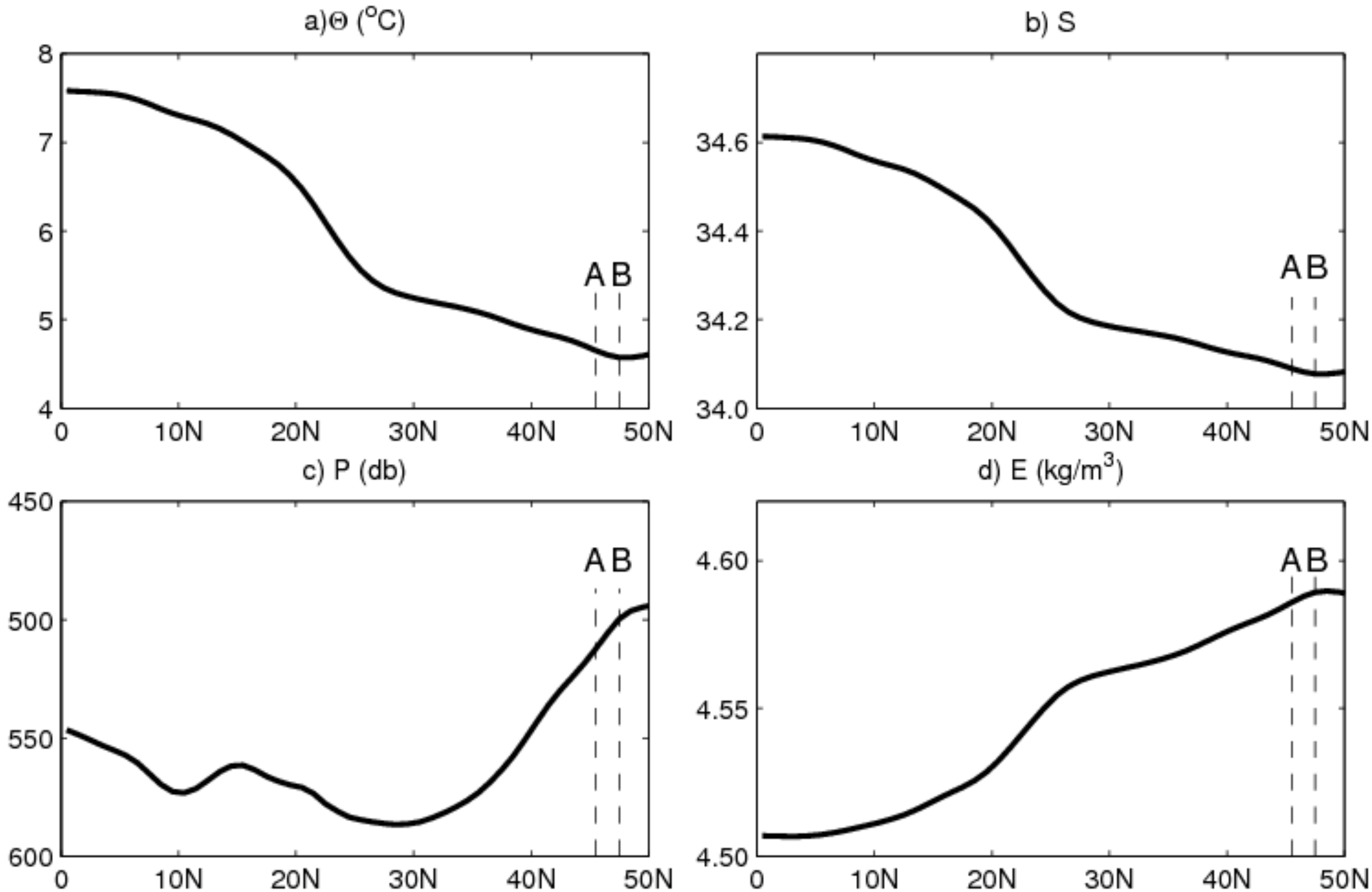
Meaning of the high gradient?

Pole-pole difference in Elasticity controls the bottom water formation

On eddy scales: Can perturbations on isopycnal surfaces grow ?



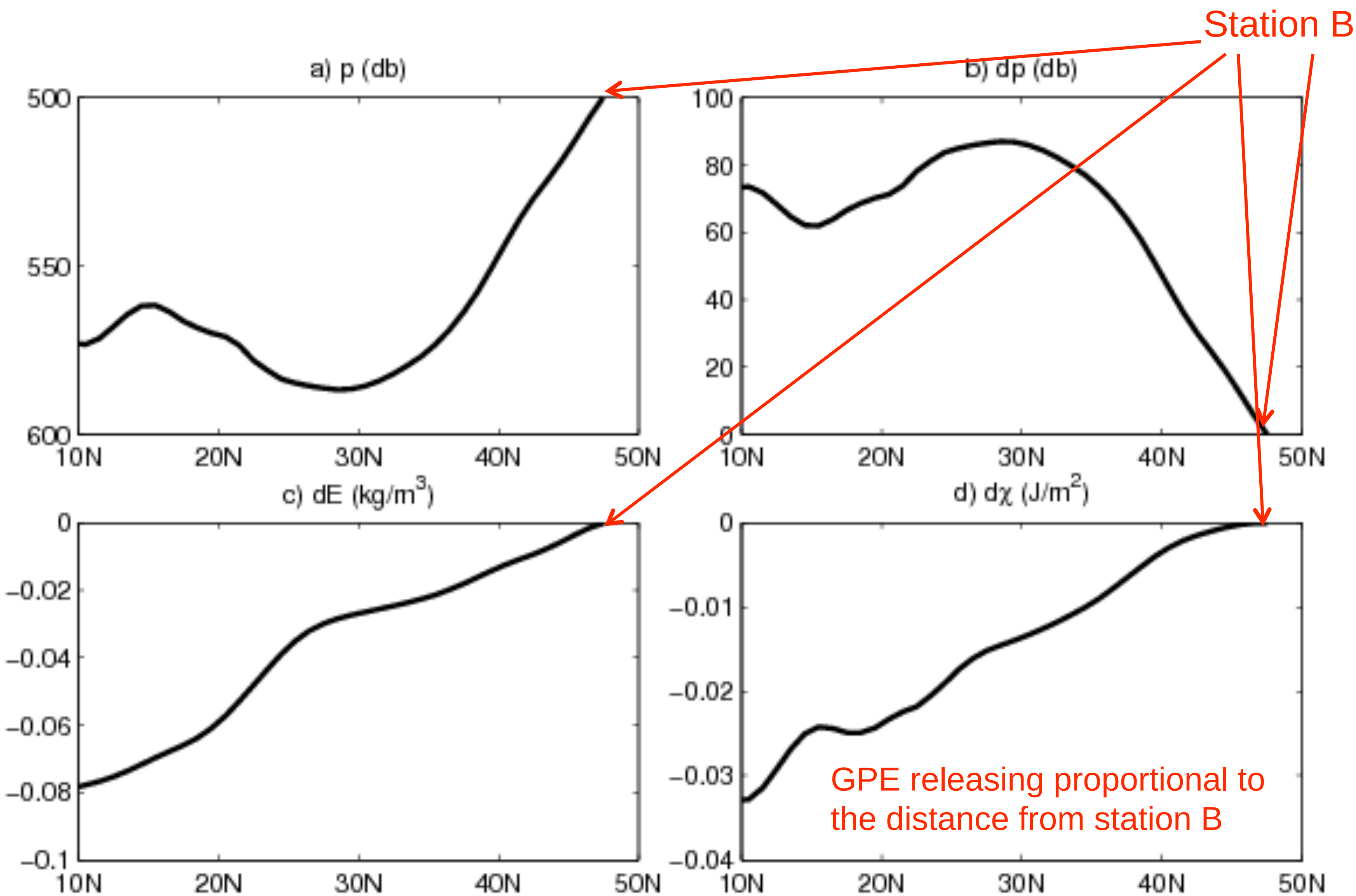
A meridional section along 129.5W



Water mass properties at station A & B

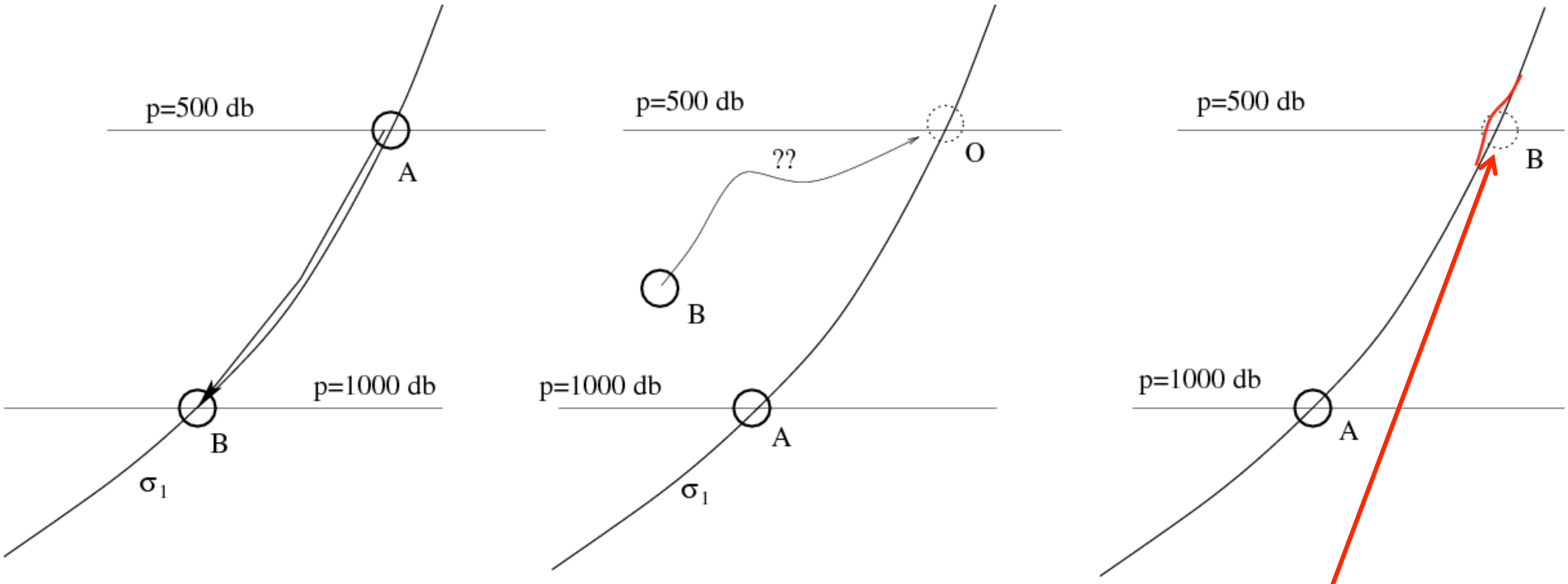
Station	Θ (°C)	S	p (db)	ΔE (kg/m^3)	$\Delta \pi$ (kg/m^3)
A (45.5N)	4.6495	34.098	512.45	4.5858	-0.3688
B (47.5N)	4.5717	34.077	499.75	4.5892	-0.6297
Difference(B-A)	-0.0378	-0.012	-12.70	0.0034	-0.2409

Amplitude of perturbations



Isopycnal exchange is no free lunch

To count GPE changes, at least two parcels should be included



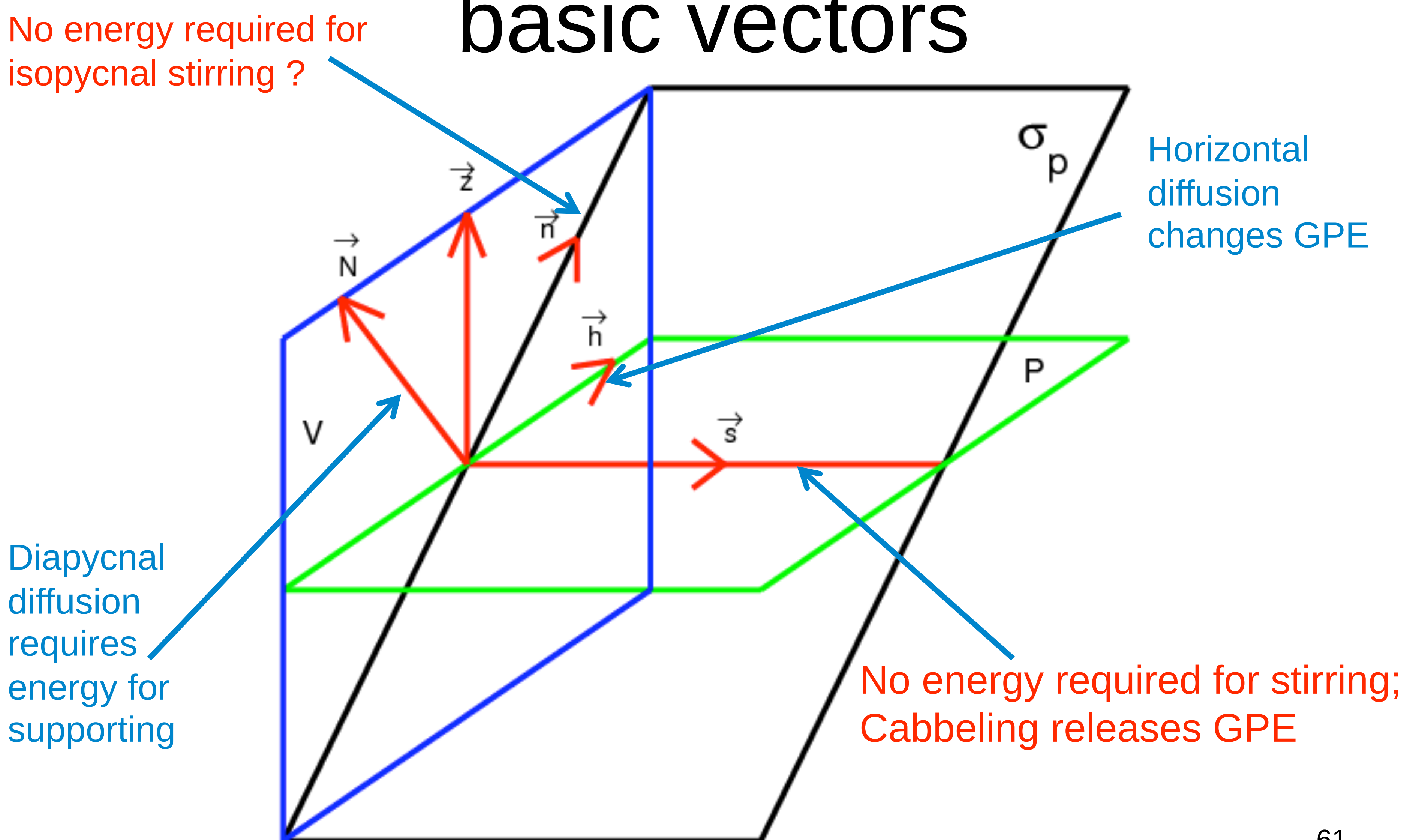
a) Two parcels before switch

b) Parcel positions after moving A

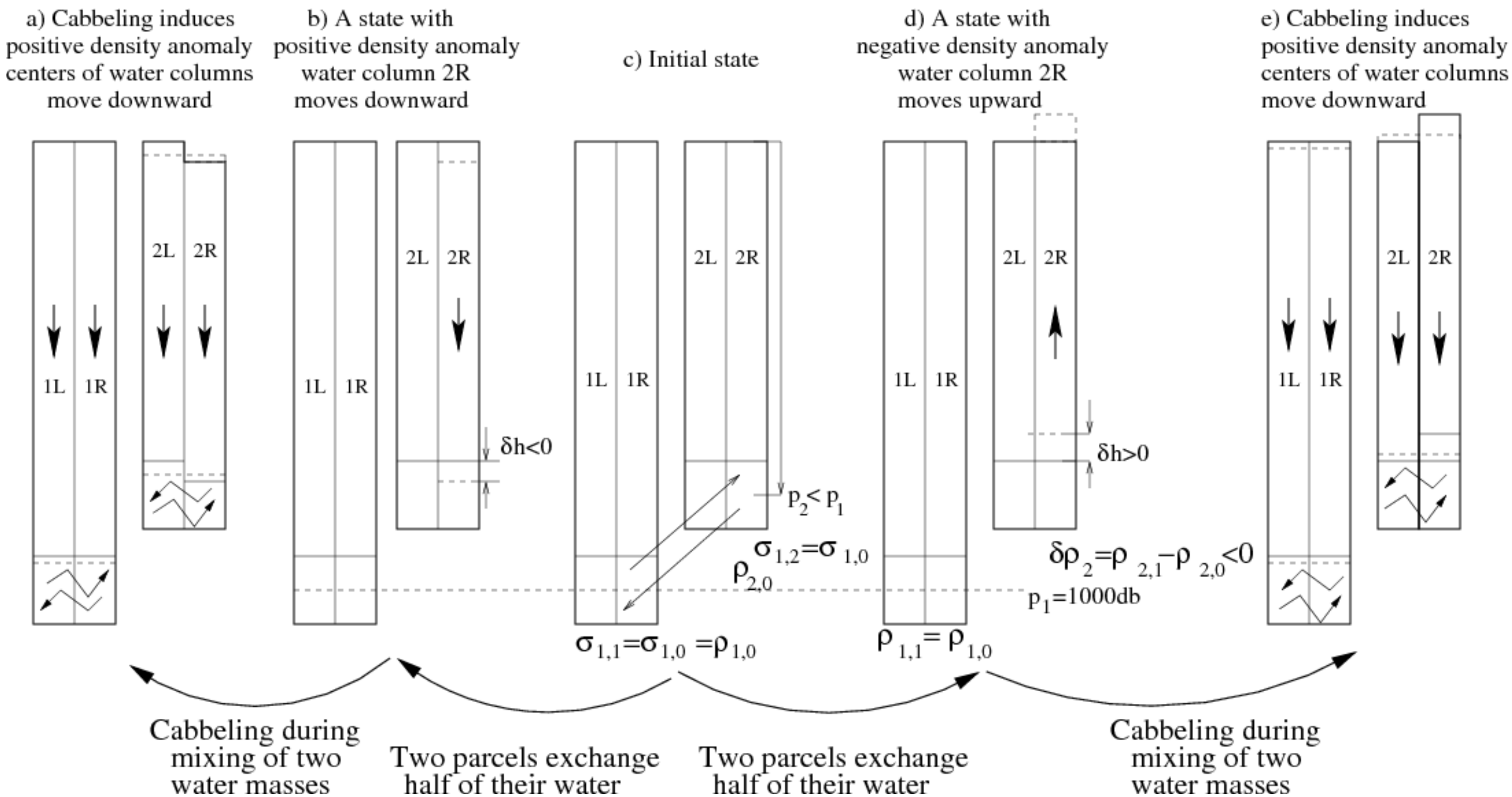
c) Density surface adjusted

Isopycnal surface may be distorted after water parcel exchange

Sketch of three surfaces and basic vectors



Exchange & mixing of two water parcels



Optimization of mixing energy for isopycnal mixing

- The optimal wedge of mixing

  It is defined by the potential density surface and the adiabatic density surface

- Within the mixing wedge, the mixing energy is optimized:

  1) When energy is released, it is maximal

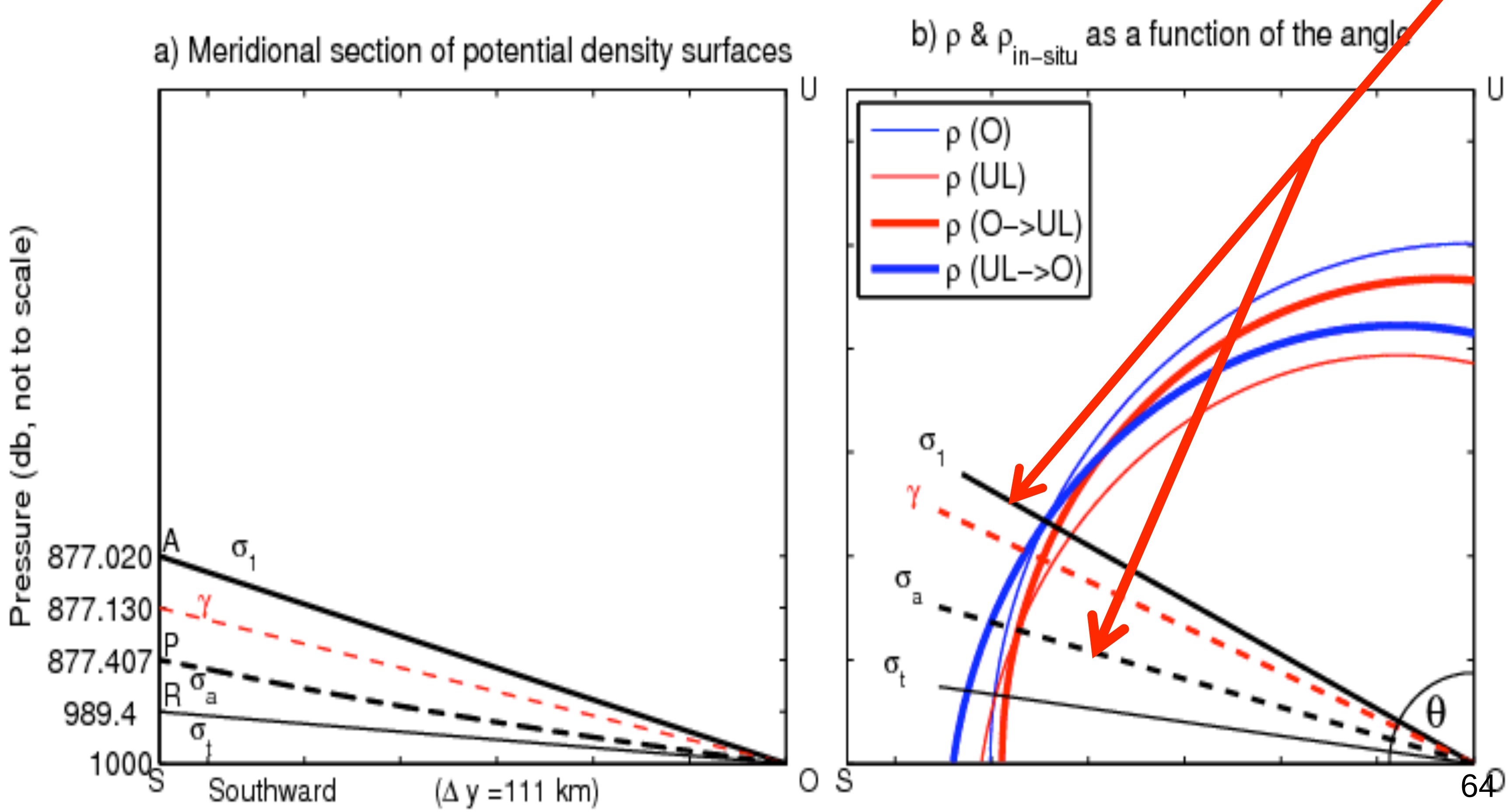
  2) When energy is required, it is minimal

- Using the mean local isopycnal slope is adequate for isopycnal mixing

A meridional section through 30.5^{o}W at 49.5^{o}S and 48.5^{o}S

(WOA01)

Within this wedge, GPE is released after exchange

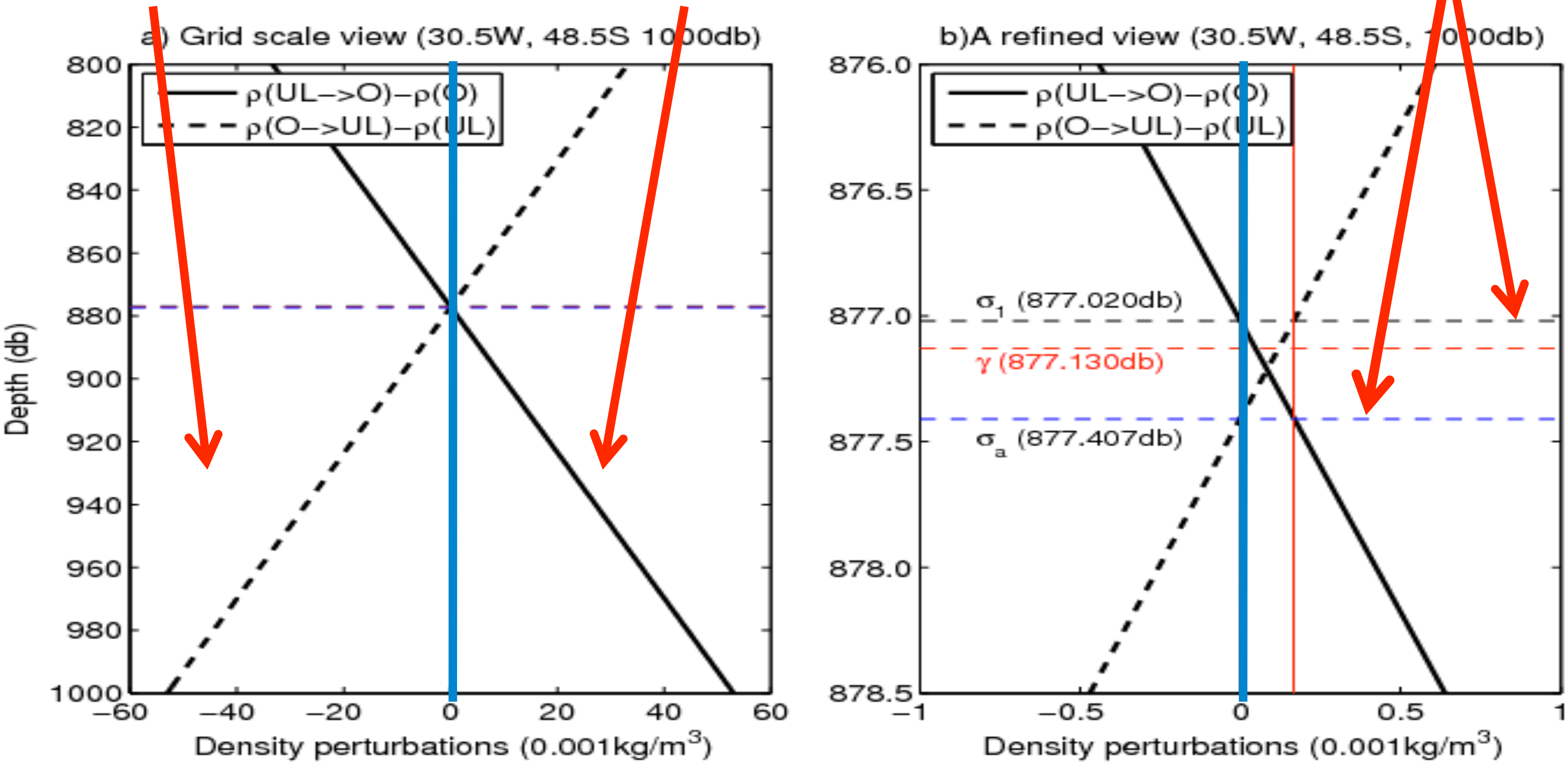


A meridional section through 30.5^{o}W at 49.5^{o}S and 48.5^{o}S

Density declines
→GPE is increased

Density increases
→GPE is released

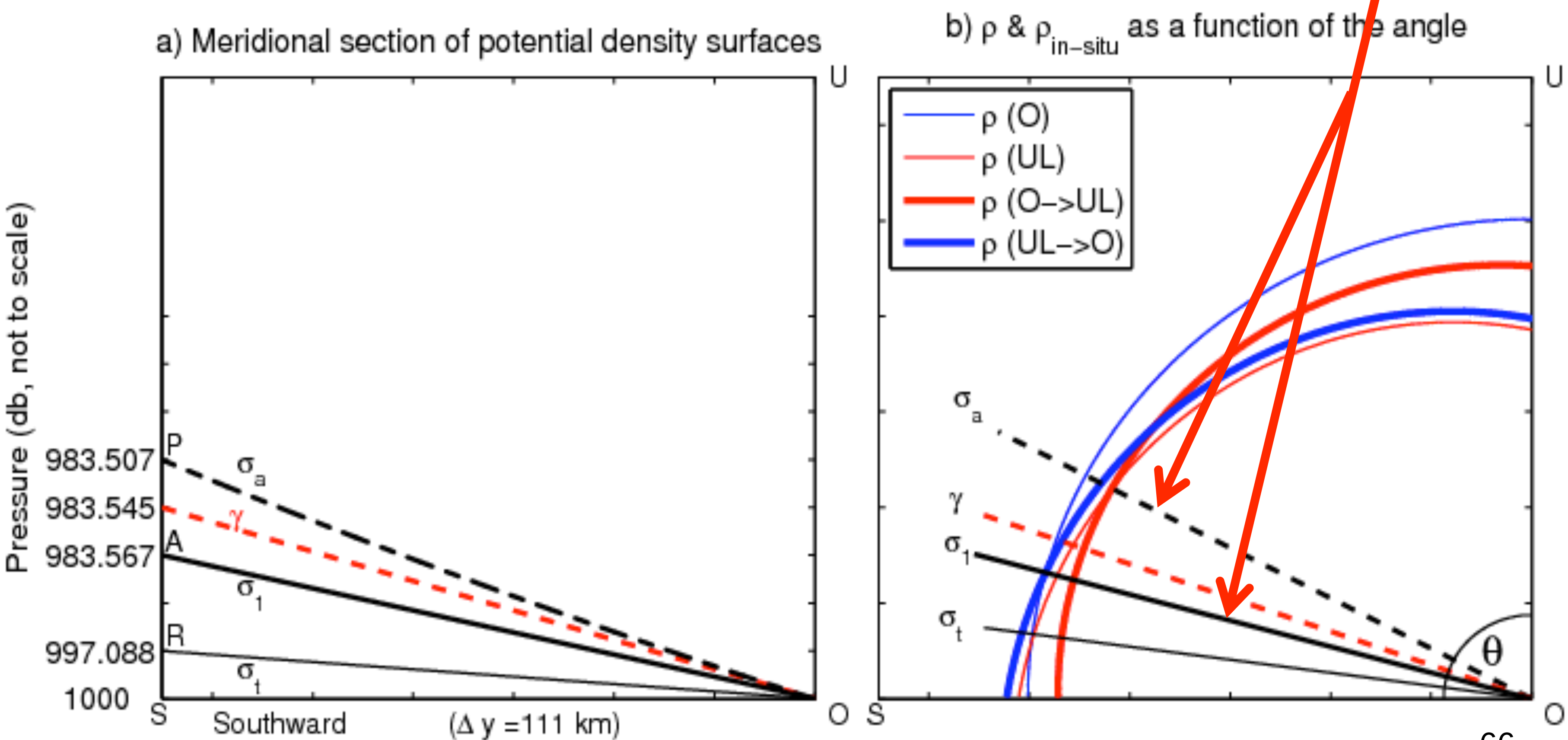
Within this wedge, GPE is released after exchange



A **meridional section through 149.5°E at 19.5°N and 20.5°N**

(WOA01)

Within this wedge, GPE is increased after exchange
→mechanical energy is required for supporting

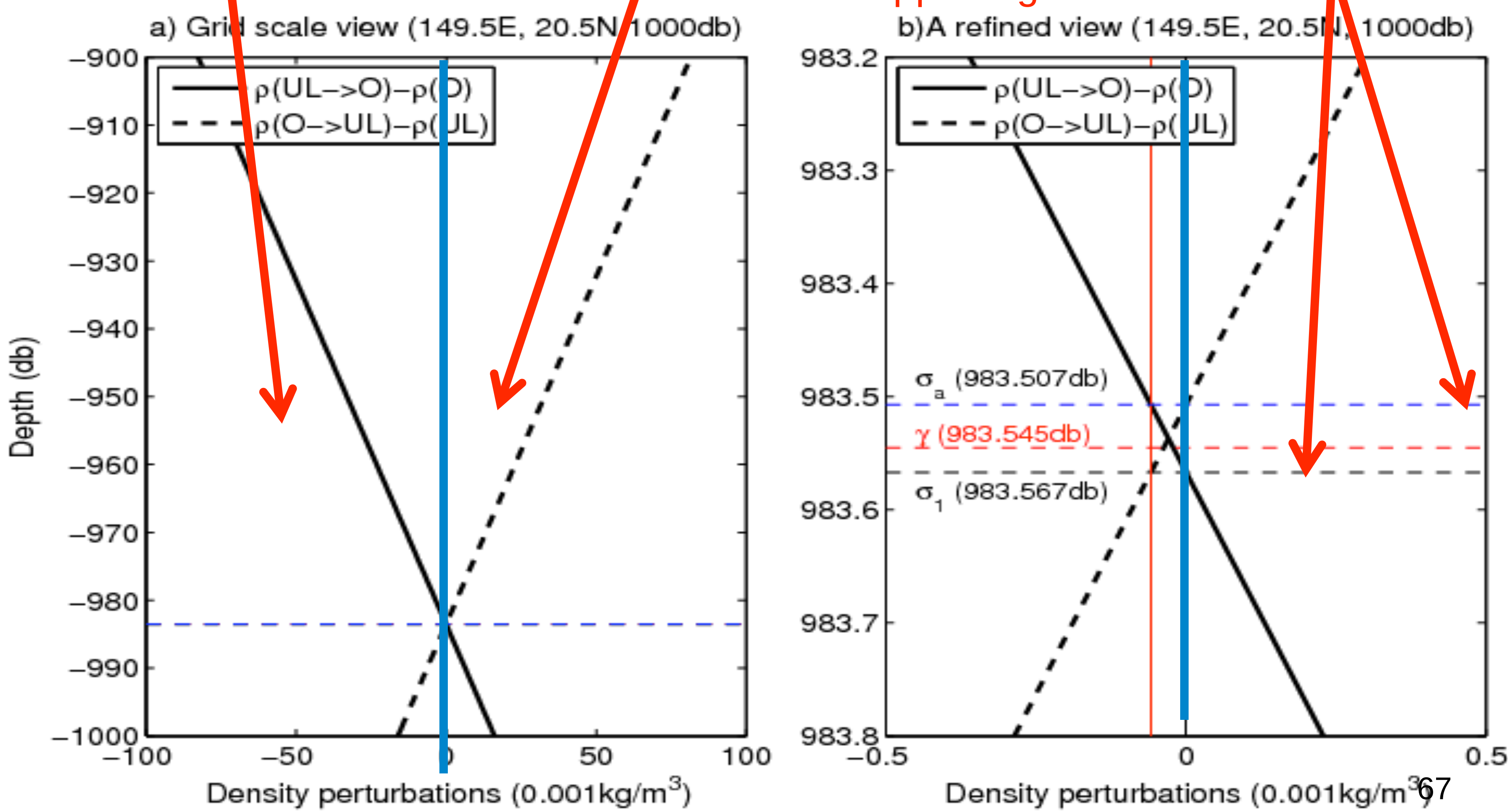


A meridional section through 149.5ºE at 19.5ºN and 20.5ºN

Density declines
→GPE is increased

Density increases
→GPE is released

Within this wedge, GPE is increased after exchange
→mechanical energy is required for supporting



Energy source/sink associated with eddy diffusion

- Energy released during diffusion cannot be very efficiently used to sustaining diffusion when energy is required.
- For the station at (30.5W, 48.5S), exchange within this wedge leads to release of GPE.
- Exchanging outside this wedge requires mechanical energy for supporting.
- Isopycnal surface can be treated as the surface where “horizontal stirring” requires the least amount of GPE, so it is the preferred surface for “horizontal diffusion”.

Density perturbation due to exchange

By definition of potential density

$$\rho(O) = \rho(A \to O)$$

$$\rho(O \to A) = \rho(O) + \rho_0 K_\eta (S_O, \Theta_O, p_O)\delta p$$

$$\rho(A \to O) = \rho(A) - \rho_0 K_\eta (S_A, \Theta_A, p_A)\delta p$$

$$\delta p = 987.578 - 1000 = -12.422\, db < 0$$

$$\begin{aligned}
\rho(O \to A) - \rho(A) &= -\left[E(S_A, \Theta_A, P_A) - E(S_O, \Theta_O, p_O)\right]\delta p \\
&= -\left[\frac{\partial E}{\partial \Theta}\delta\Theta + \frac{\partial E}{\partial S}\delta S + \frac{\partial E}{\partial p}\delta p\right]\delta p \\
&= -\left[\frac{\partial E}{\partial \Theta}\frac{\partial \Theta}{\partial p}\bigg|_{\vec{n}} + \frac{\partial E}{\partial S}\frac{\partial S}{\partial p}\bigg|_{\vec{n}} + \frac{\partial E}{\partial p}\bigg|_{\vec{n}}\right]\delta p^2
\end{aligned}$$

Non-homogeneous and anisotropic horizontal diffusivity

GPE change for a control volume

$$\delta\chi = p\delta hA = p\Delta hAE_l\delta l^2 > 0$$

$$E_l = \left[\left.\frac{\partial E}{\partial \Theta}\frac{\partial \Theta}{\partial l}\right|_{\vec{n}} + \left.\frac{\partial E}{\partial S}\frac{\partial S}{\partial l}\right|_{\vec{n}} + \left.\frac{\partial E}{\partial p}\frac{\partial p}{\partial l}\right|_{\vec{n}} \right] \left.\frac{\partial p}{\partial l}\right|_{\vec{n}} = \nabla E \cdot \nabla p > 0$$

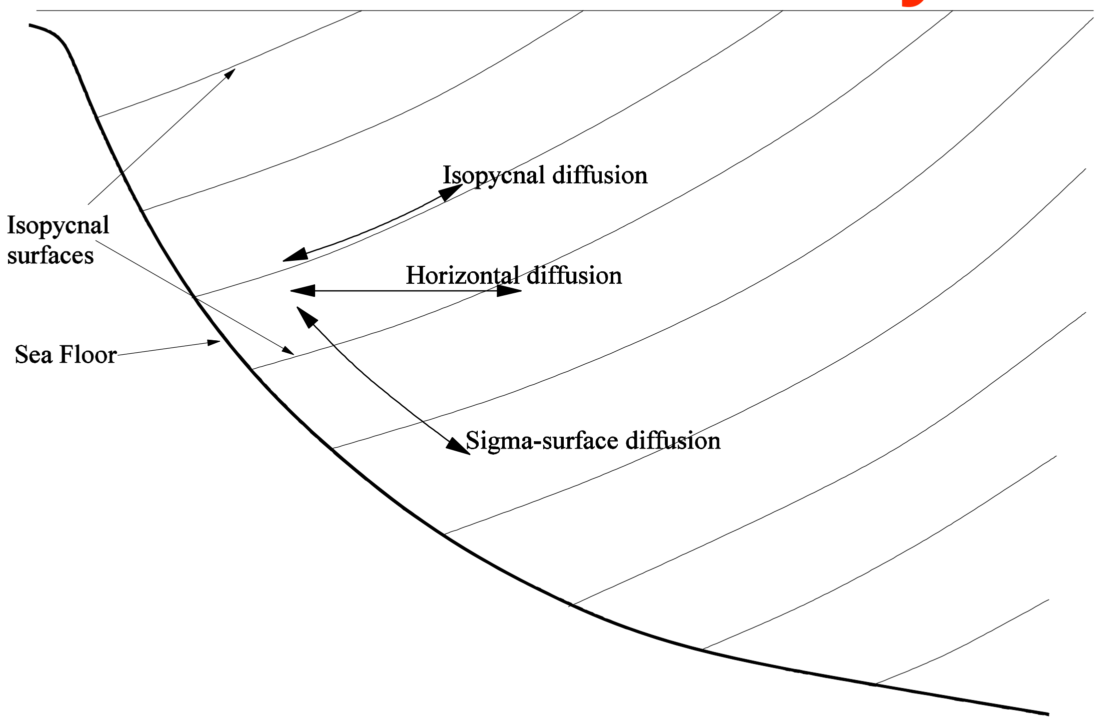
Rate of GPE change for a control volume

$$\Delta\chi / \Delta T = p\Delta hAE_l\delta l^2 / \Delta T$$

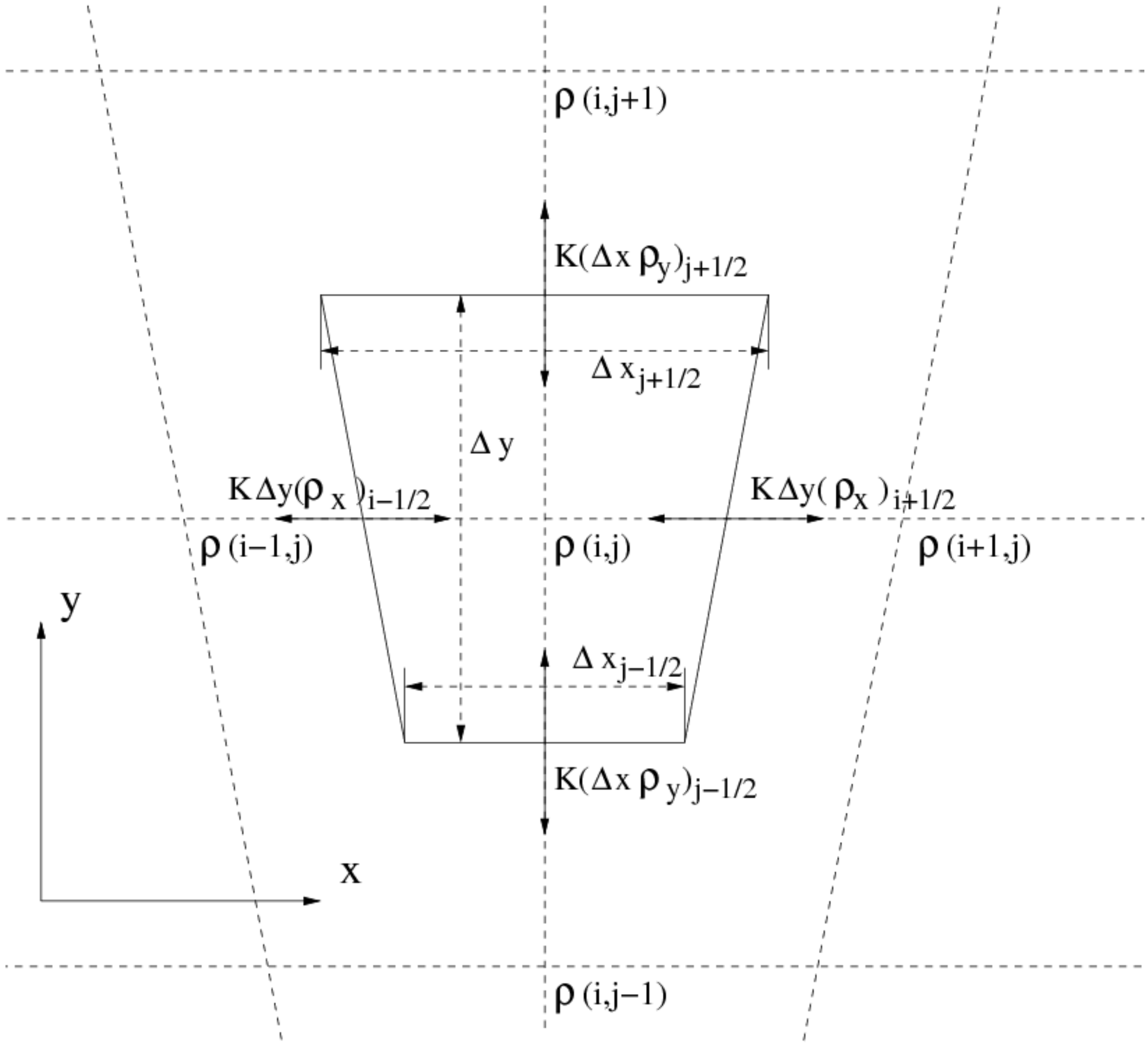
Rate of GPE change per unit volume

$$\frac{1}{A\delta h}\frac{\delta\chi}{\Delta T} = K_{H,n}p_S E_l$$

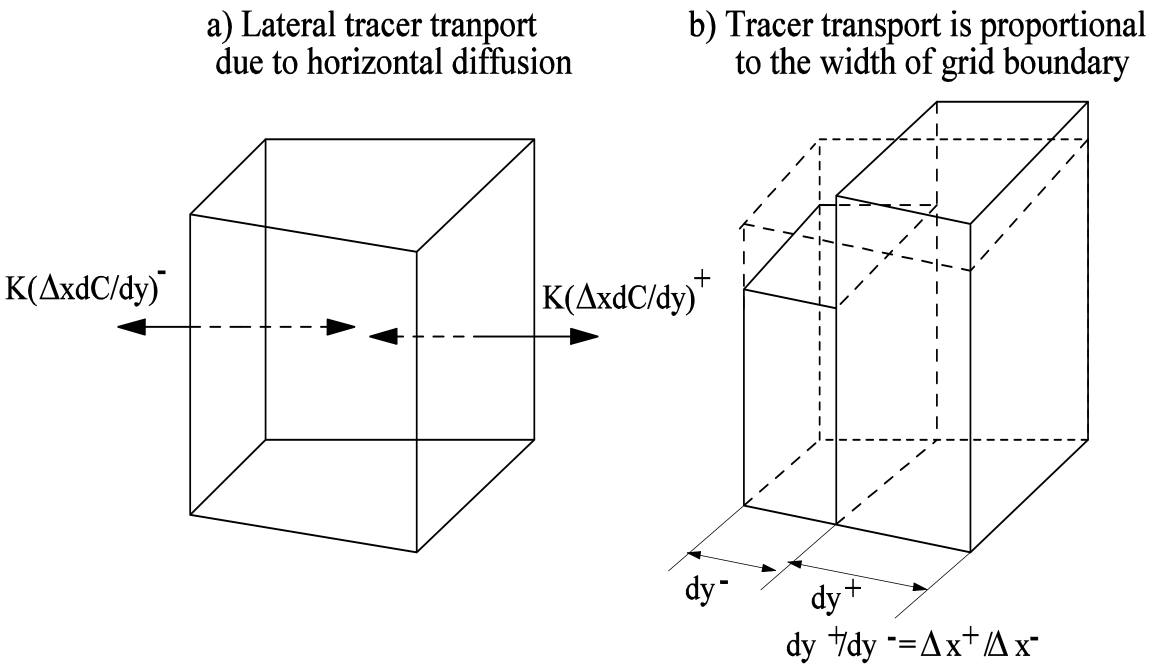
Isopycnal, horizontal and sigma-surface eddy diffusion



Tracer transport in a spherical grid



Transport is proportional to boundary width



GPE change due to horizontal stirring

$$\Delta x_j \Delta y \frac{1}{\rho_{i,j,k}} \frac{\partial \rho_{i,j,k}}{\partial t} = K\left[\frac{\Delta y}{\Delta x_j}\left(\frac{\rho_{i+1,j,k}}{\rho_{i,j,k}} - 1\right) + \frac{\Delta y}{\Delta x_j}\left(\frac{\rho_{i-1,j,k}}{\rho_{i,j,k}} - 1\right) + \frac{\Delta x_{j+1/2}}{\Delta y}\left(\frac{\rho_{i,j+1,k}}{\rho_{i,j,k}} - 1\right) + \frac{\Delta x_{j-1/2}}{\Delta y}\left(\frac{\rho_{i,j-1,k}}{\rho_{i,j,k}} - 1\right)\right]$$

Changes in layer thickness and GPE:

$$\delta h_{i,j,k} = -\Delta h_{i,j,k} \delta\rho_{i,j,k} / \bar{\rho}$$

$$\delta\chi = p_k A_S \delta h_{i,j,k} = -p_k A_S \Delta h_{i,j,k} \delta\rho_{i,j,k} / \bar{\rho}$$

$$\dot{\chi}_{stir}^{hori} = K p_k \Delta h_{i,j,k}\left[\frac{\Delta y}{\Delta x_j}\left(1 - \frac{\rho_{i+1,j,k}}{\rho_{i,j,k}}\right) + \frac{\Delta y}{\Delta x_j}\left(1 - \frac{\rho_{i-1,j,k}}{\rho_{i,j,k}}\right) + \frac{\Delta x_{j+1/2}}{\Delta y}\left(1 - \frac{\rho_{i,j+1,k}}{\rho_{i,j,k}}\right) + \frac{\Delta x_{j-1/2}}{\Delta y}\left(1 - \frac{\rho_{i,j-1,k}}{\rho_{i,j,k}}\right)\right]$$

For the case of square grid:

$$\dot{\chi}_{stir}^{hori} = K p_k \Delta h_{i,j,k}\left[4 - \left(\rho_{i+1,j,k} + \rho_{i-1,j,k} + \rho_{i,j+1,k} + \rho_{i,j-1,k}\right) / \rho_{i,j,k}\right]$$

GPE change due to horizontal mixing

$$\Delta T_{i,j,k} = B\left[\frac{\Delta y}{\Delta x_j}\left(T_{i+1,j,k} - T_{i,j,k}\right) + \frac{\Delta y}{\Delta x_j}\left(T_{i-1,j,k} - T_{i,j,k}\right) + \frac{\Delta x_{j+1/2}}{\Delta y}\left(T_{i,j+1,k} - T_{i,j,k}\right) + \frac{\Delta x_{j-1/2}}{\Delta y}\left(T_{i,j-1,k} - T_{i,j,k}\right)\right]$$

We select:

$$B = \frac{K\Delta t}{\Delta x_j \Delta y}$$

$$\Delta t = \frac{\Delta x_j \Delta y}{K D_j}, D_j = \frac{\Delta y}{\Delta x_j} + \frac{\Delta y}{\Delta x_j} + \frac{\Delta x_{j+1/2}}{\Delta y} + \frac{\Delta x_{j-1/2}}{\Delta y}$$

$$\Delta T_{i,j,k} = D_j^{-1}\left[\frac{\Delta y}{\Delta x_j}\left(T_{i+1,j,k} - T_{i,j,k}\right) + \frac{\Delta y}{\Delta x_j}\left(T_{i-1,j,k} - T_{i,j,k}\right) + \frac{\Delta x_{j+1/2}}{\Delta y}\left(T_{i,j+1,k} - T_{i,j,k}\right) + \frac{\Delta x_{j-1/2}}{\Delta y}\left(T_{i,j-1,k} - T_{i,j,k}\right)\right]$$

$$\bar{T}_{i,j,k} = T_{i,j,k} + \Delta T_{i,j,k} = \left[\frac{\Delta y}{\Delta x_j} T_{i+1,j,k} + \frac{\Delta y}{\Delta x_j} T_{i-1,j,k} + \frac{\Delta x_{j+1/2}}{\Delta y} T_{i,j+1,k} + \frac{\Delta x_{j-1/2}}{\Delta y} T_{i,j-1,k}\right] D_j^{-1}$$

$$\dot{\chi}_{cabb}^{hori} = p_k\left(\bar{\rho}_{i,j,k} / \rho_{i,j,k,mix} - 1\right)\Delta h_{i,j,k}\Delta x_j \Delta y / \Delta t$$

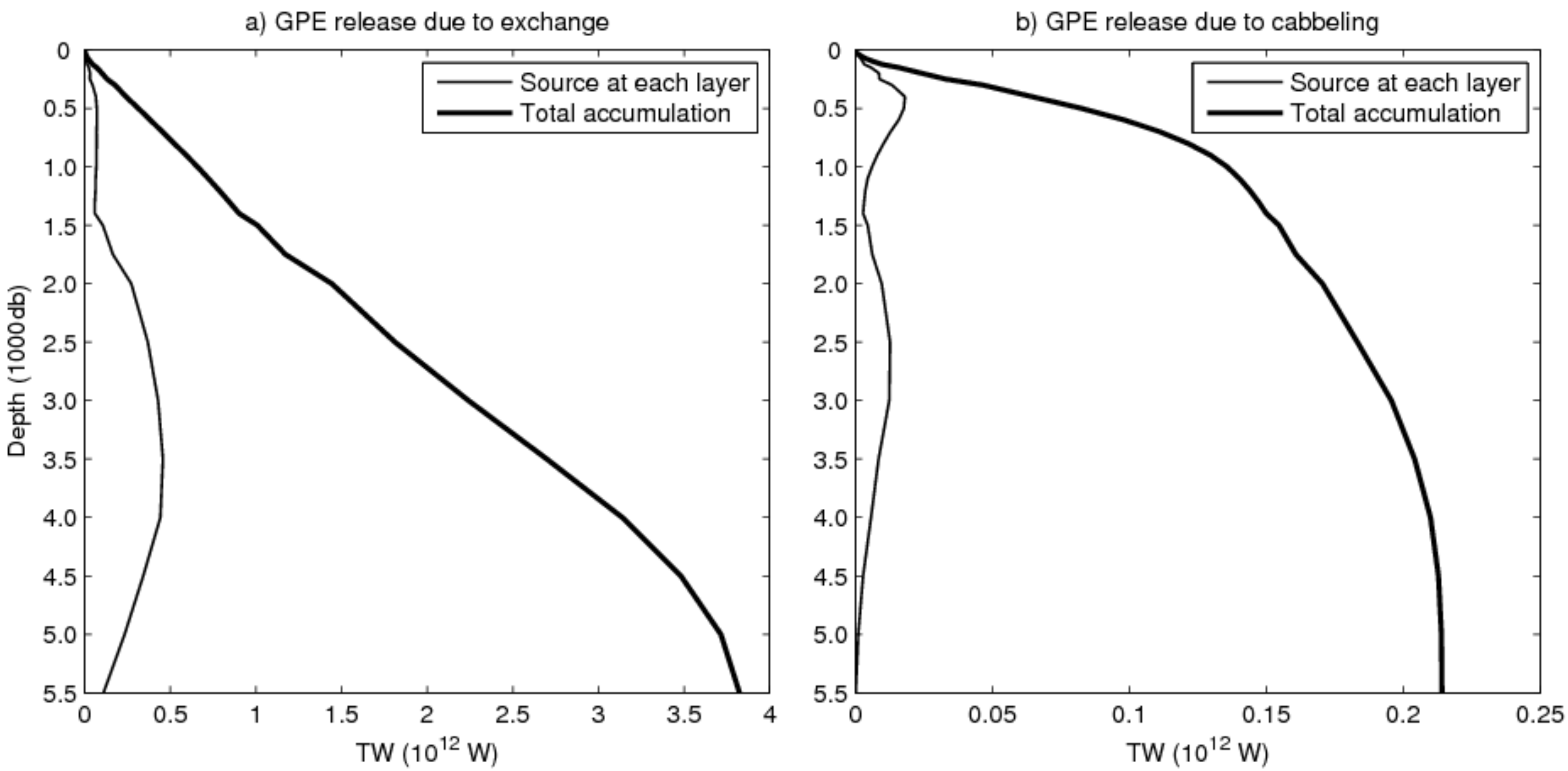
$$= K p_k\left(\bar{\rho}_{i,j,k} / \rho_{i,j,k,mix} - 1\right) D_j \Delta h_{i,j,k}$$

GPE source/sink (TW)

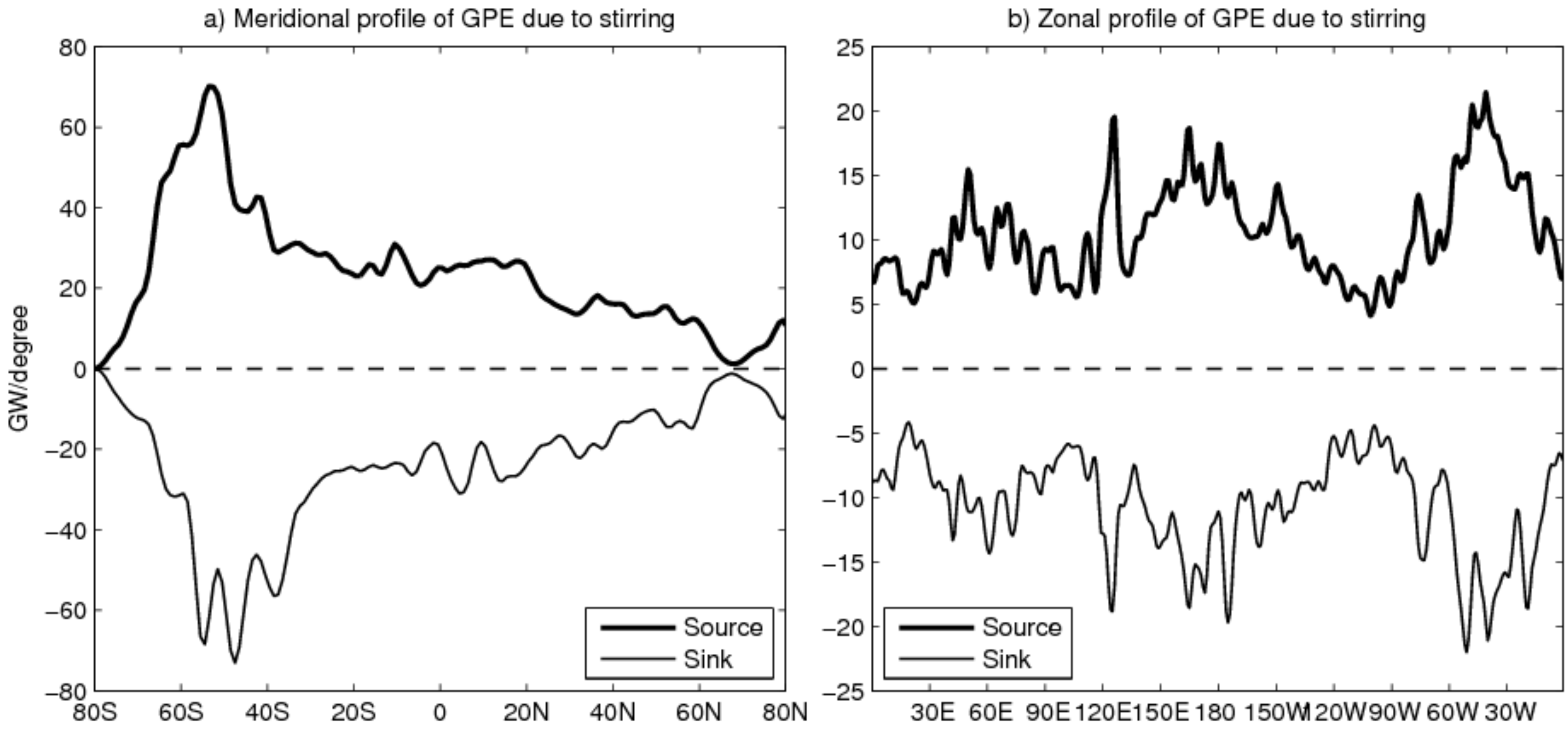
Type of eddy diffusion	Stirring		cabbeling due to mixing	Sum of the absolute values
	Source	Sink	Sink	
horizontal	3.8	-3.8	-0.21	7.8
sigma	150	-58	-17	225
Isopycnal	0.031	0.025	-0.052	0.108

Assuming K=1000m^2/s, 1x1 degree grid for the world ocean

GPE, horizontal stirring & cabbeling

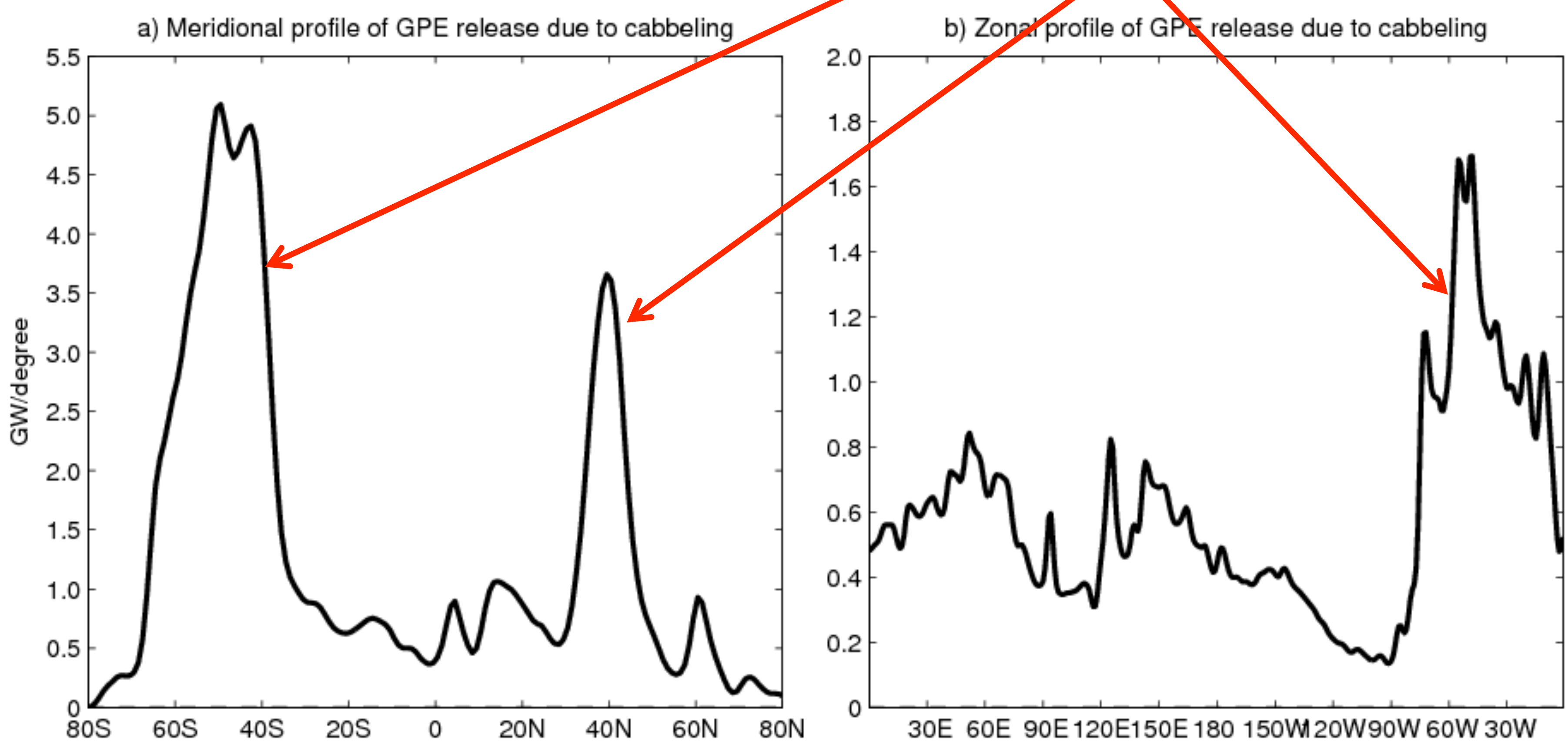


GPE source/sink due to horizontal stirring

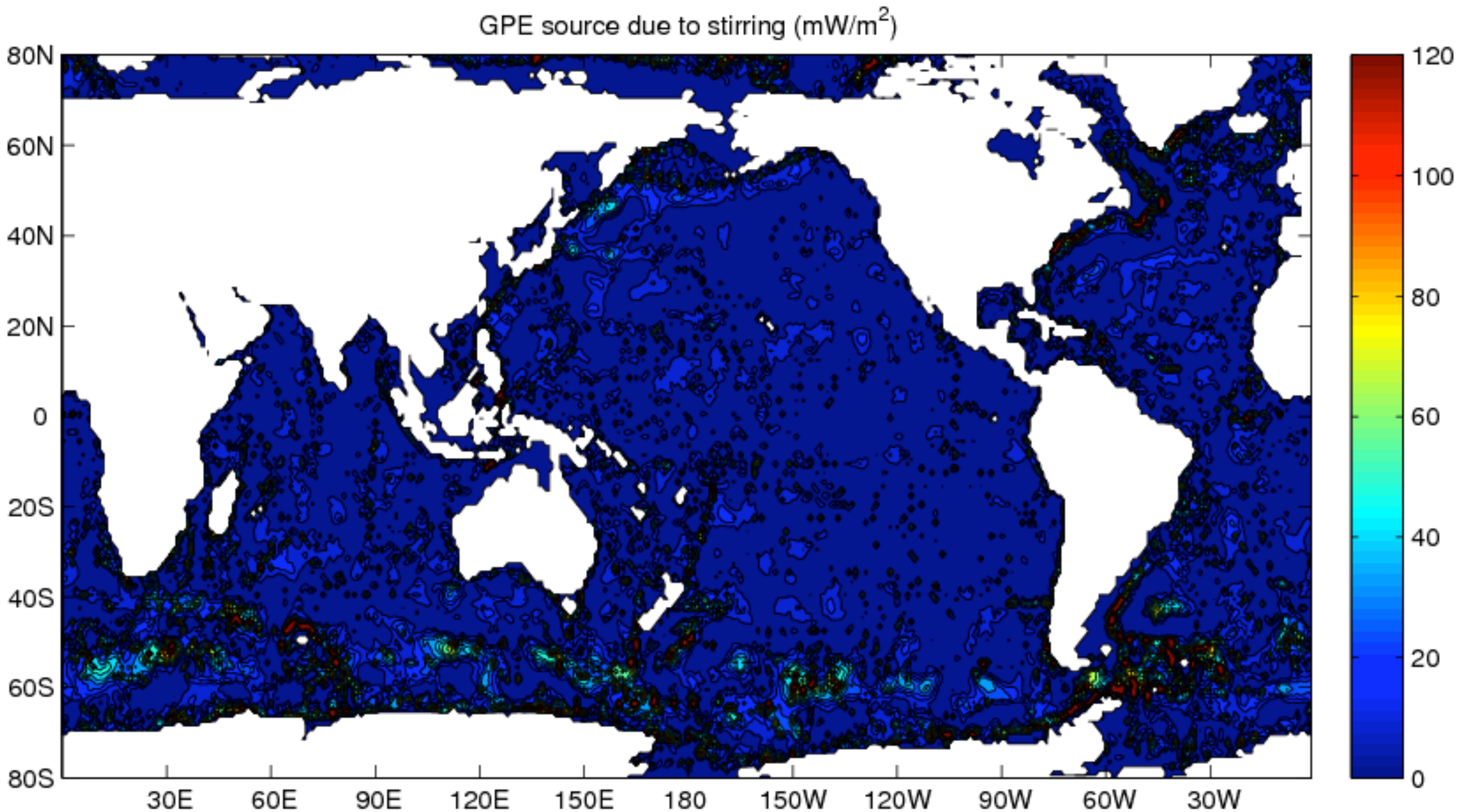


GPE due to cabbeling associated with horizontal mixing

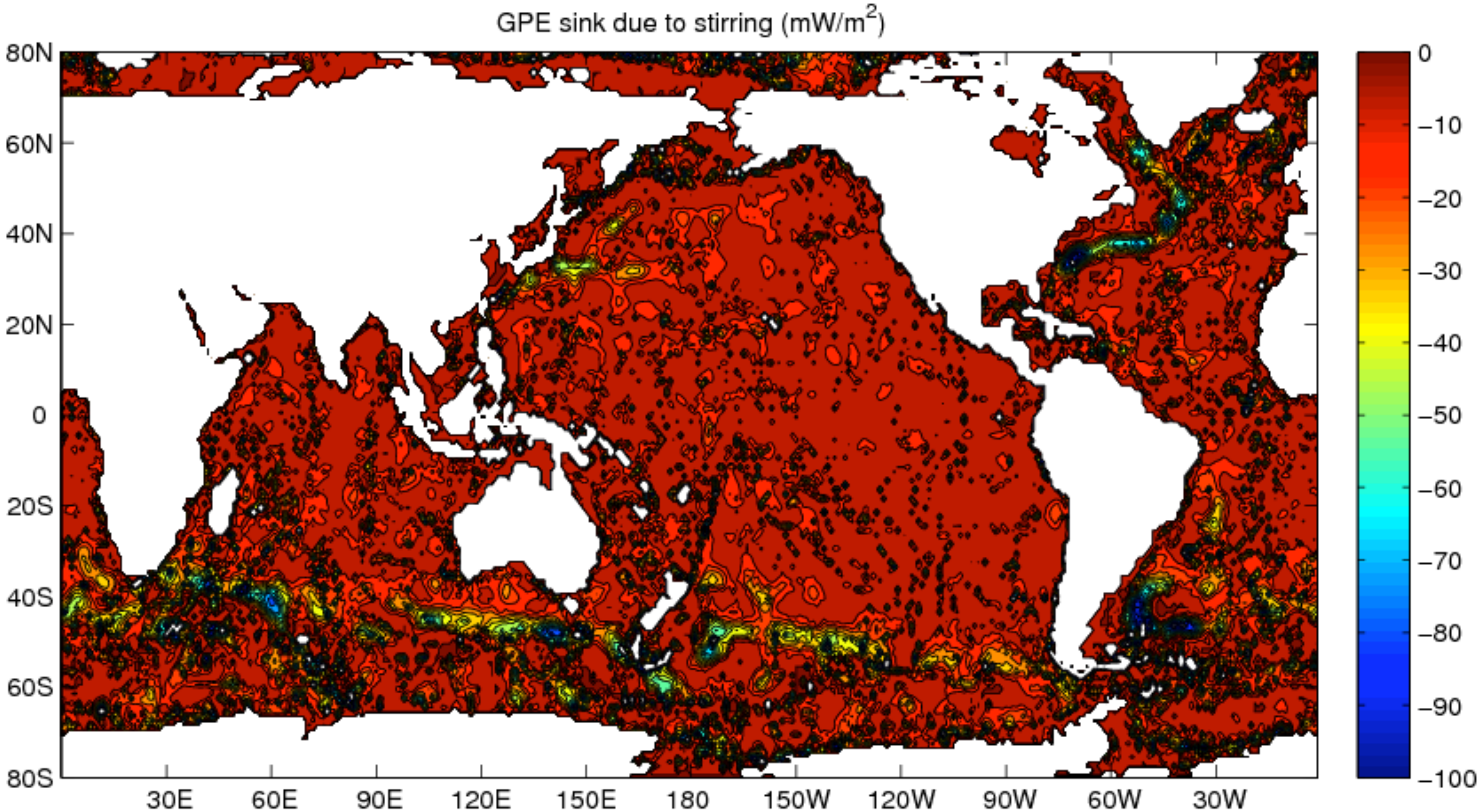
Strong fronts exist in ACC and Gulf Stream



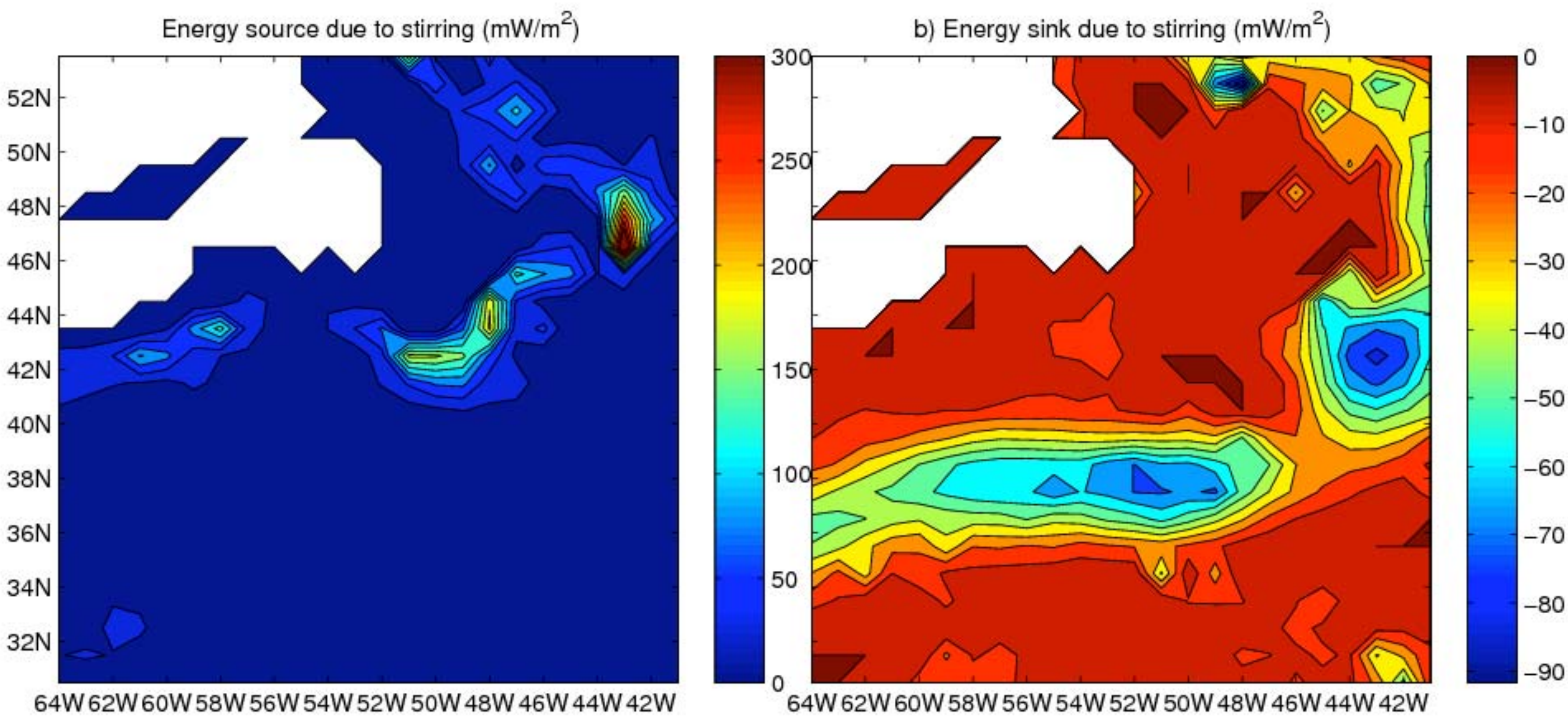
GPE source due to horizontal stirring



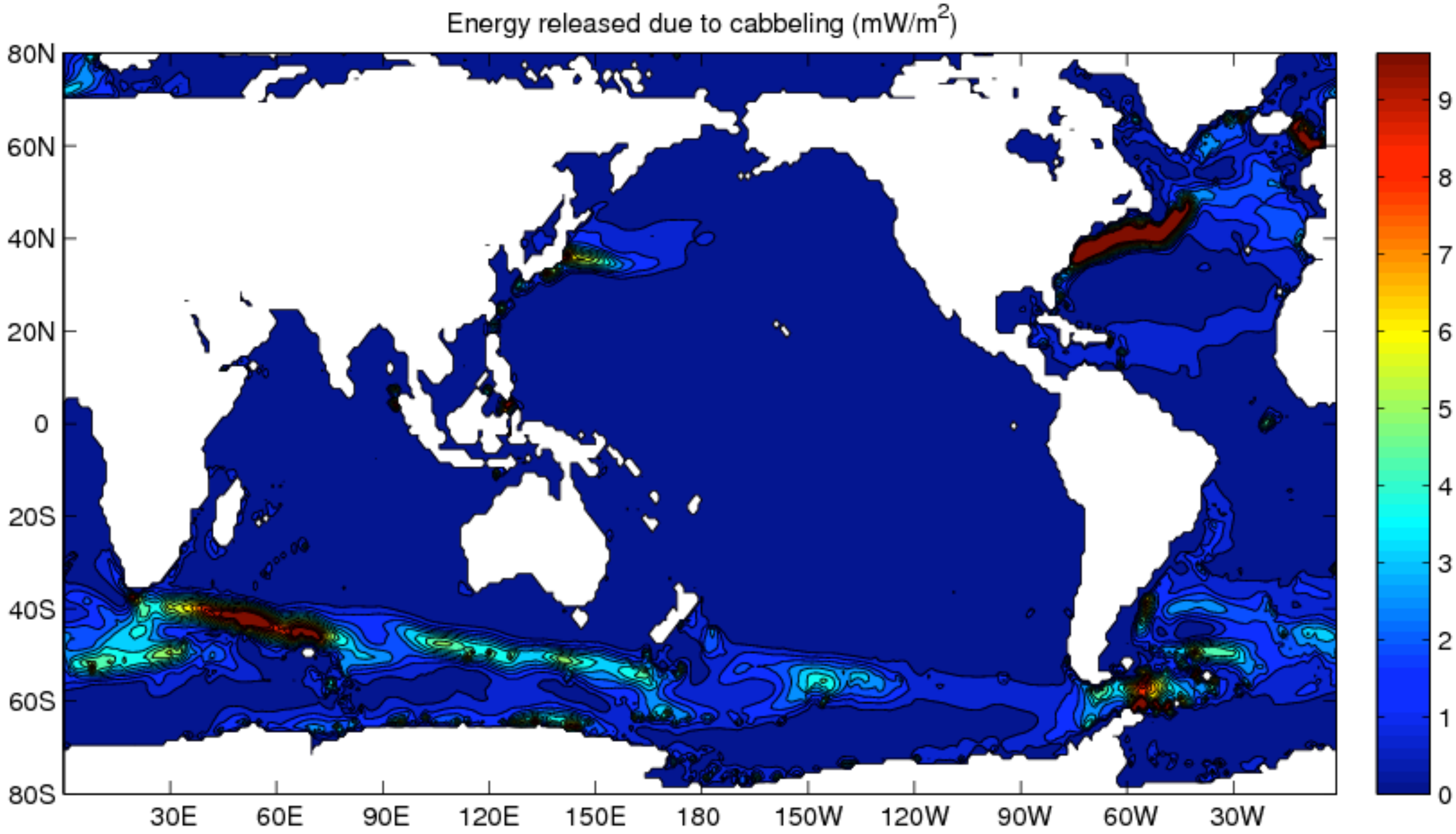
GPE sink due to horizontal stirring



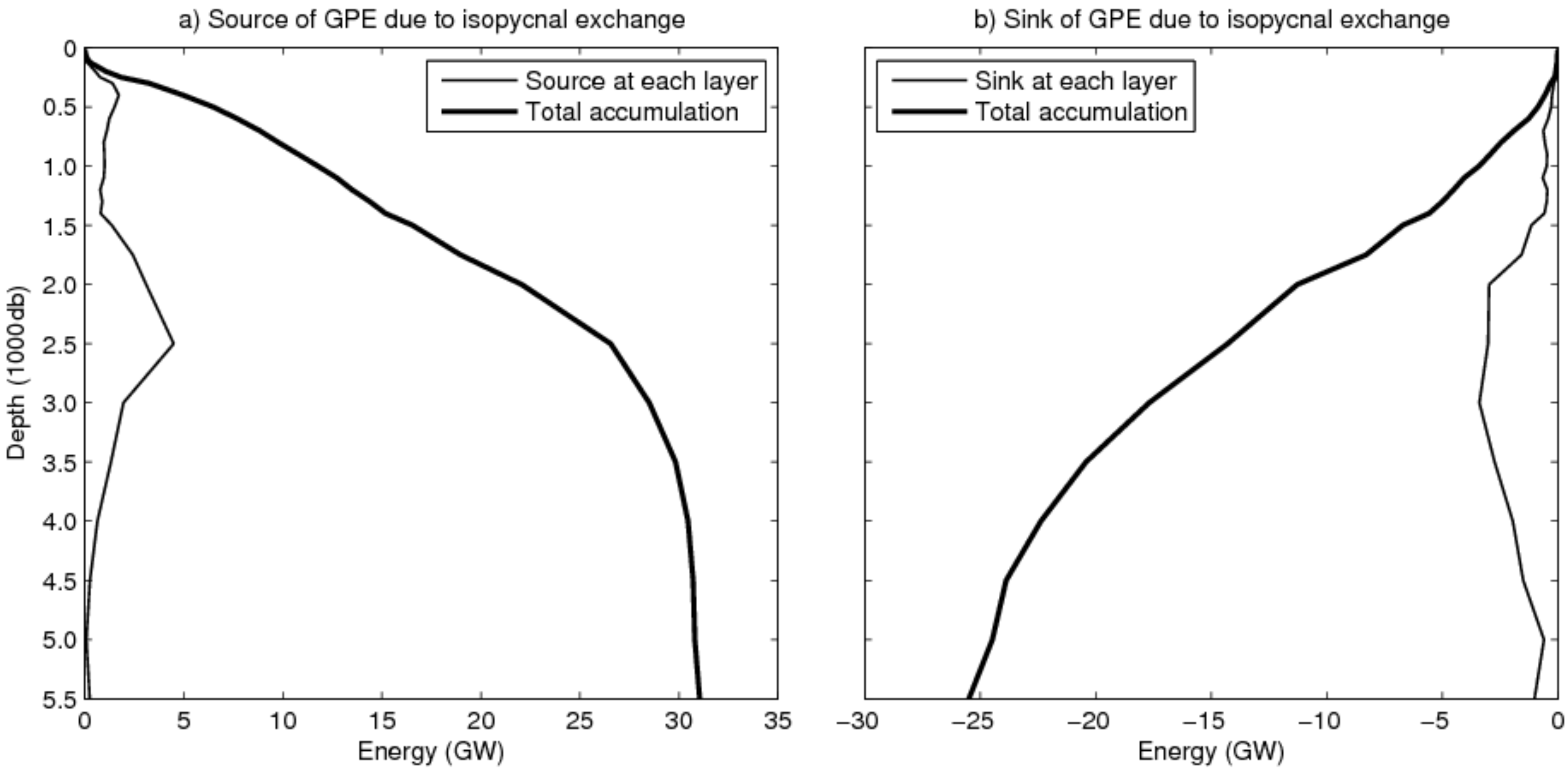
GPE sink due to horizontal stirring



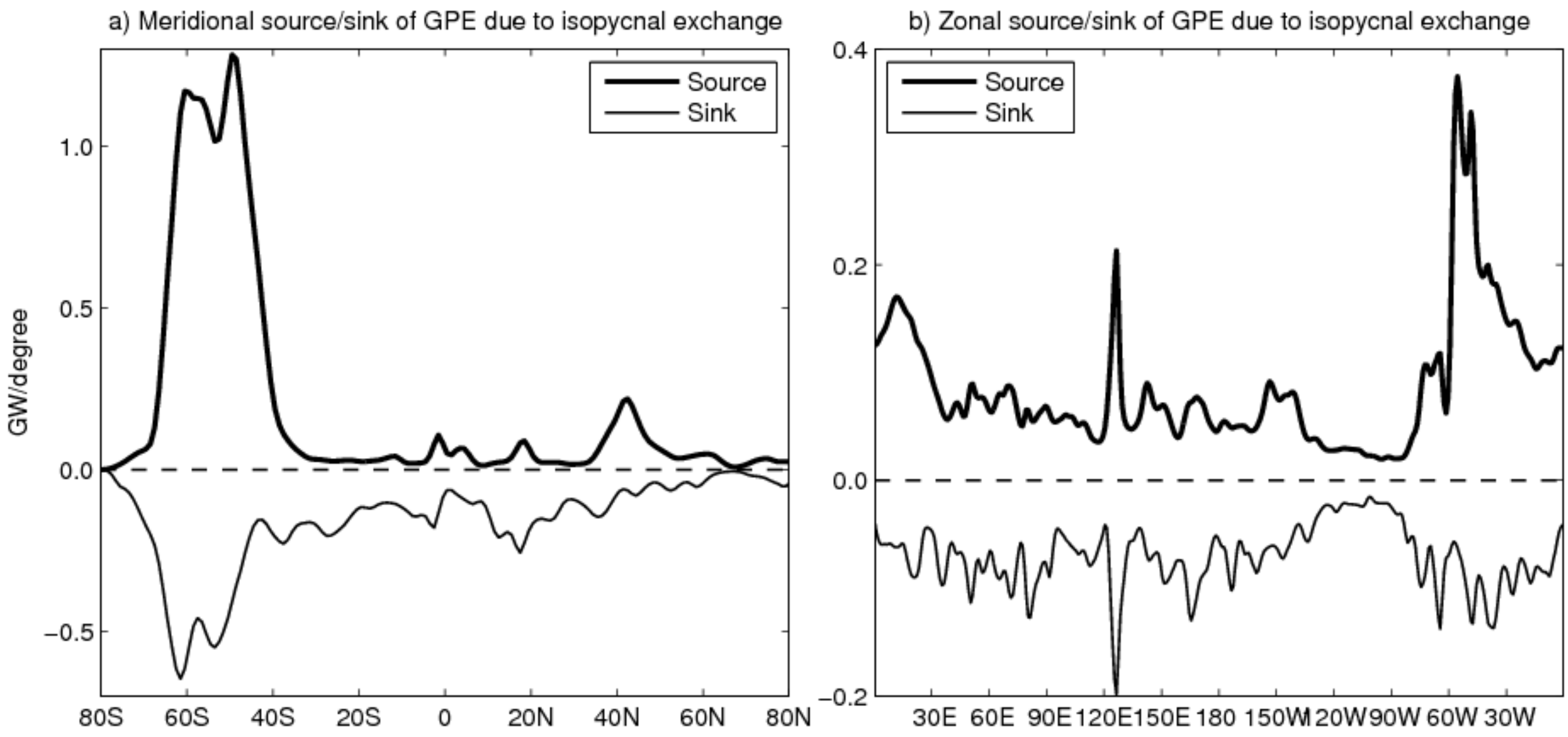
GPE sink due to cabbeling associated with horizontal mixing



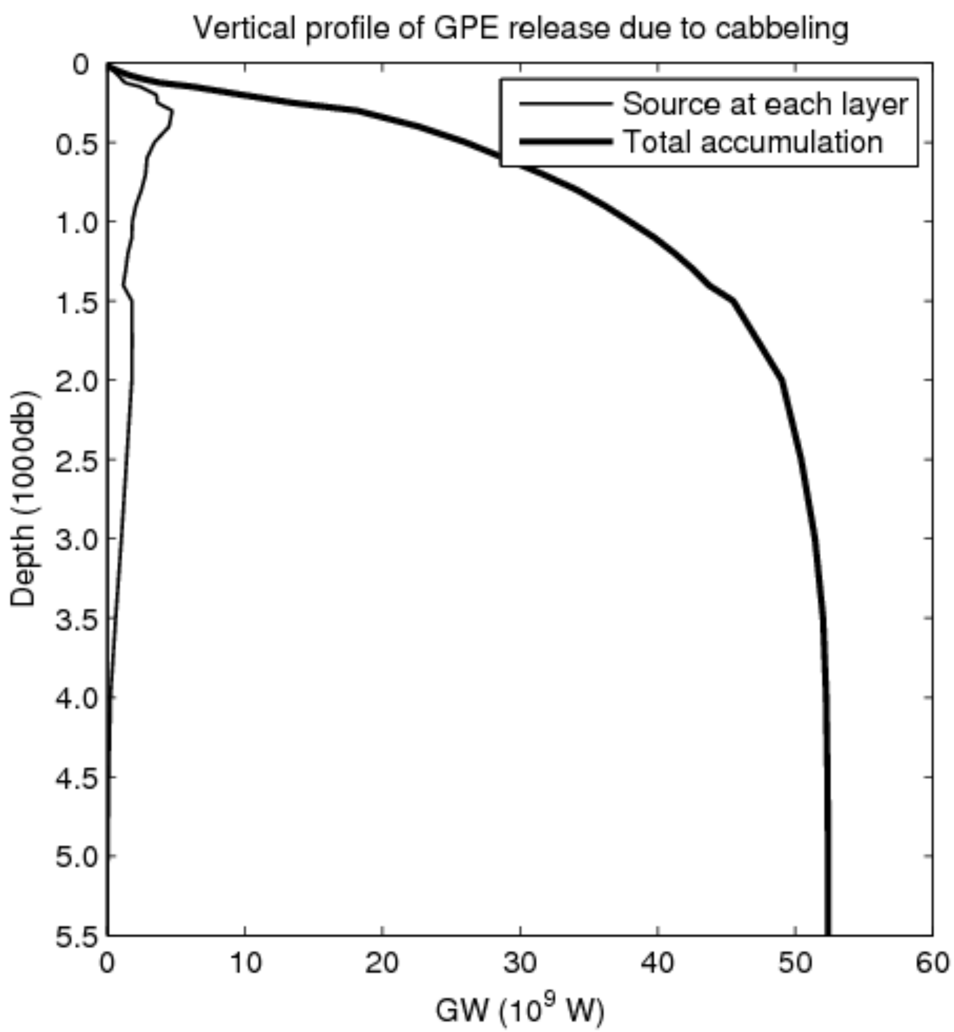
GPE source/sink due to isopycnal stirring



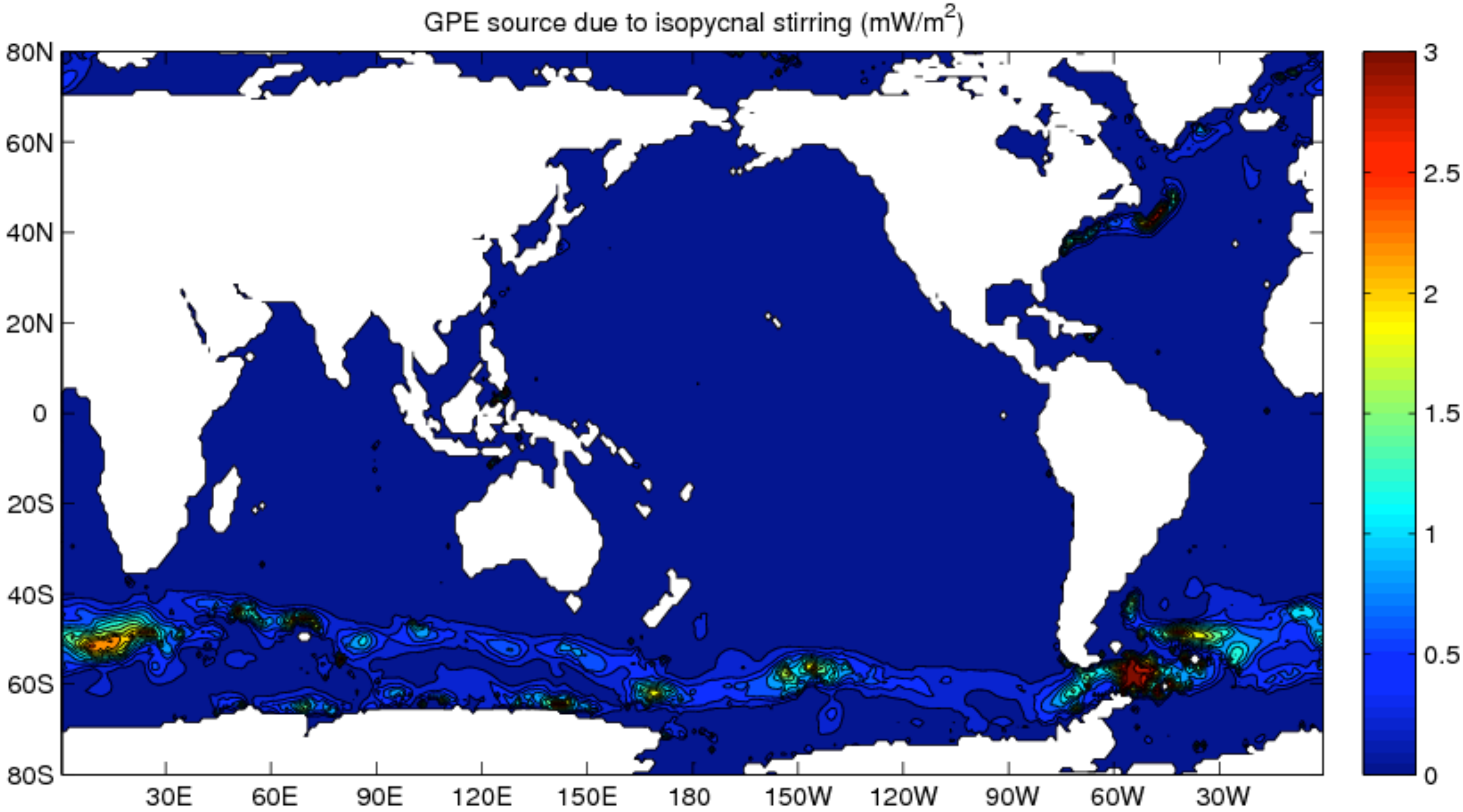
GPE source/sink due to isopycnal stirring



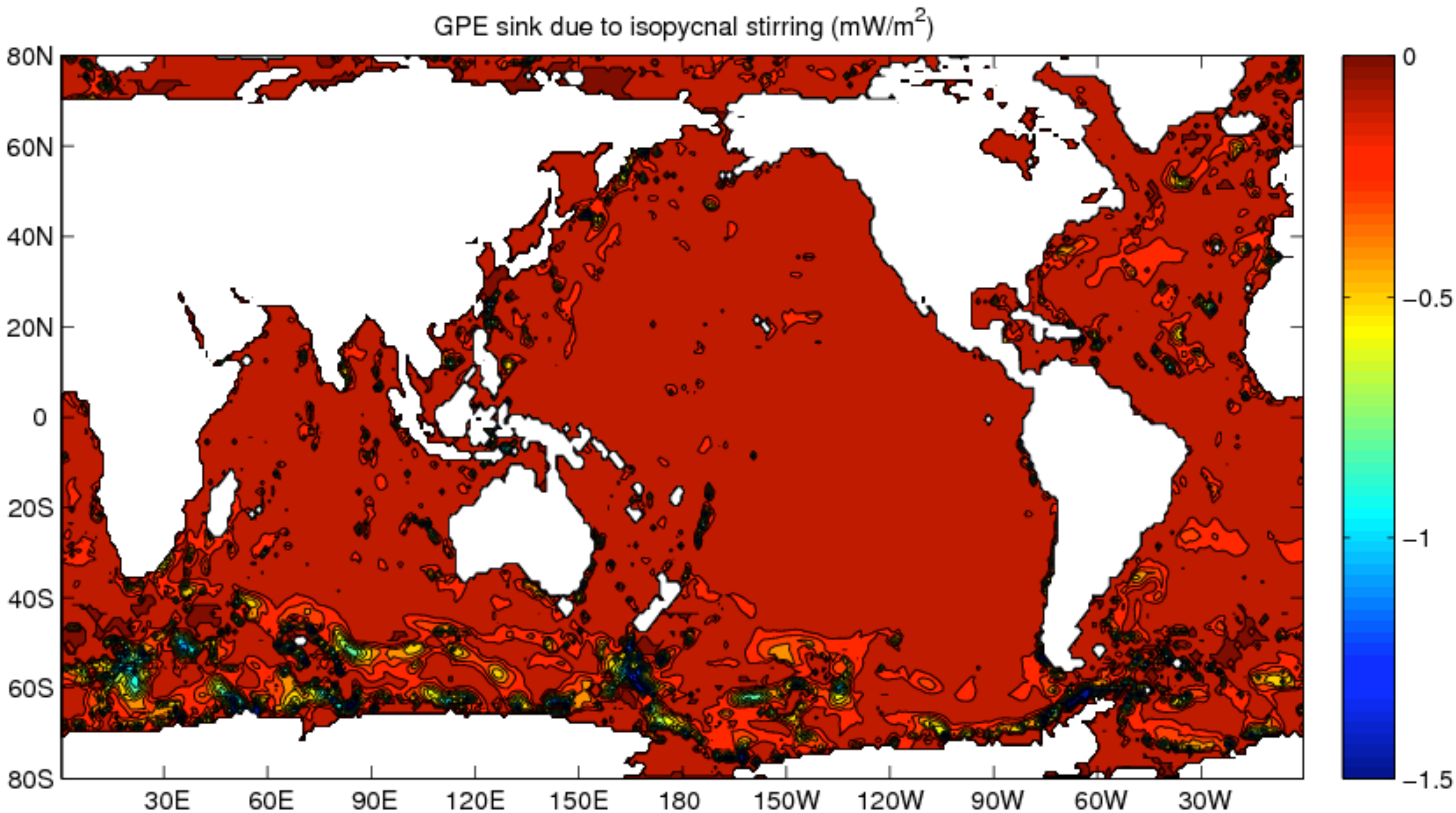
GPE sink due to cabbeling in connection with isopycnal mixing



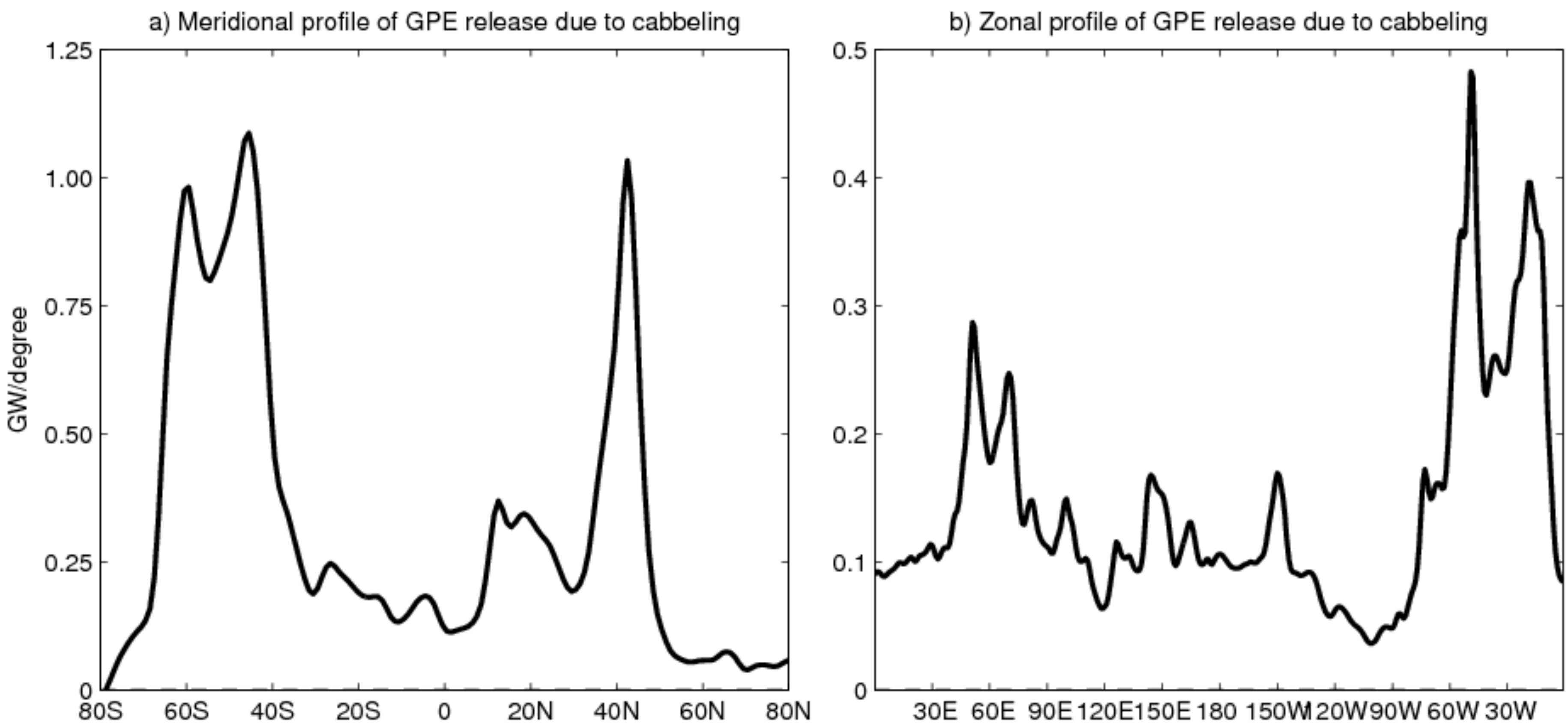
To count GPE source due to isopycnal stirring



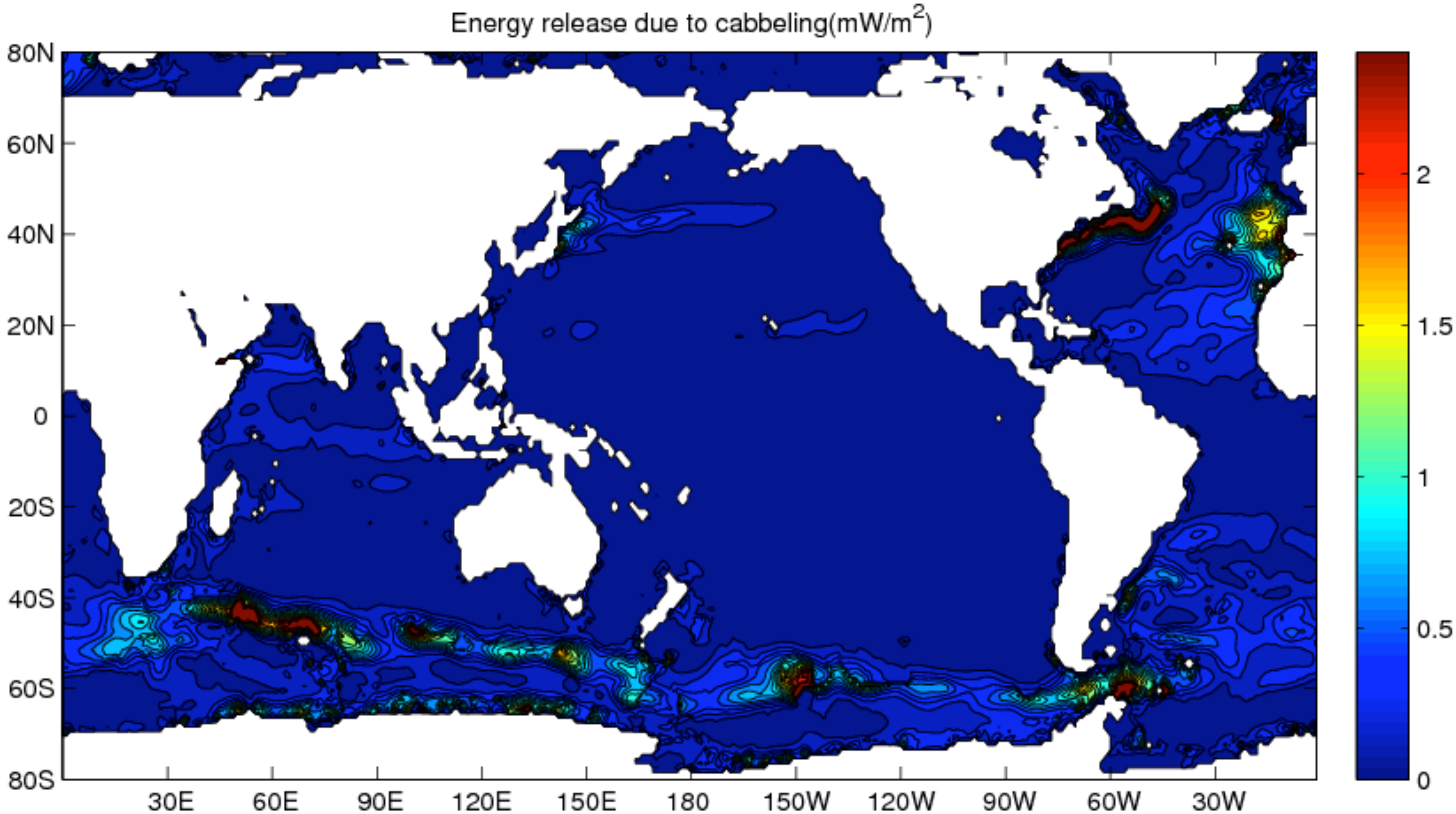
GPE sink due to isopycnal stirring



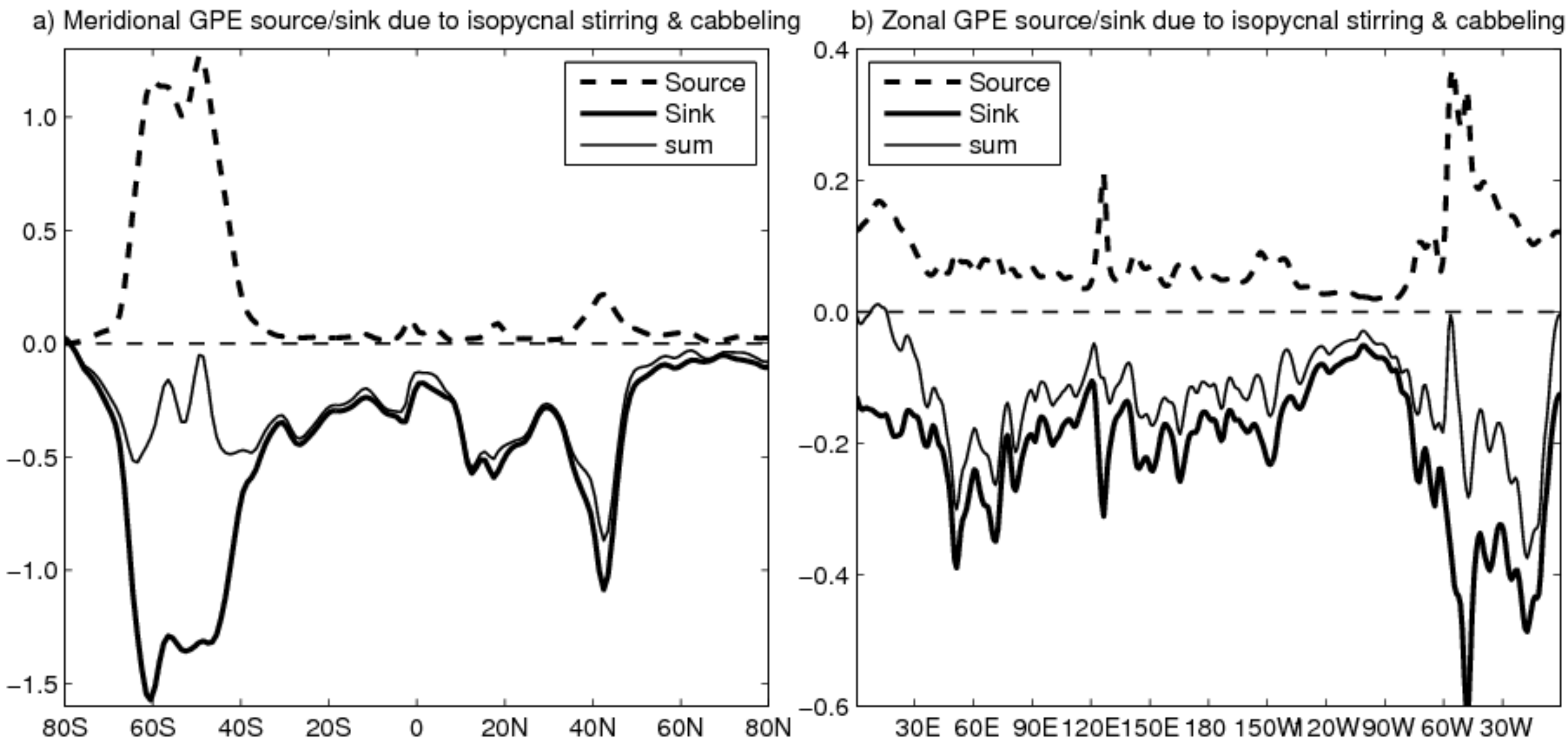
GPE sink due to cabbeling in connection with isopycnal mixing



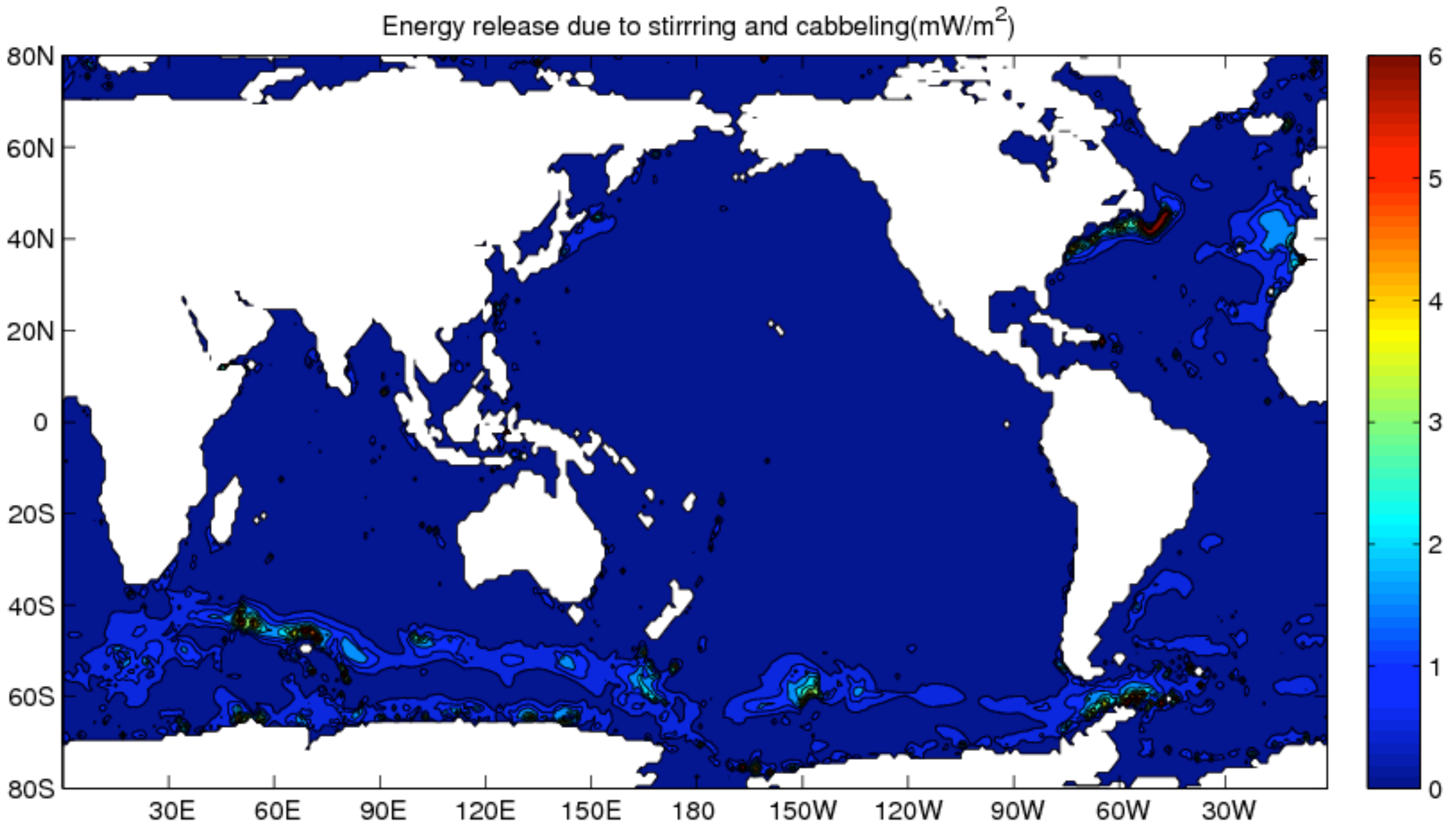
GPE sink due to cabbeling in connection with isopycnal mixing



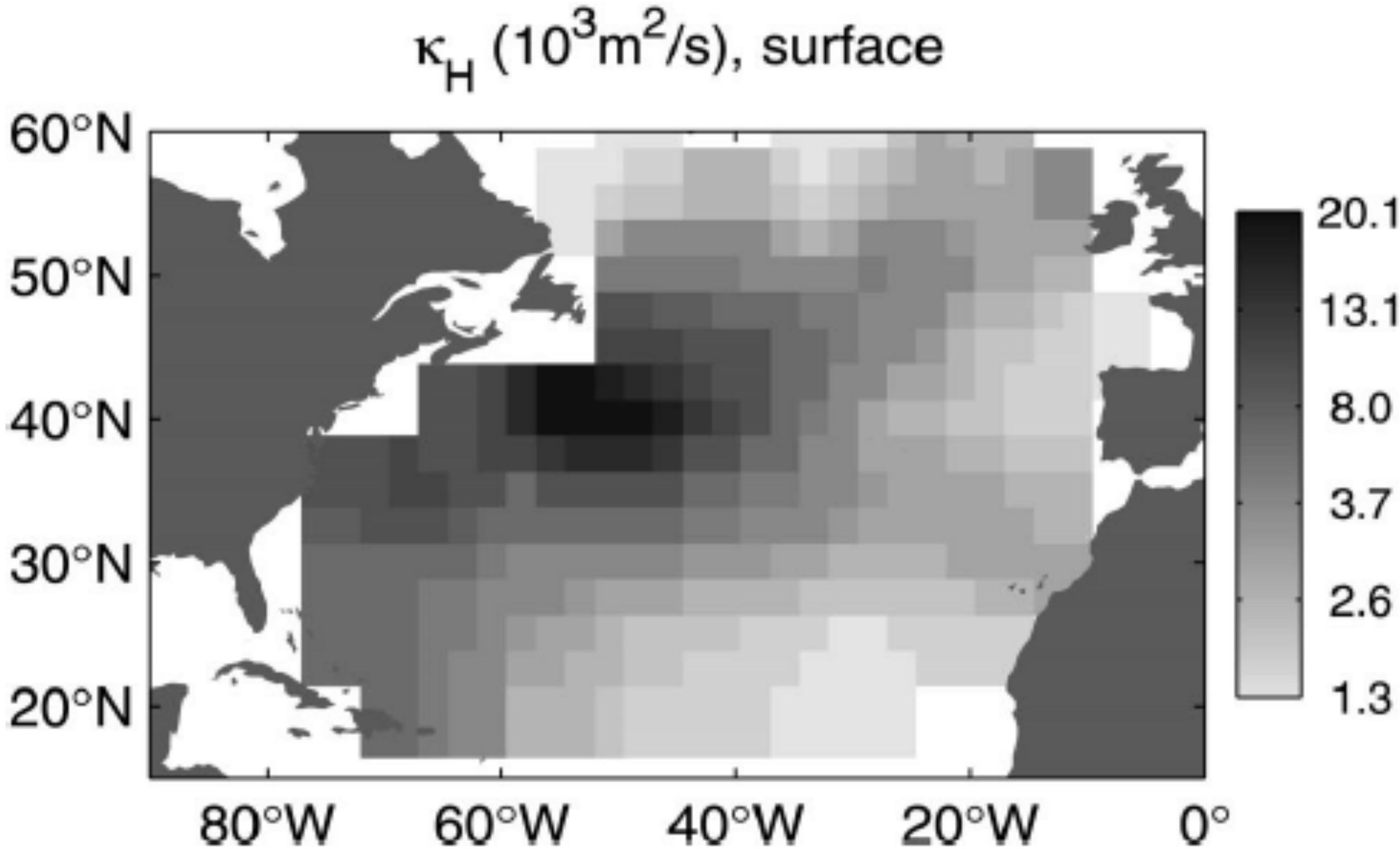
Net GPE source & sink due to isopycnal stirring/mixing



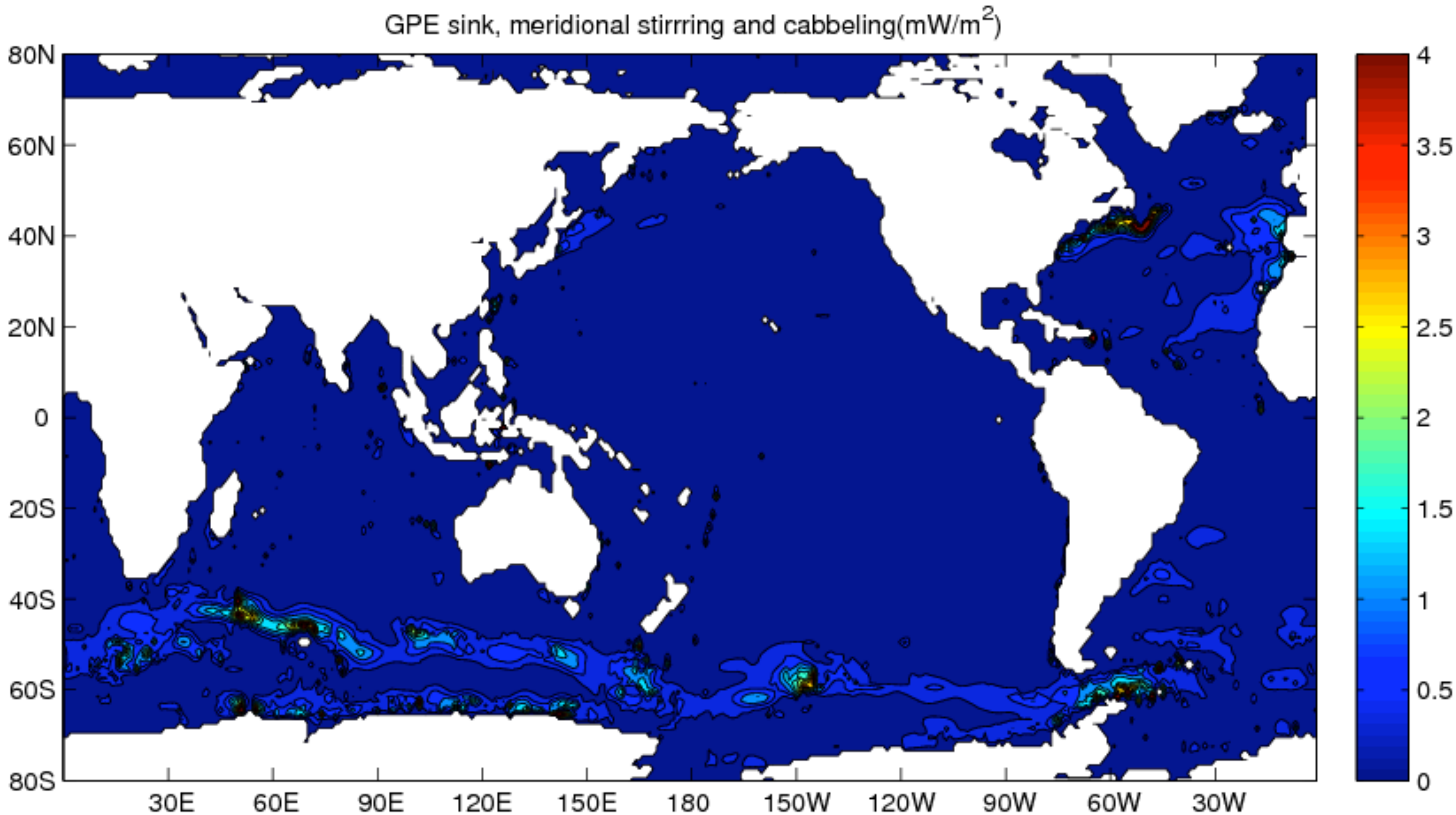
GPE sink due to isopycnal stirring and cabbeling



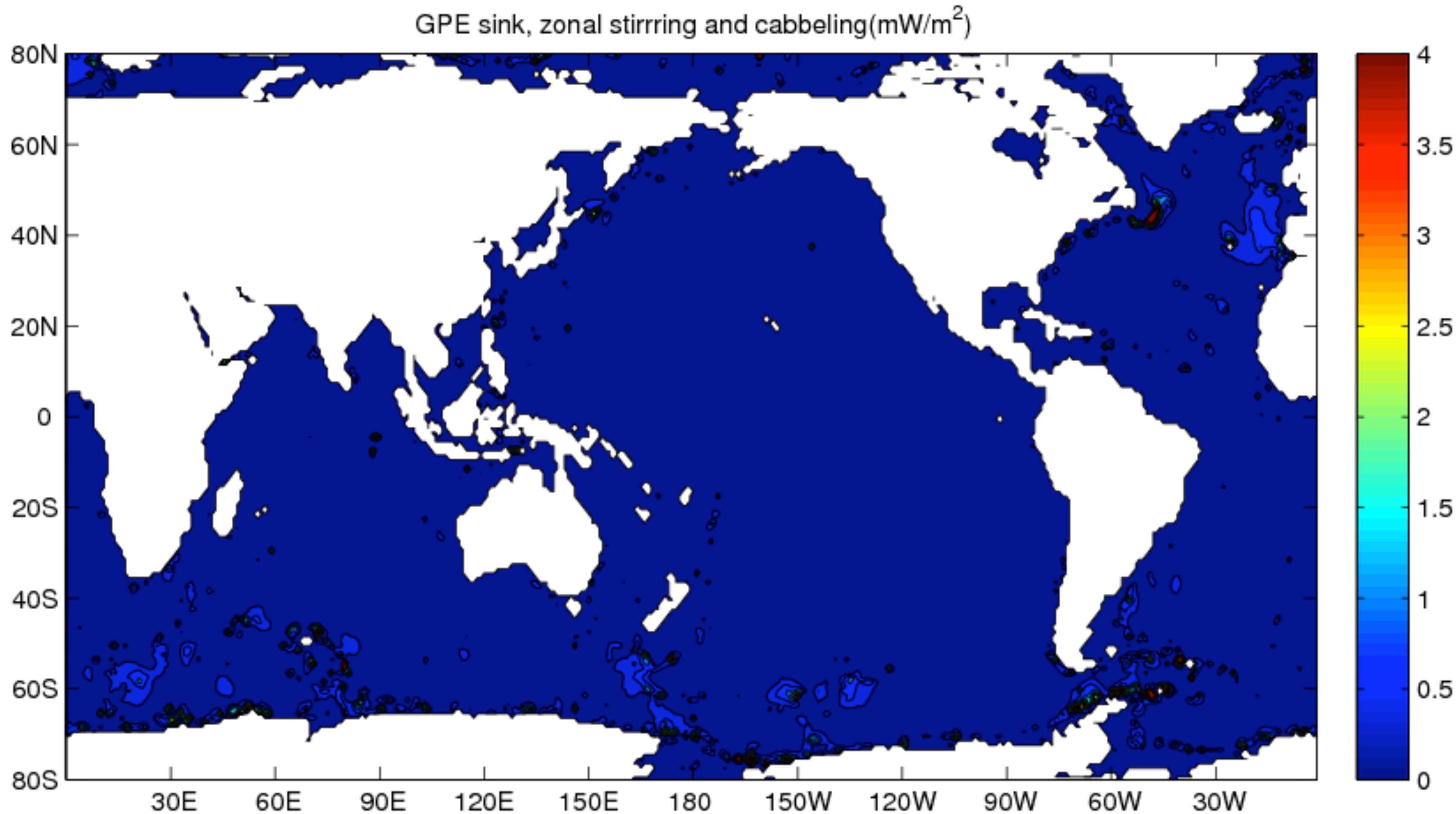
Eddy diffusivity inferred from drifter trajectories (Lumpkin et al., 2002)



GPE sink due to meridional isopycnal stirring/cabbeling

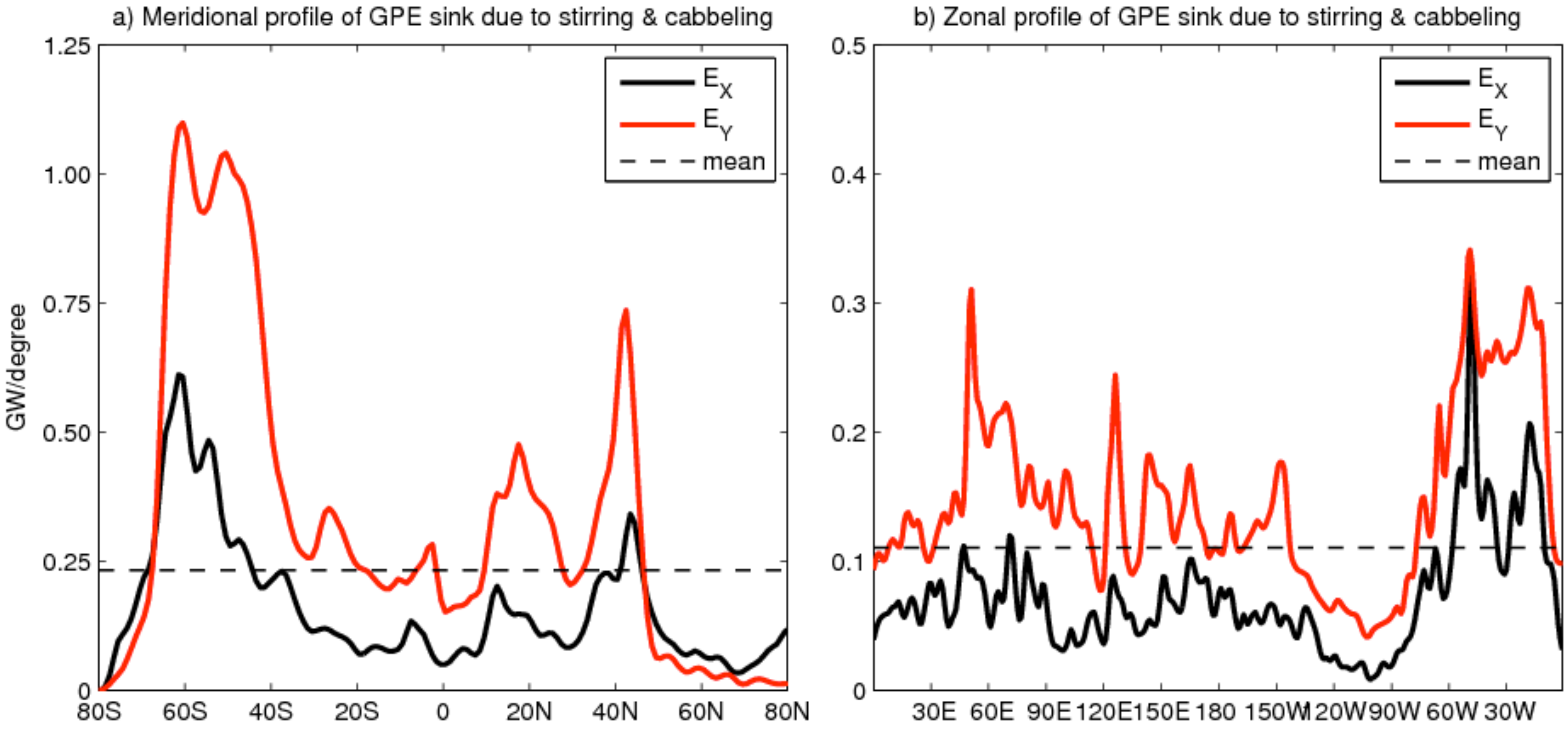


GPE sink due to zonal isopycnal stirring/cabbeling

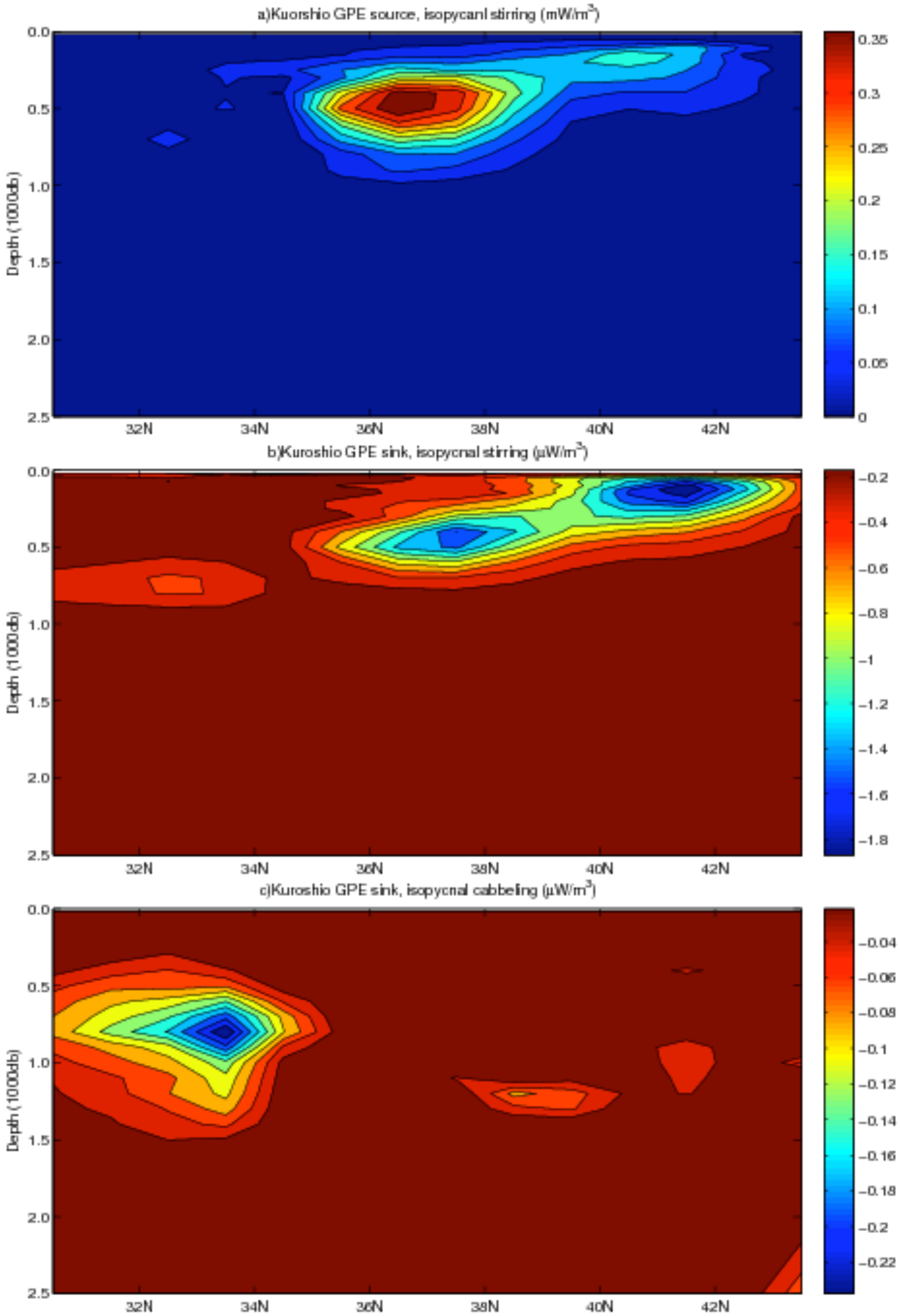


GPE sink due to isopycnal stirring/cabbeling

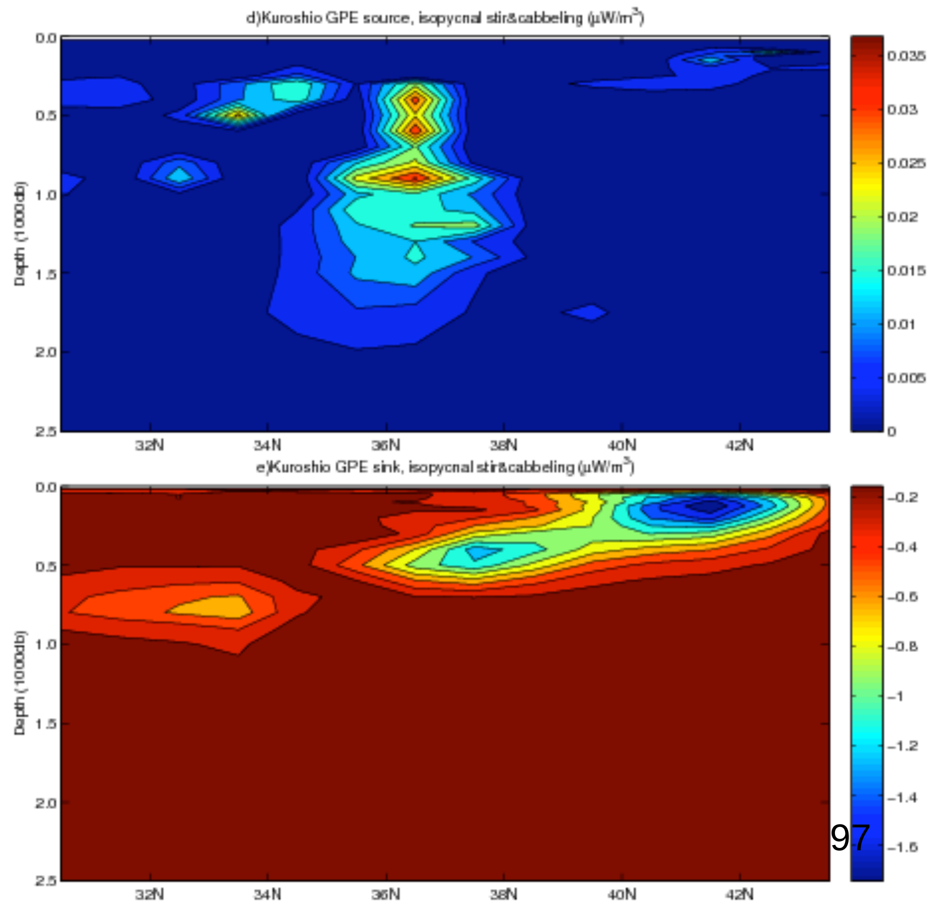
Diffusion is not homogeneous or isotropic



Kuroshio

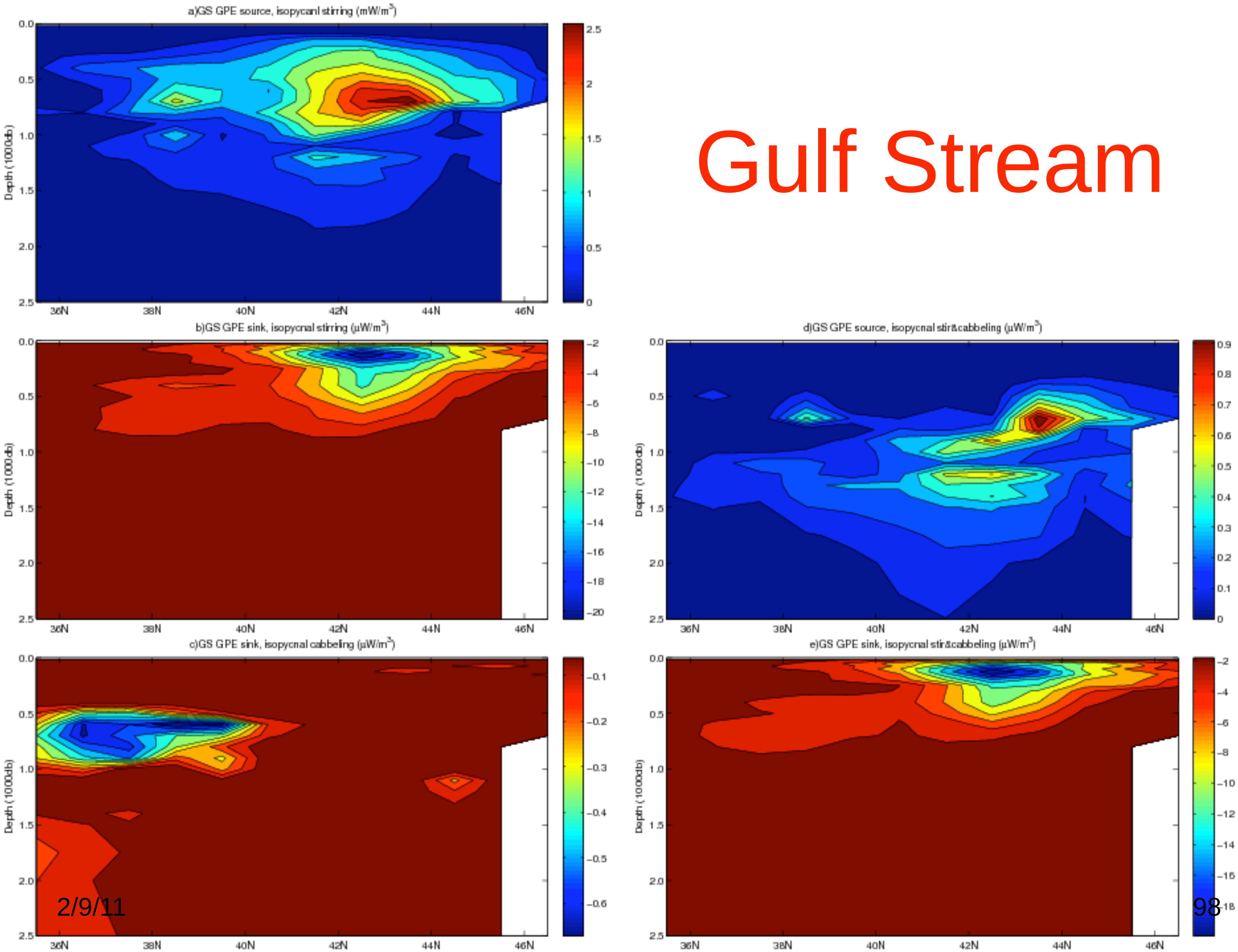


a)Kuorshio GPE source, isopycanl stirring (mW/m³)
b)Kuroshio GPE sink, isopycnal stirring (μW/m³)
c)Kuroshio GPE sink, isopycnal cabbeling (μW/m³)
Depth (1000db)
32N 34N 36N 38N 40N 42N



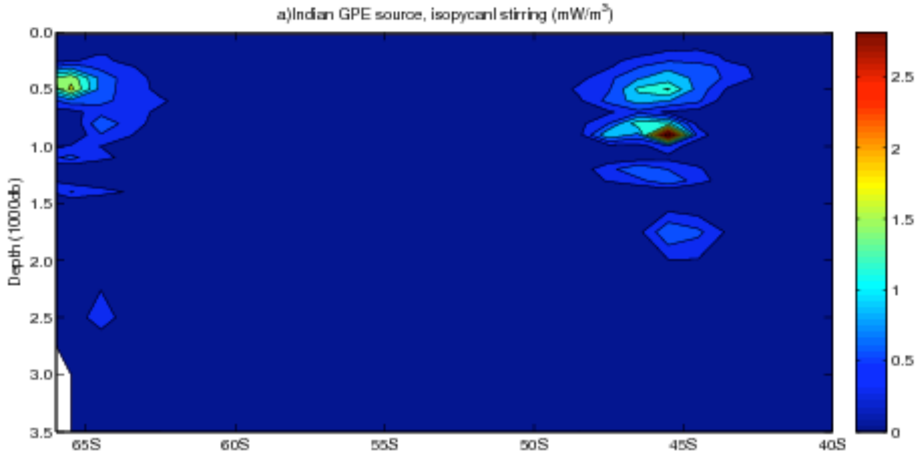
d)Kuroshio GPE source, isopycnal stir&cabbeling (μW/m³)
e)Kuroshio GPE sink, isopycnal stir&cabbeling (μW/m³)
Depth (1000db)
32N 34N 36N 38N 40N 42N

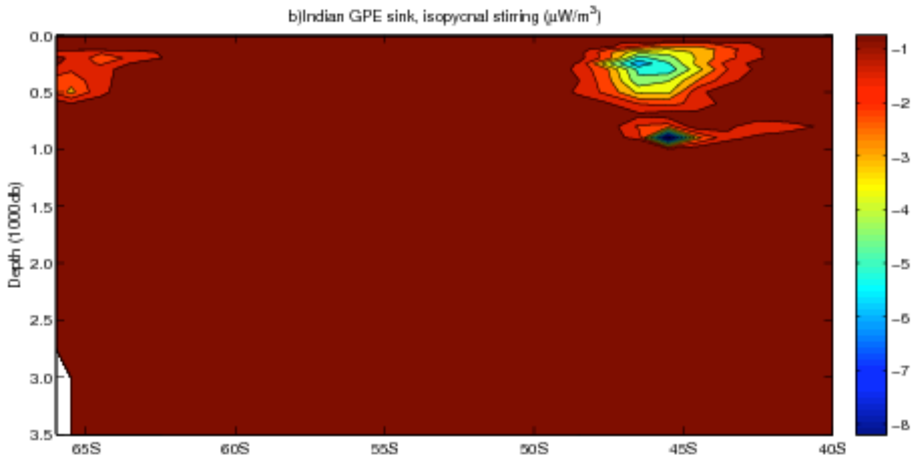
Gulf Stream

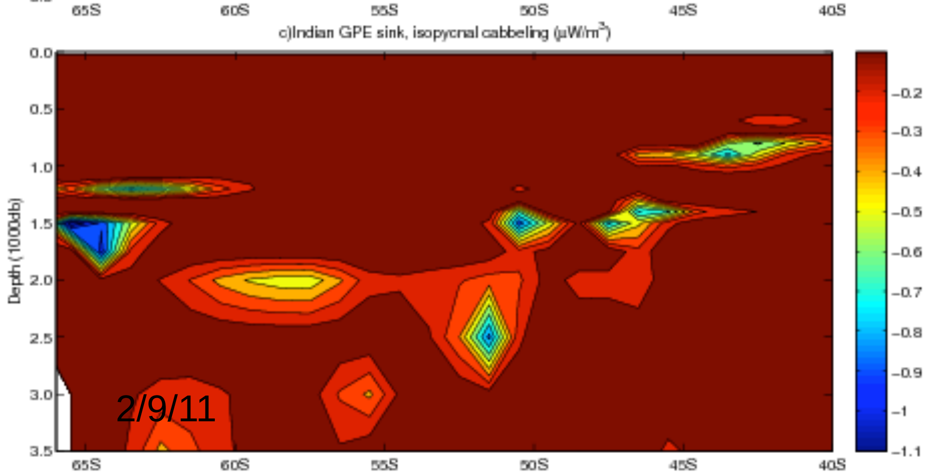


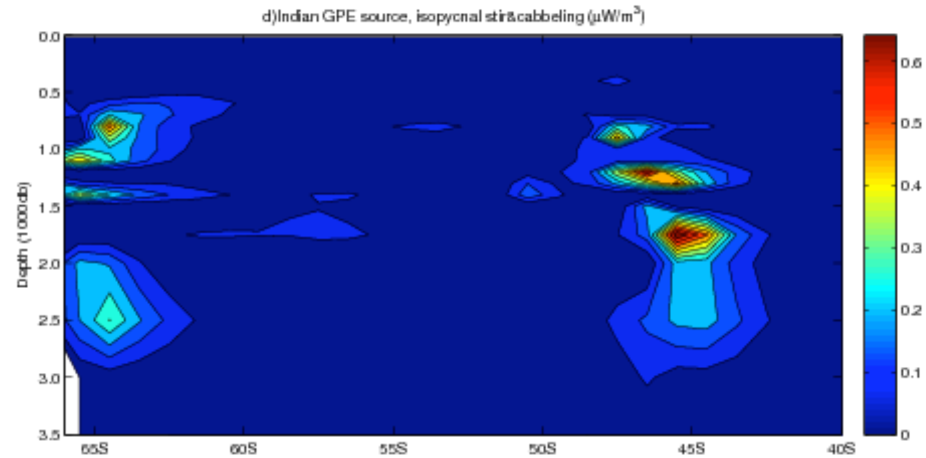
a)GS GPE source, isopycanl stirring (mW/m^3)
b)GS GPE sink, isopycnal stirring (μW/m^3)
c)GS GPE sink, isopycnal cabbeling (μW/m^3)
d)GS GPE source, isopycnal stir&cabbeling (μW/m^3)
e)GS GPE sink, isopycnal stir&cabbeling (μW/m^3)
Depth (1000db)
36N
38N
40N
42N
44N
46N

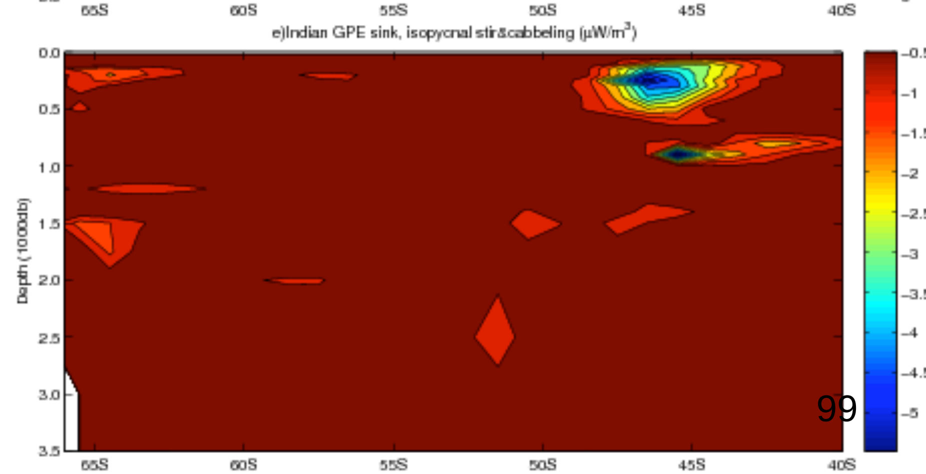
Indian Ocean



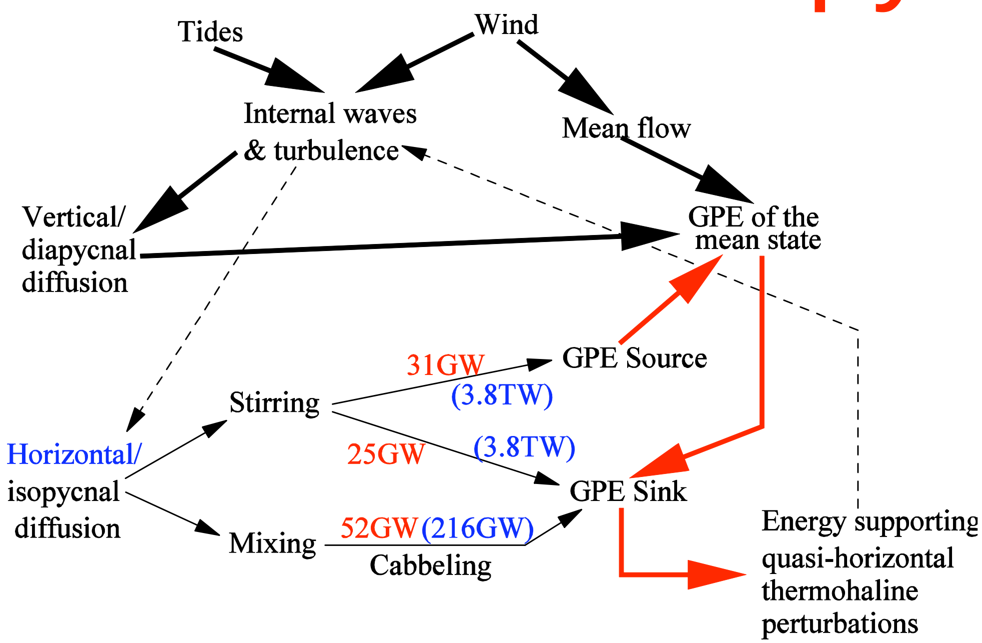






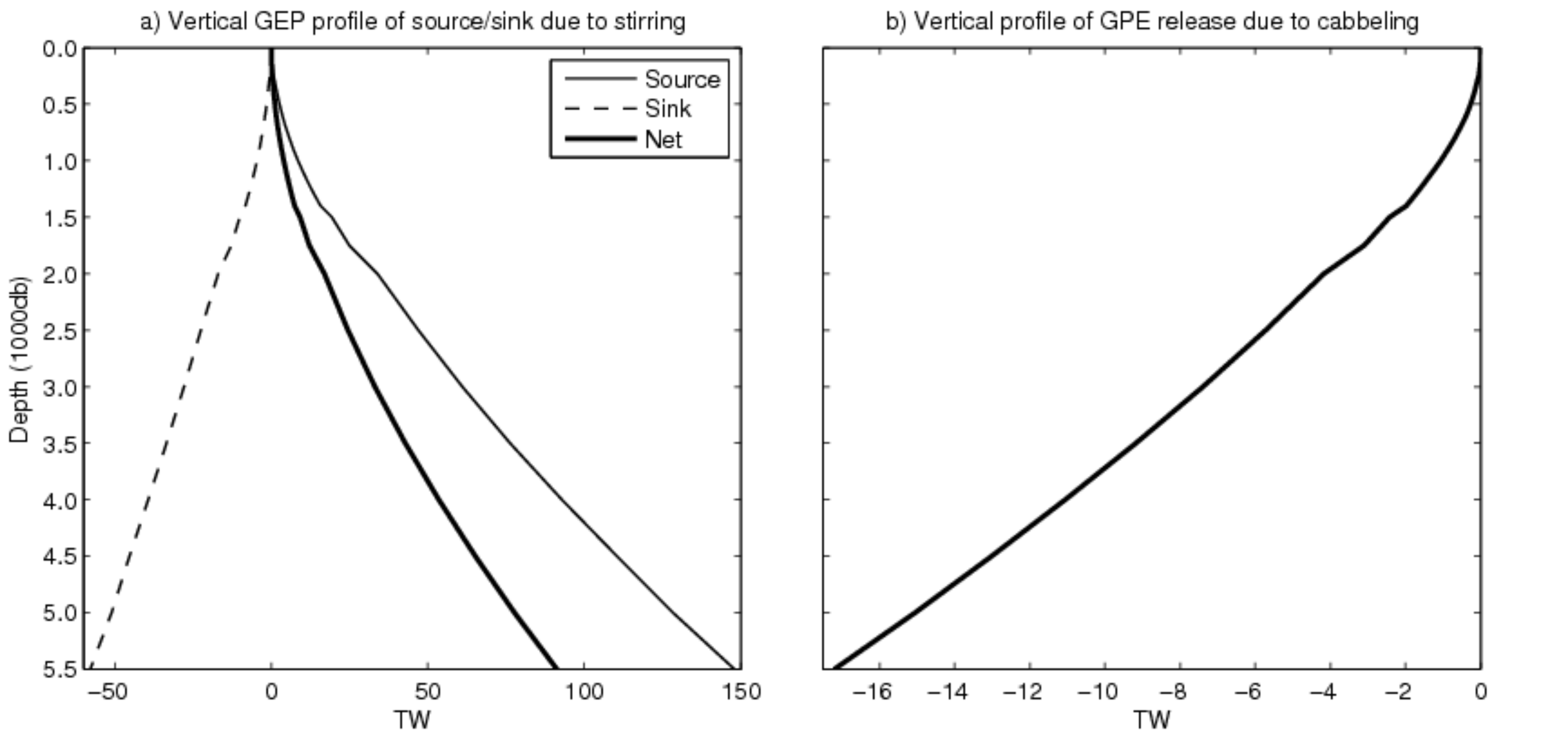


A sketch of GPE balance associated with isopycnal diffusion

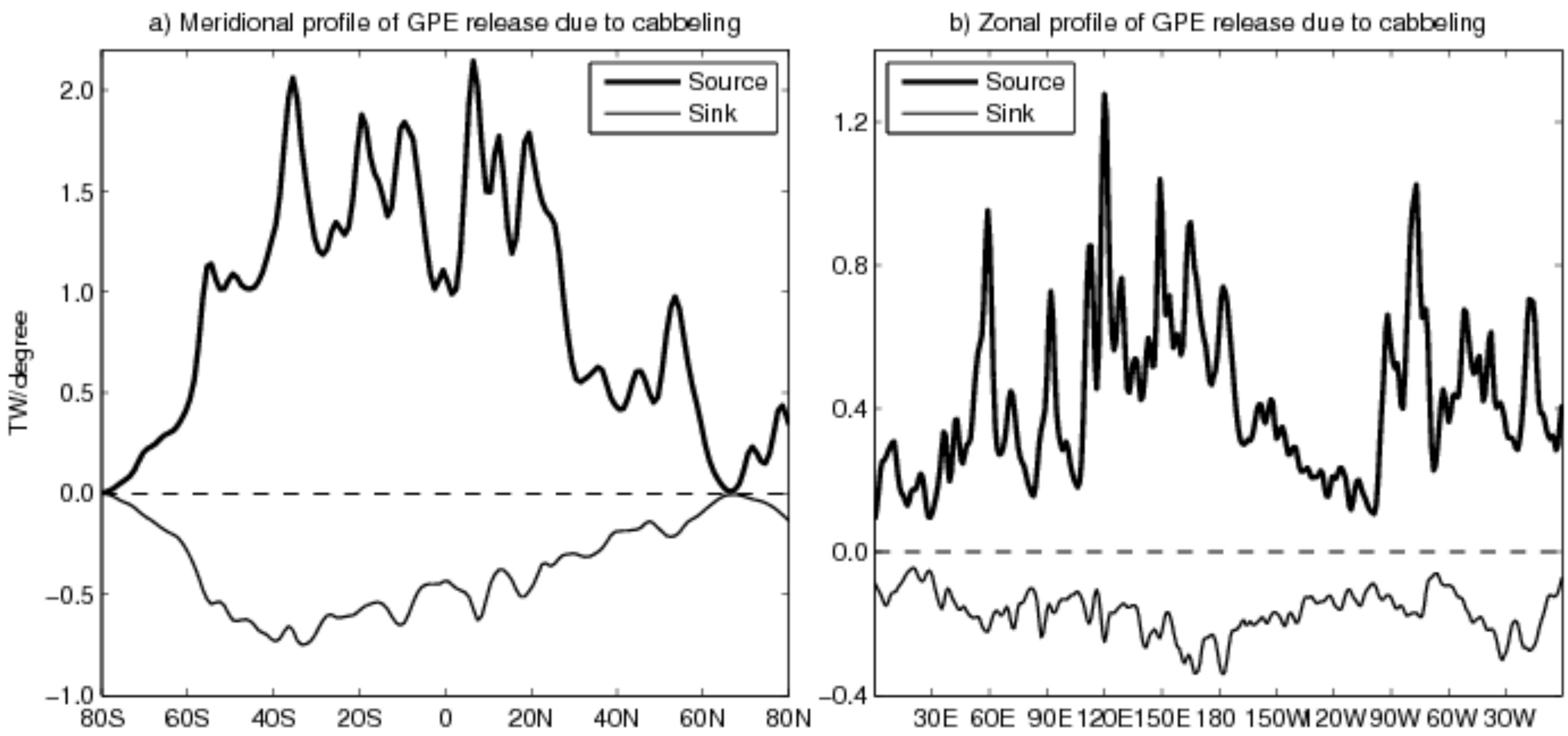


Energetics of quasi-horizontal stirring/mixing

GPE source/sink due to sigma stirring/cabbeling

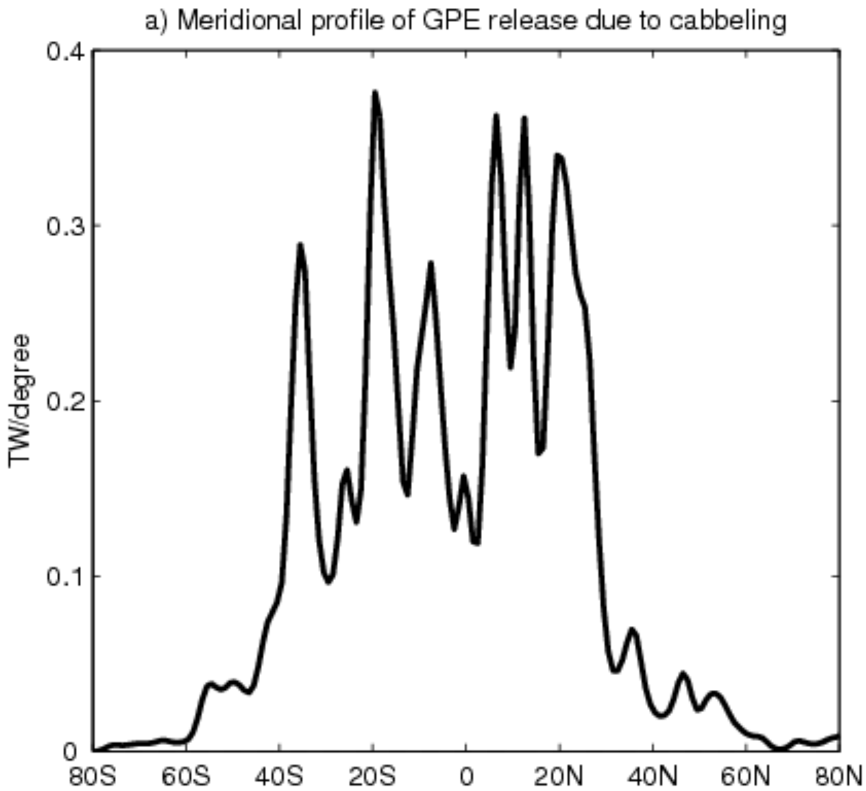


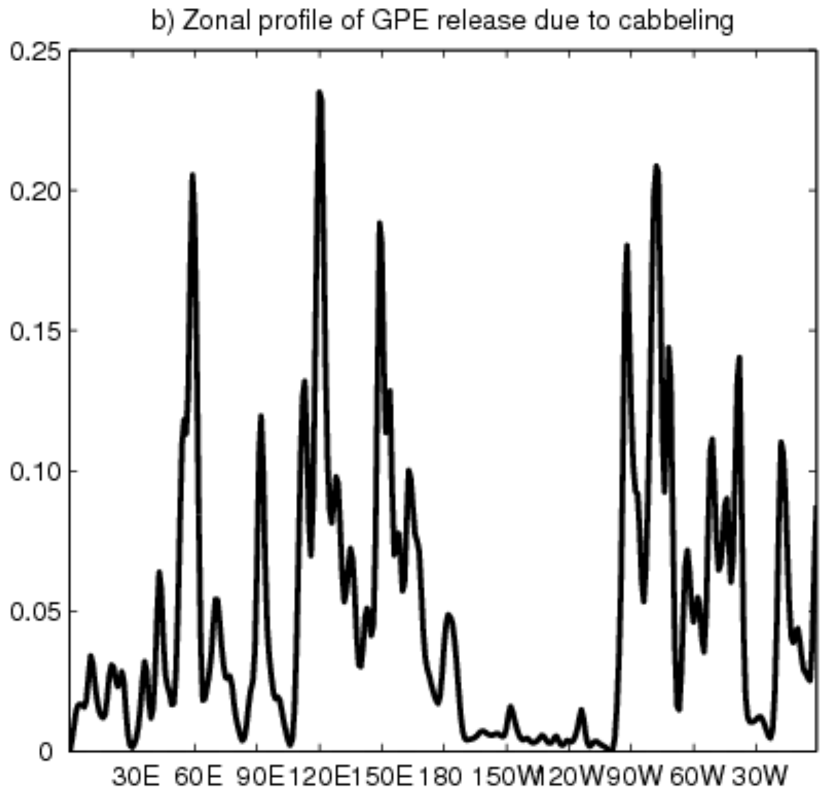
GPE sink due to cabbeling associated with sigma mixing



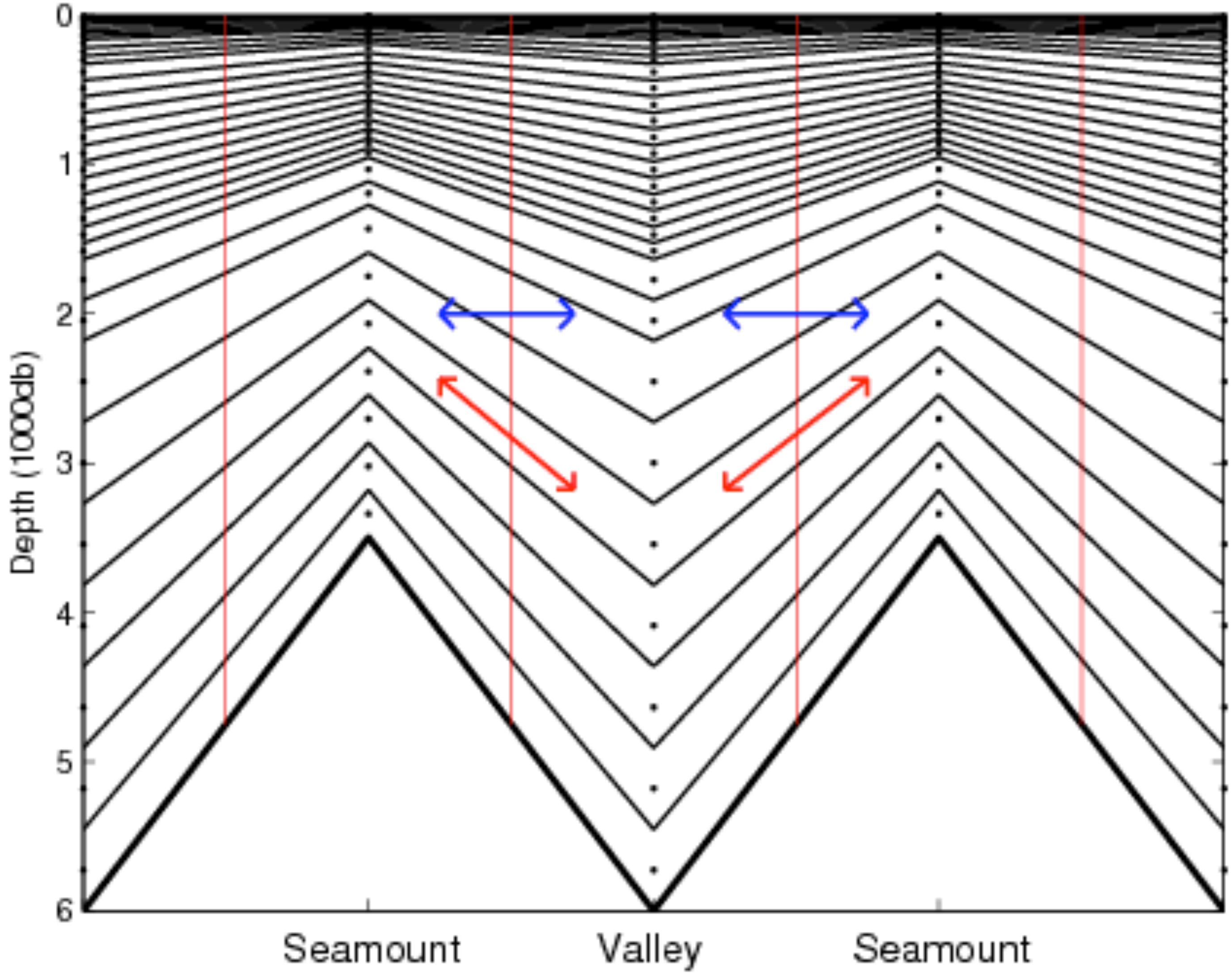
GPE sink due to cabbeling associated with sigma mixing

High rate of sink is not related to strong fronts in the ocean; the sea floor topography is more relevant.

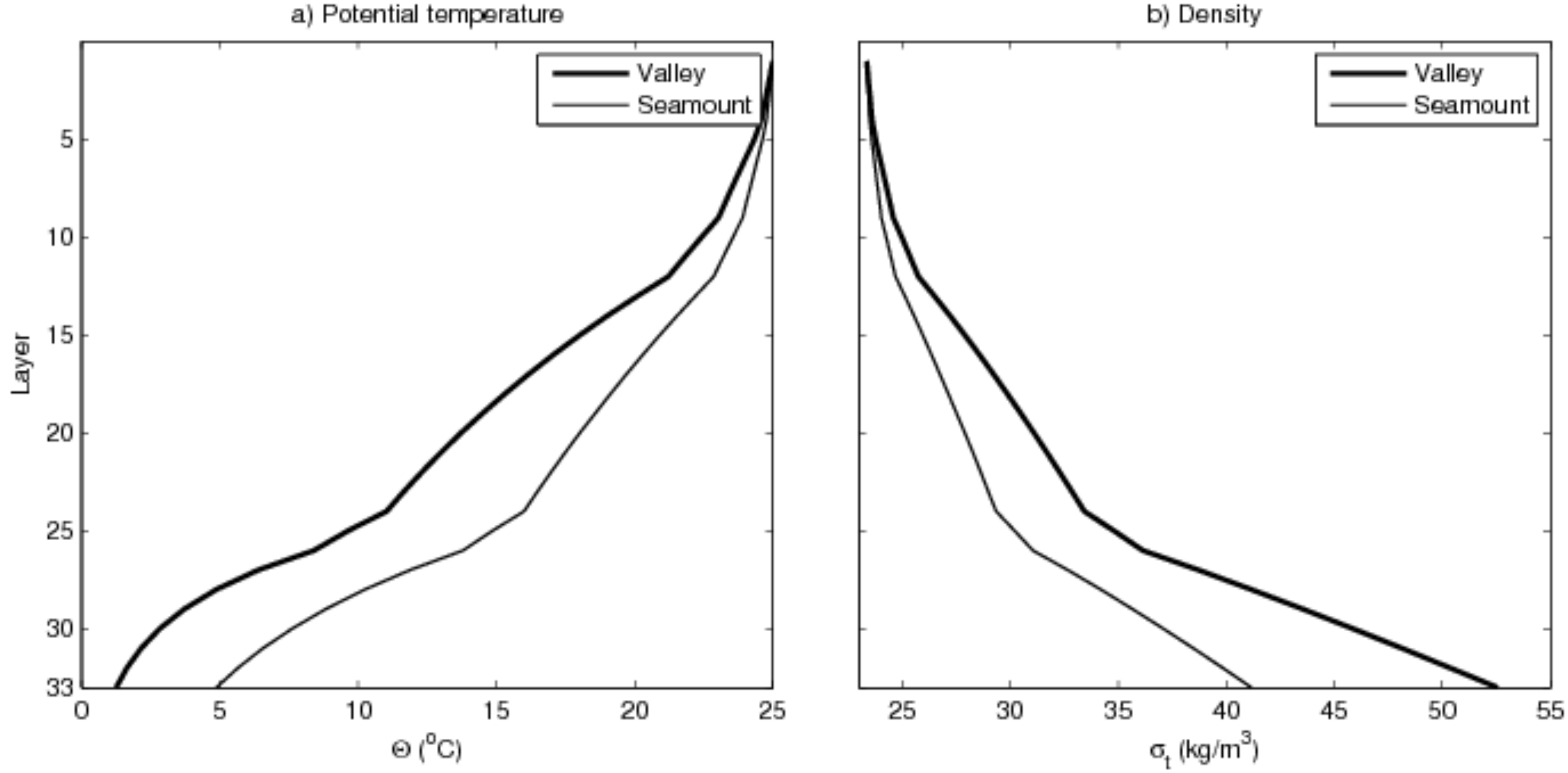




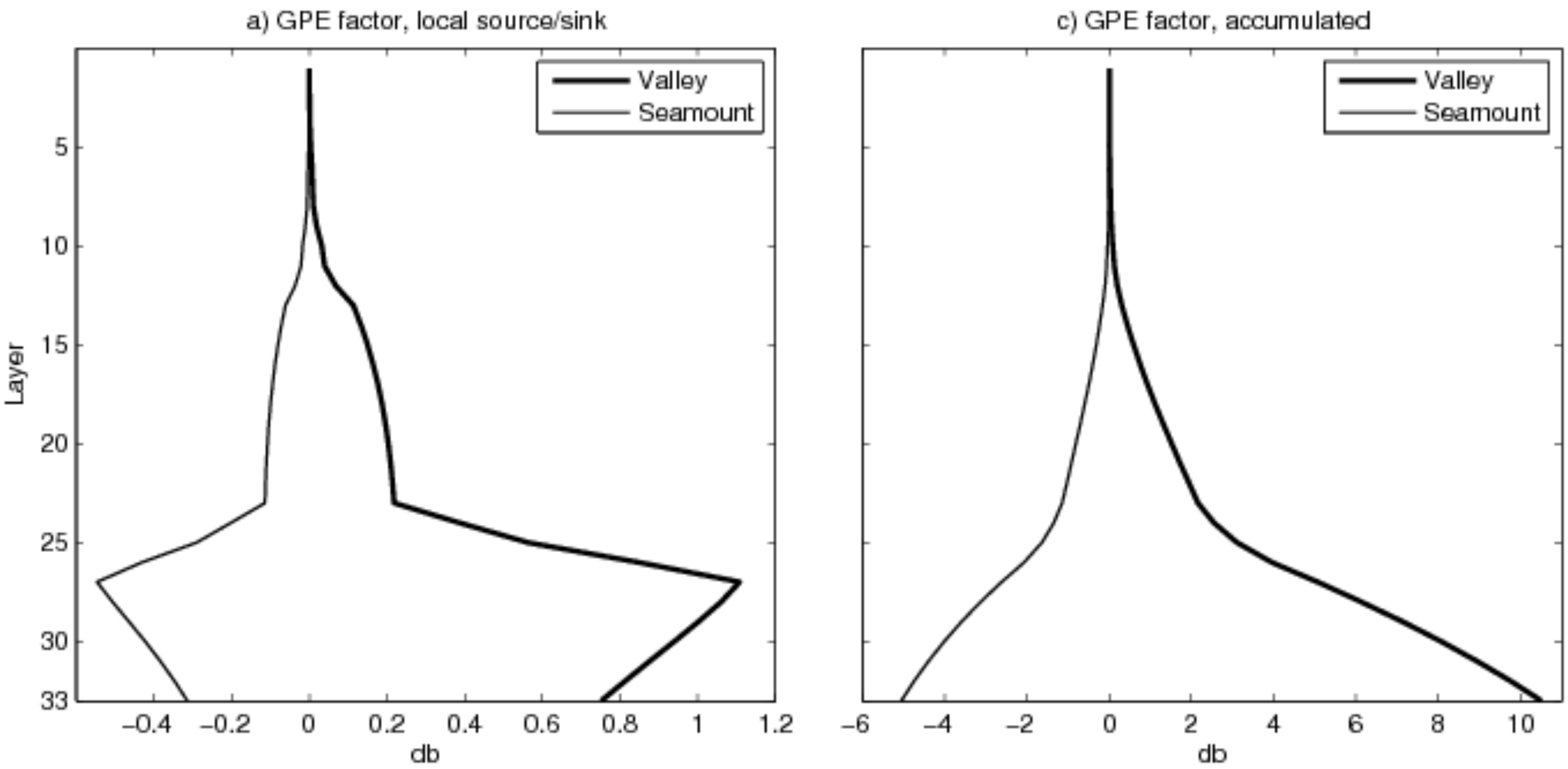
One-dimensional topography & sigma coordinate



Vertical profiles of potential temperature and density



Vertical distribution of source/sink and their accumulation



Conclusion

- Quasi-horizontal tracer diffusion changes GPE:

  1) Isopycnal surface diffusion may be the best choice. Diffusion is non-uniform and an-isotropic

  2) Horizontal diffusion is no good, sigma surface diffusion is completely non-physical

- Thermohaline perturbations on isopycnal surfaces may grow with energy released from mean state --- this is a new type of instability.
- Our analysis may suggest where to observe such instability.