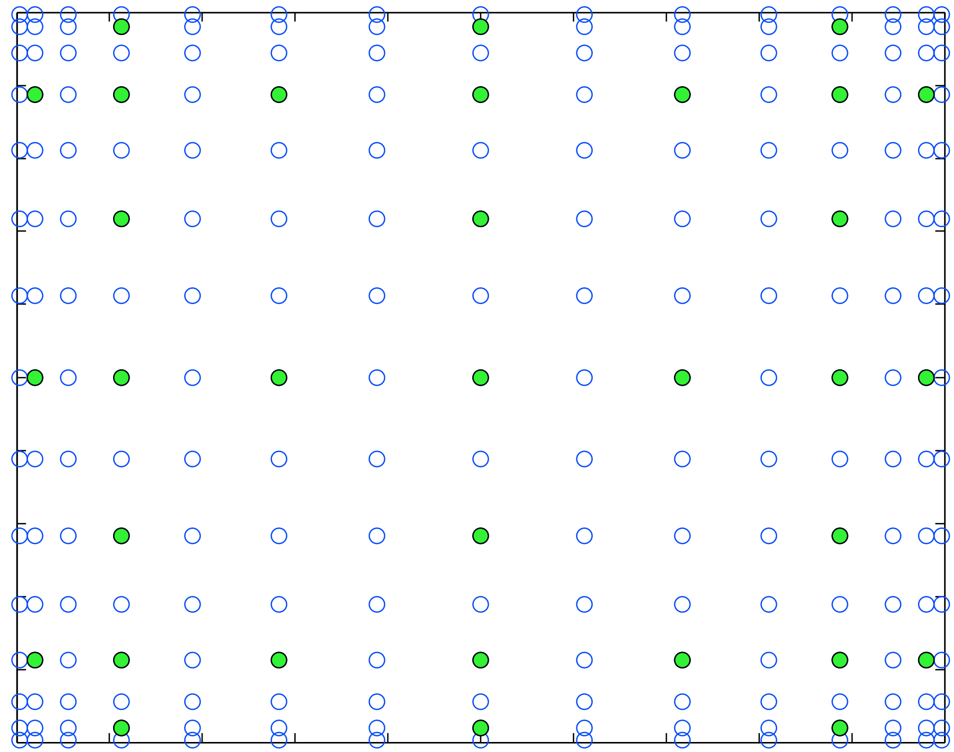
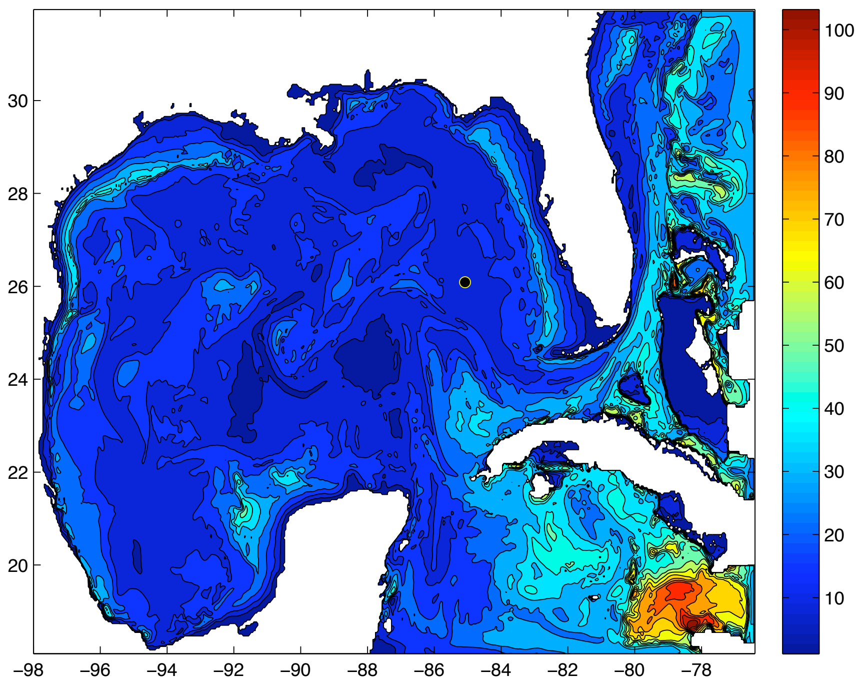
Sparse Quadratures: Motivation

- Fully tensorized quadratures are particularly prone to the so-called curse of dimensionality. With q quadrature points along each axis, one has q^N quadrature points in N dimensions.
- Sparse quadratures aim at mitigating this curse by relying on partially-tensored formulas.

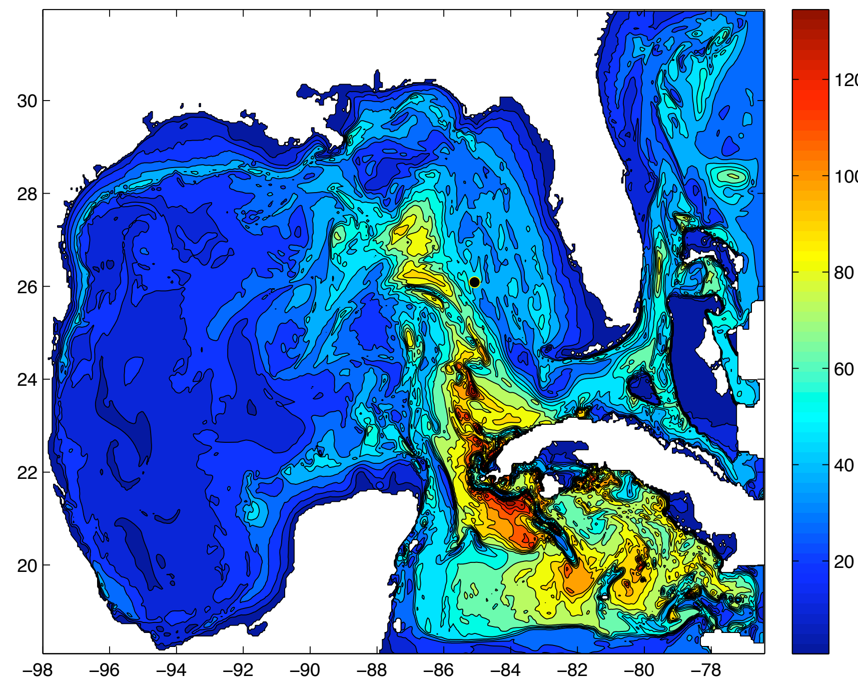


Solid – partially tensored
Hollow – full tensorization

Computational Experiment



t=69 hrs

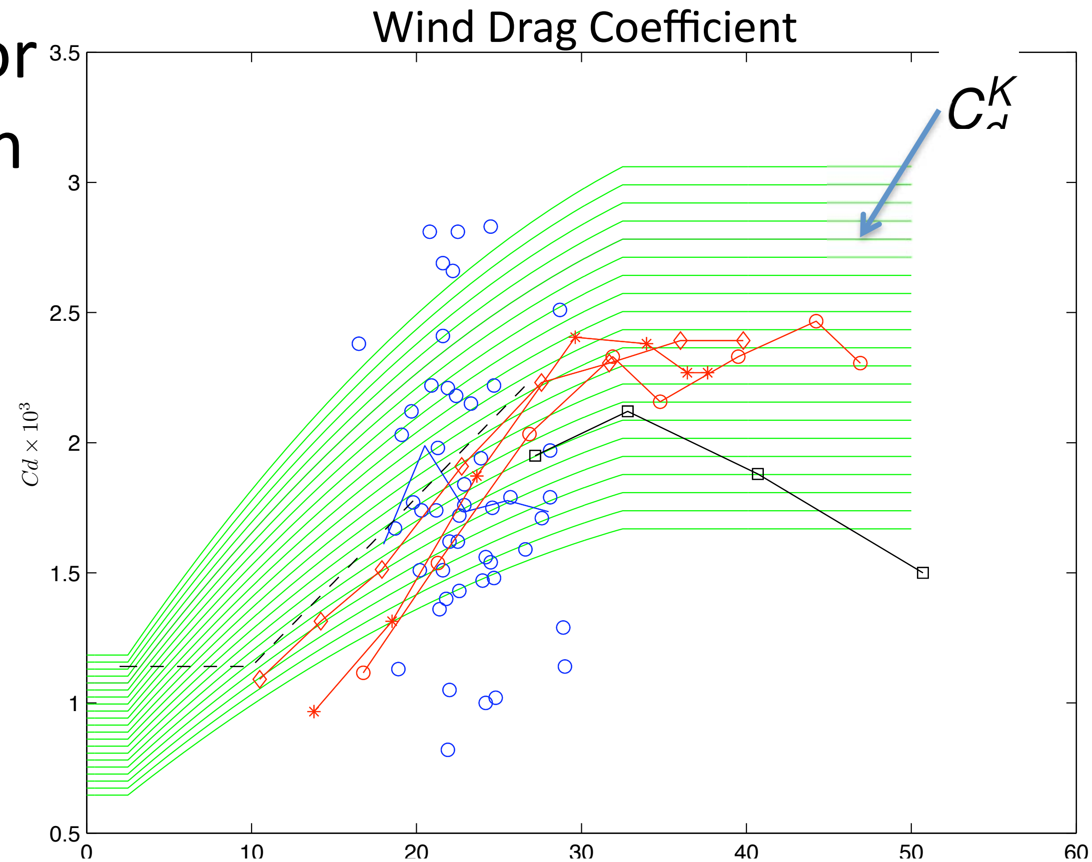


T=159 hrs

- Circulation in GOM. Analyze impact of uncertainty in (a) subgrid stress paramterization, and (b) wind coupling.
- High resolution (1/25°) simulations. Time period: Sep 1 – 10, 2004 (t=0 corresponds to 9/1). Track of Hurricane Ivan

Input uncertainties

- Uniform distributions for mixing parameterization and wind coupling
- KPP Model
  - Richardson number
    [0.1,0.7] ξ_1
  - Background viscosity
    [10^{-4},10^{-3}] ξ_2
  - Background diffusivity
    [10^{-5},10^{-3}] ξ_3



Factor [0.6,1.1] ξ_4

Blue circles: aircraft (French et al., 2007); Black: drop sonds (Powell et al., 2003); Dashed: drop sonds (Powell, 1981); Red: lab data (Donelan et al., 2004); Pink Line: HYCOM (Kara et al., 2000)

Analysis

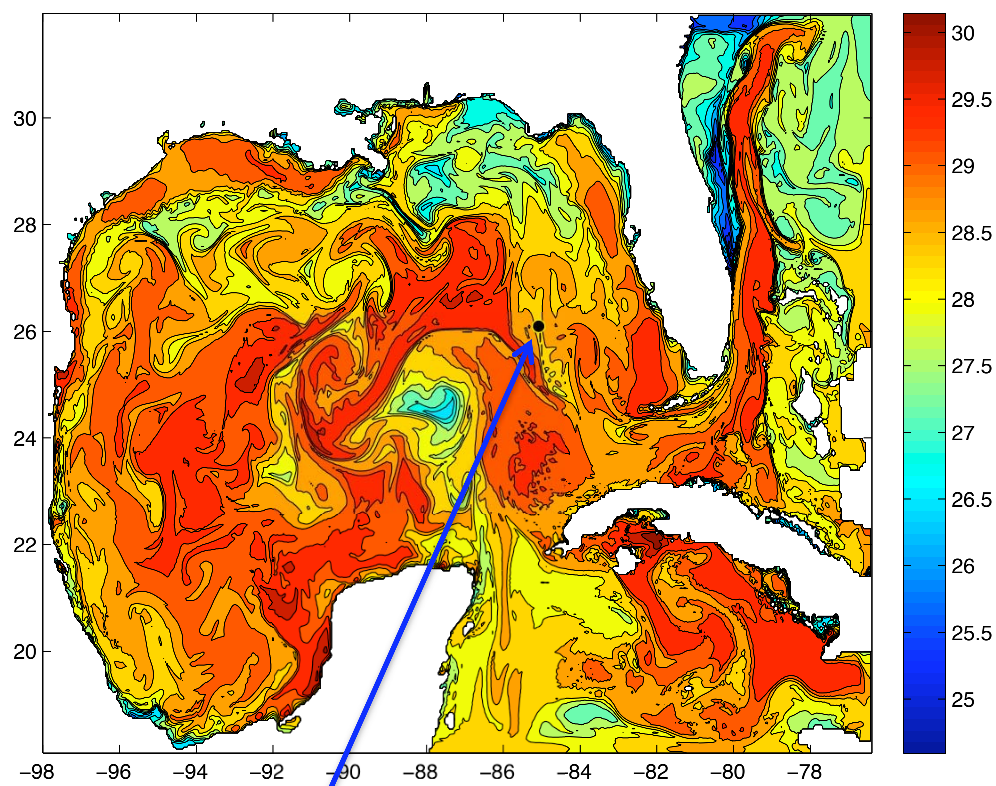
- Systematic Refinement
- 4-dimensional space

	l=1	l=2	l=3	l=4
d=2	5	9	17	33
d=3	7	19	39	87
d=4	9	33	81	193

193 Nested Realizations

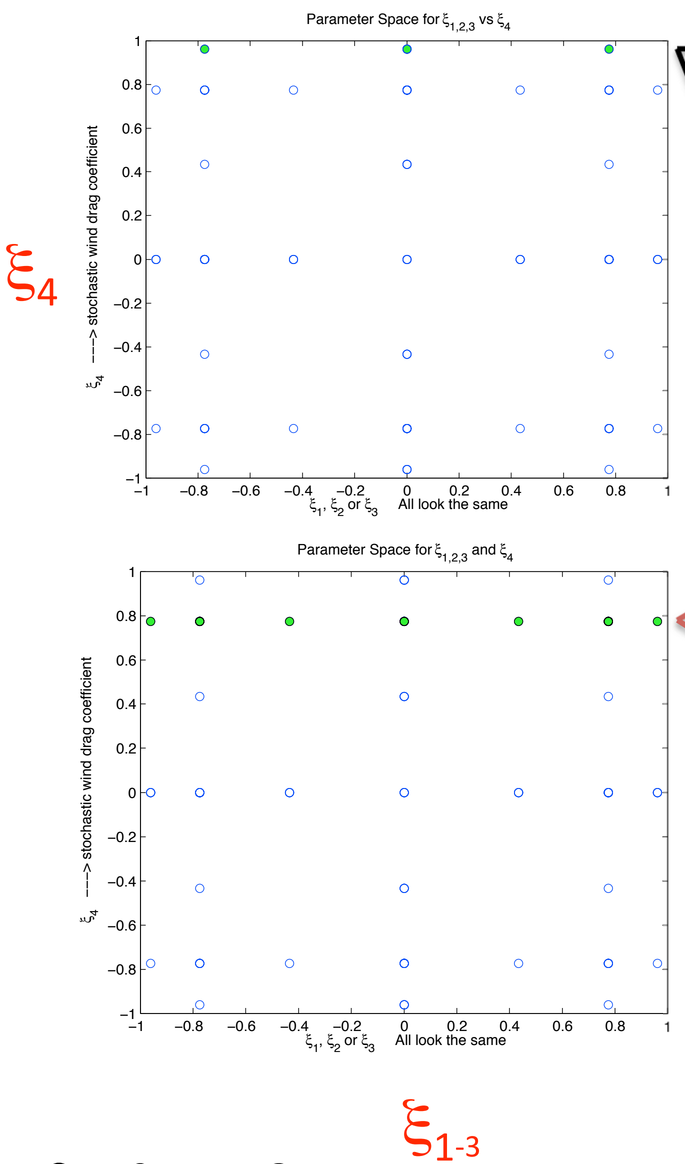
Quadrature exact up to p=l+1

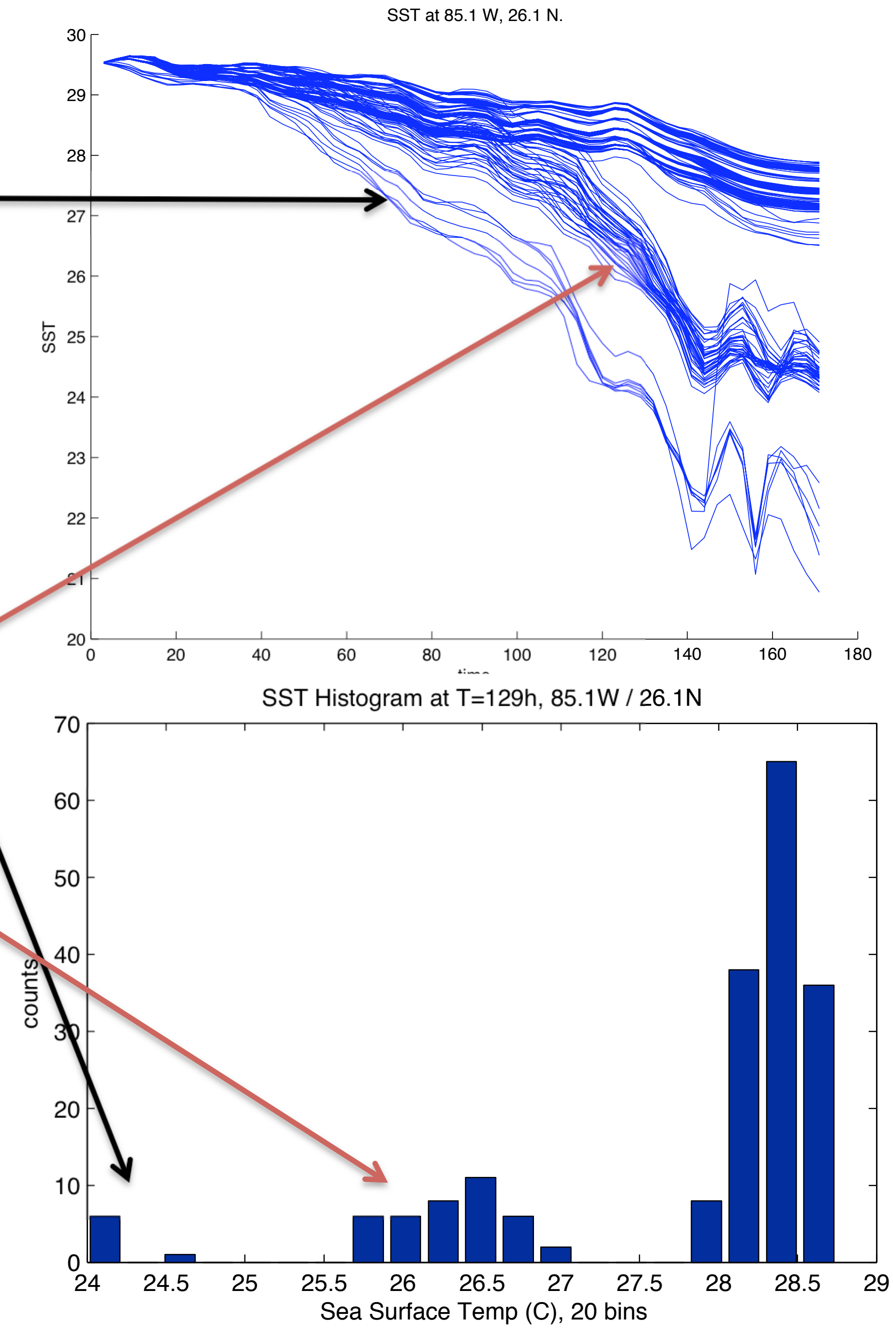
Sea Surface Temperature (SST)



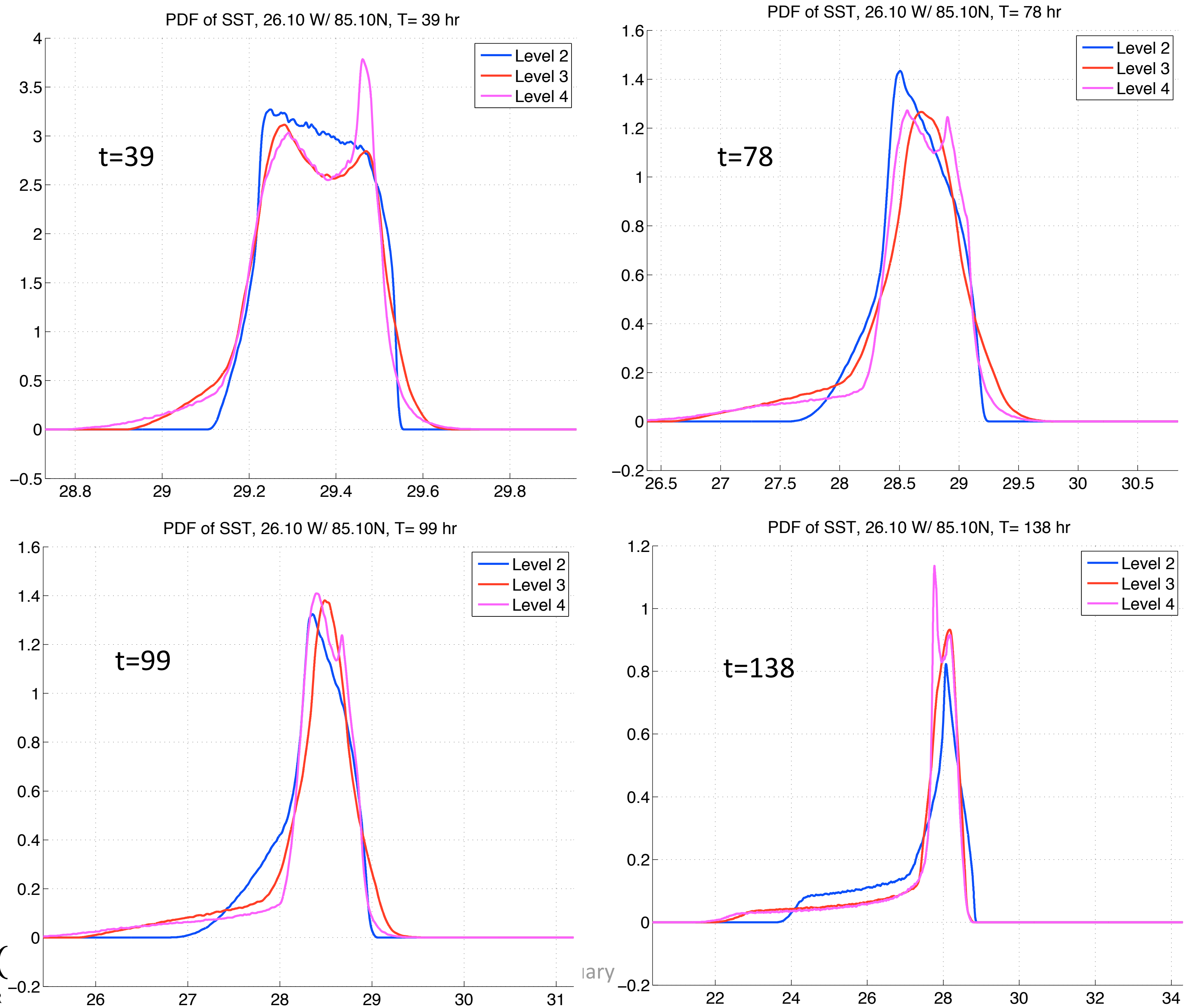
Probe

Stochastic Database





PDFs at different levels and order (p=l)



Bayesian Inference

Let H be a set of hypotheses made to model a set of data **D.** Bayes theorem states:

$$p(H|\mathbf{D}) \propto p(\mathbf{D}|H)\, p(H)$$

where p(H | **D)** is the posterior probability, p(**D | H)** is the likelihood function and p(H) is the prior of H.

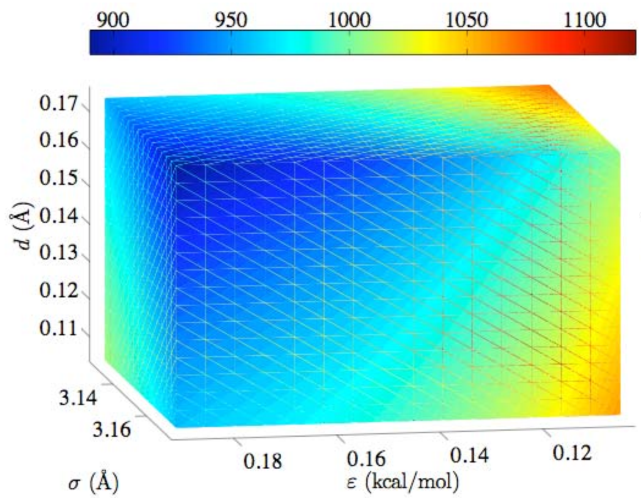
In our case H is the set of coefficients we want to infer:

- Model parameters
- PC coefficients themselves. This may be essential when model realizations are noisy!

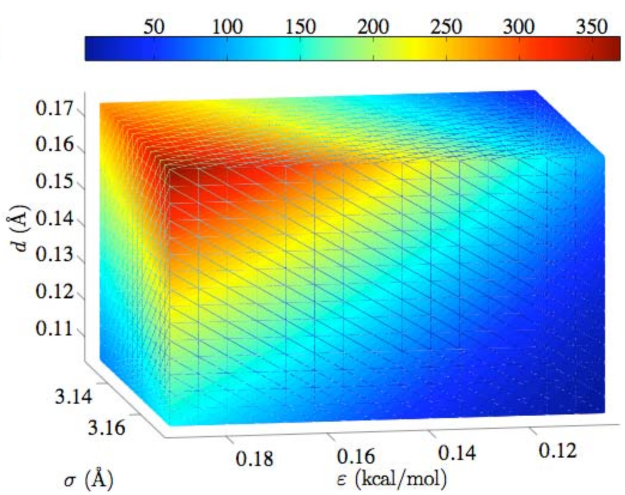
When PC representation of stochastic system is available, inference exercise can be greatly accelerated because it is MUCH CHEAPER to sample the PC representation than the forward model itself.

3D Example

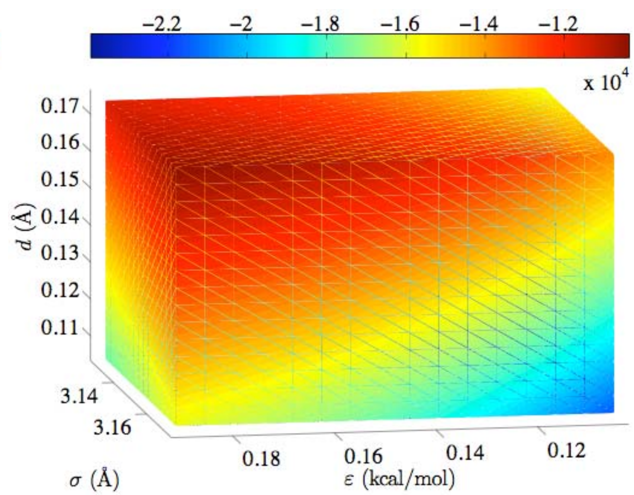
- True model: Synthetic molecular dynamics simulations of water.
- Three hidden parameters: $\varepsilon_t = 0.17$; $\sigma_t = 3.15$, $d_t = 0.14$
- Goal: model inference based on 5 noisy realizations
- Surrogate model for density, mean-square displacement (self-diffusion) and enthalpy:
  - 3rd-order expansion, obtained based on 7-point Gauss quadrature with multiple realizations at each quadrature point
  - Illustration based on MAP estimate of PC coefficients



density

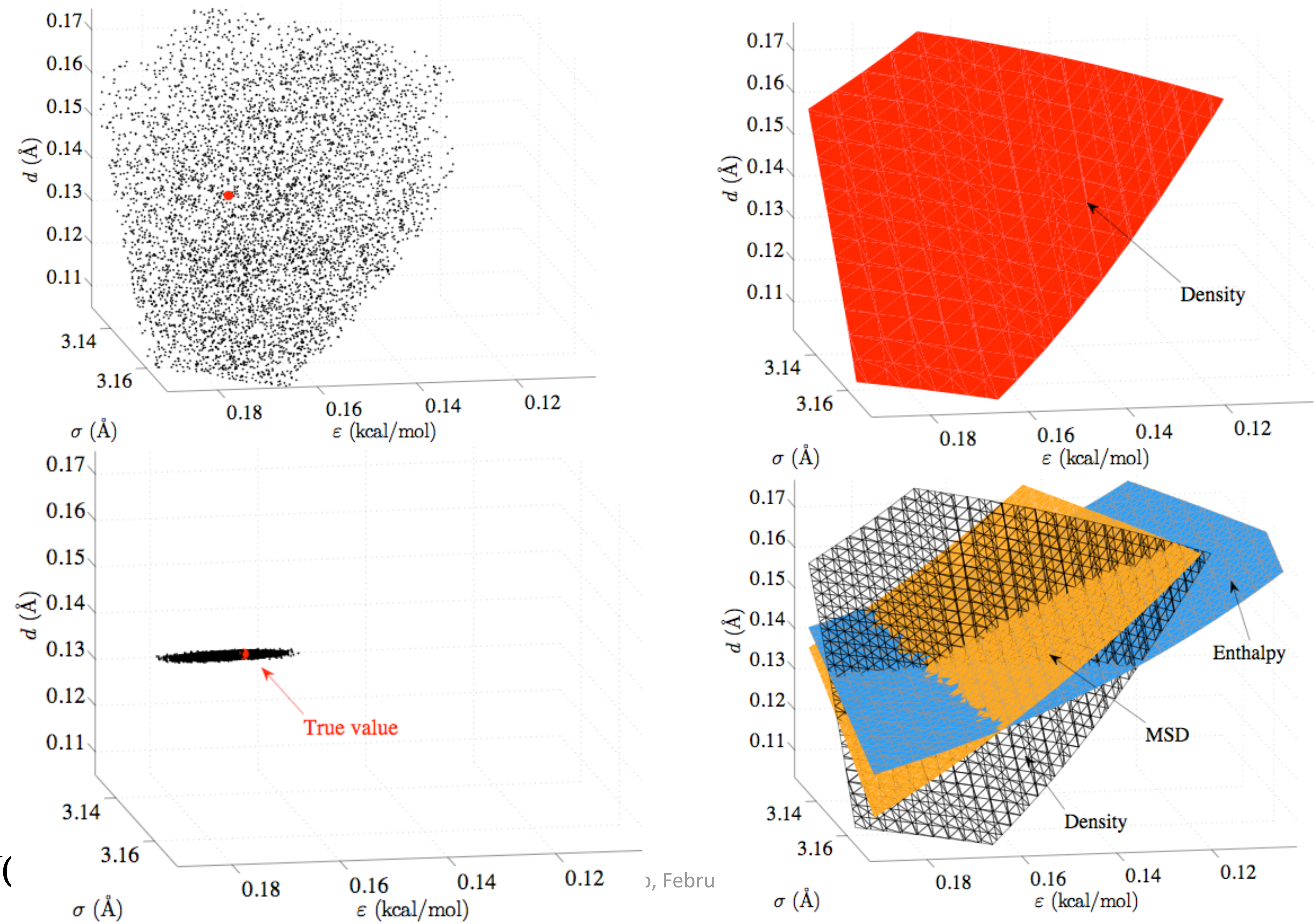


MSD



enthalpy

Inference based on 1 and 3 observables



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