Acoustic thermometry data compared with two ocean models: the importance of Rossby waves and ENSO in modifying the ocean interior

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Abstract

An interpretation is made of interannual changes in acoustic travel time between Oahu and seven receivers at distances of 3000–4000 km. Measurements were made in late 1983, and over two 5-month intervals between 1987 and 1989. Previous publications demonstrated that these changes stem from variations in temperature. Two hydrodynamic ocean models are used to identify plausible oceanic features that could cause these variations. They are from the Naval Research Laboratory and the Florida State University at (1/8)° and (1/6)° resolution, respectively, and are forced with different interannual wind sets for more than a decade. Modelled El Niño’s and La Niña’s generate poleward travelling Kelvin waves on the eastern boundary of the Pacific. These excite Rossby waves that propagate westward at mid-latitudes. Rossby waves are the dominant model features which affect the modelled acoustic travel times, and hence section-averaged temperatures in the eastern North Pacific. These waves yield travel times whose standard deviations and rates of changes are similar to the measurements. In the observations, some sections separated by less than 500 km exhibit trends in heat content with opposite signs. Similar variability can be explained with modelled Rossby waves. Model wavelengths less than 500 km, eddies, and seasonal cycles induced by seasonal winds yield travel times that are two orders of magnitude too small to account for the data. Published by Elsevier Science B.V.

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1. Introduction

Changes in atmospheric climate are potentially affected by exchanges of heat with the ocean. Oceanic observations are being collected by ships, satellites, and autonomous drifters. Observations are also made using artificially generated sounds. The travel times of pulse-like signals are measured between widely separated points and related to the temperature using tomographic techniques (Munk and Wunsch, 1979). The travel time depends primarily on the speed of sound which increases with temperature. If the temperature increases, the travel time decreases. Although changes in travel time over basin-scales are accurate measures of path-integrated temperature change (Spiesberger et al., 1992; Spiesberger and Metzger, 1992; Spiesberger and Tappert, 1996), specific features of the circulation responsible for these changes have not been identified (Fig. 1). Our concern is to identify plausible features. There are insufficient crossing sections to map the relevant features with tomography or to investigate temporal evolution on time scales of more than 5 months at a time. Therefore, we turn to ocean models for additional information.

There are two other reasons for comparing acoustic thermometry data with models. First, models need to be validated. Ocean models have not been checked with data having the accuracy obtained with these acoustic thermometers which are able to measure daily changes in the spatially-averaged temperature in the upper kilometer within 0.02°C (Spiesberger and Metzger, 1992; Spiesberger et al., 1992; Spiesberger and Tappert, 1996).

Secondly, claims have been made that acoustical arrays, like the Heard Island experiment, could detect anthropogenic changes in climate in one decade (Munk and Forbes, 1989; Semtner and Chervin, 1990) or two decades (Mikolajewicz et al., 1993). These modelling studies accounted for natural variability associated with eddies, seasonal cycles, and thermohaline cycles with periods near 300 years. Apparently, these studies did not account for components of the natural variability such as El Niño/Southern Oscillation (ENSO) or other apparent multi-decadal changes (Levitus, 1989; Roemmich and Wunsch, 1984; Jacobs et al., 1994; Deser et al., 1996; Clarke and Levedev, 1996) which have basin-scale impact with time scales similar to the claimed detection times. Spiesberger (1993) and Meyers et al. (1996) suggest that Rossby waves linked to ENSO are important in modifying heat contents at mid-latitudes. Indeed, we find that these Rossby waves lead to large changes in heat content and acoustic travel time, e.g., O(1) s, that dominate all other modelled fluctuations, such as eddies. If anthropogenic

Fig. 1. Plan view of the experiment with the acoustic source, S1, and the seven receivers, R1 to R7. The indicated positions of R3 and R4 are exact. The positions of the other receivers are inexact, and are placed to show the approximate coverage obtained with the sections. Other panels show the change in acoustic travel-time at each receiver up to June of 1989 except at R1 and R2 for which no data exist in 1989. Travel time change is set to zero on the first measurement taken in November 1987. The right ordinates provide an approximate guide (to within about 30%) as to how much the average temperature need change in the upper kilometer to yield changes in acoustic travel time. Adapted from Spiesberger et al. (1992).
changes are to be detected in a decade or two, it may be necessary to subtract out effects from ENSO and other decadal changes.

The Kaneohe source near Oahu transmitted acoustic signals at 133 Hz and 0.06 s resolution intermittently between 1983–1989 for the purpose of detecting and understanding climate variability (Fig. 1). The receivers are at distances of 3000 to 4000 km. It was hoped that these transmissions would continue through the next 50 to 100 years so that temperature variations due to human activity might be detected (Spiesberger et al., 1983). Travel times changed by about one part in 10,000 or ±0.2 s in 40 min of travel. The changes were dominated by interannual rather than seasonal variability.

This interannual variability is interpreted with ocean models which exhibit interannual variability. They were developed by the Florida State University (FSU) and the Naval Research Laboratory (NRL). Following an El Niño or La Niña, both models show a Kelvin wave propagating poleward along the eastern boundary of the Pacific. This excites a westward propagating Rossby wave at mid-latitudes (Meyers et al., 1996). A Rossby wave linked to the 1982–1983 El Niño travelled from the western coast of North America to Japan over a decade in the NRL model. It was also found in the sea-surface height signature measured with satellite altimeters (Jacobs et al., 1994). This independent confirmation of the model strengthens the findings in this paper that Rossby waves

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**Fig. 2.** Changes in travel time for the NRL model at receivers R1–R7. Effects from both temperature and currents are included, although the effects of currents are negligible.
are important in modifying both the acoustic travel times and heat contents over basin-scales in the eastern North Pacific.

As is true of many studies of climate change, new issues are raised which remain for future investigations. A new issue raised and partially addressed is to understand why the models yield travel times with characteristics similar to measurements, but whose correlation coefficients with the data are sometimes low.

It is meaningful to see if the measured and modelled travel times overlie one another because the modelled variations are overwhelmingly due to large-scale Rossby waves whose overall characteristics are insensitive to slight changes in the model's initial wind forcing. The modelled variations in travel time are much greater than the measurement error (e.g., back cover in Spiesberger and Metzger, 1992) or the non-deterministic components from the model. Models can be checked even though the data span times shorter than an ENSO cycle.

A high correlation with data can only be obtained if the model generates an accurate westward propagating Rossby wave following an El Niño. This wave can only be generated if the model output has an accurate poleward travelling Kelvin wave which is forced by El Niño. The accuracy of modelled Kelvin waves is investigated by comparing model sea levels with those from twelve tidal stations from Ecuador to Alaska. The NRL model, forced by daily winds from the European Center for Medium-range Weather Forecasts (ECMWF, 1995), agrees well with the sea level data for the 1982–1983 and 1991–1992 El Niños but not for the 1986–1987 El Niño, which is too weak in the model results. The ocean model physics is time independent. But the archived operational ECMWF wind product, available data, data assimilation technique, and atmospheric model physics and resolution changed many times over the 1981–1993 time frame of the ocean simulation (Trenberth, 1992; ECMWF, 1994). Therefore, the poor ocean model simulation of the 1986–1987 El Niño may be a consequence of the atmospheric forcing product. It is desirable to have ocean model simulations driven by atmospheric forcing that are derived using consistent techniques through time. In the

Table 1
Statistics of the changes in acoustic travel time between the Kaneohe source and the indicated receivers as computed with the NRL model

<table>
<thead>
<tr>
<th>Receiver</th>
<th>Complete</th>
<th>Non-seasonal &gt; 500 km</th>
<th>&lt; 500 km</th>
<th>Seasonal</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>(0.82, 0.17)</td>
<td>0.078, 0.030</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>(0.83, 0.17)</td>
<td>0.076, 0.027</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R3</td>
<td>(0.74, 0.17)</td>
<td>0.053, 0.018</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R4</td>
<td>(0.78, 0.20)</td>
<td>0.064, 0.021</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R5</td>
<td>(0.84, 0.17)</td>
<td>0.10, 0.039</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R6</td>
<td>(0.51, 0.11)</td>
<td>0.062, 0.023</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R7</td>
<td>(0.92, 0.18)</td>
<td>0.056, 0.015</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The first and second entries in parentheses indicate the mean change in and standard deviation of travel time, respectively in seconds. Statistics are evaluated for model output generated for the 12 years 1981 through 1992.
past such products were available only for certain regions (e.g., Stricherz et al., 1992). Global data sets are now becoming available (ECMWF, 1996; Kalnay et al., 1996). In any case, because most of the acoustic data were measured in 1987–1989, a high correlation between the model and data cannot occur without an accurate Kelvin/Rosby wave generated from the 1986–1987 El Niño.

The introduction concludes with a summary of other tomographic experiments designed to study climatic variations. Spiesberger and Metzger (1992) used 3000-km transmissions to study non-seasonal variations of heat content in the North Pacific. The Heard Island transmissions provided a demonstration of acoustic propagation and estimation of ship velocity from transmissions over global distances (Birdsall et al., 1994; Munk and Baggeroer, 1994). Connections have not yet been established between these transmissions and temperatures along the sections. Along with the GAMOT program (Freitag et al., 1995), the ATOC program has been underway since 1993 to measure temperature variations in the Pacific over long distances. Comparisons with models from ATOC should be forthcoming. Mikhailovsky et al. (1995) present evidence that temperature can be measured along a 2000 km section in the Arctic from 20 Hz transmissions. The THETIS experiment used transmissions at 100 km to estimate deep convection in the Western Mediterranean (Send et al., 1995). Morawitz et al. (1996)
estimated deep water production in the Greenland Sea from many kinds of data including tomographic transmissions at 200 km.

2. Ocean models

The FSU and NRL models solve primitive equations at mesoscale resolution using layers of constant density spun into motion by surface windstress (Hurlburt and
Thompson, 1980; Wallcraft, 1991; Hurlburt et al., 1996; Kubota and O'Brien, 1988; Pares-Sierra and O'Brien, 1989). These models have been compared with data including satellite altimetry, hydrography, drifters, infra-red images from satellites, tide gauge time series, current meter data, and ship drift climatologies (Johnson and O'Brien, 1990; Hurlburt et al., 1992, 1996; Metzger et al., 1992; Shriver and Hurlburt, 1997). The NRL model has six layers, a free surface, realistic bottom topography confined to the lowest layer, no slip boundary conditions, and an eddy viscosity of 100 m² s⁻¹. It has a horizontal grid resolution of (1/8)° between like variables and (1/16)° resolution between unlike variables on an Arakawa C grid. The domain extends from 20°S to 62°N in the Pacific. Daily averages are made from the ECMWF winds which are provided at 12 h intervals and at 2.5° resolution. The model is spun up for 137 years at (1/4)° resolution, then another 12 years at (1/8)° resolution using climatological winds at monthly resolution (Hellerman and Rosenstein, 1983). The transition to ECMWF winds is as follows. At each spatial location, the mean wind is computed from ECMWF from the beginning of 1981 to the end of 1991. The 1981–1991 mean at each location is removed and replaced by the annual mean over all time, from Hellerman and Rosenstein (1983) for that same location. This is simulation BT6i in Hurlburt et al. (1996).

The 1.5 layer reduced gravity FSU model has (1/6)° resolution between like variables and (1/12)° resolution between unlike variables on an Arakawa C grid. Topographic effects are included by modifying the local phase speed (Cushman-Roisin and O'Brien, 1983). The eastern boundary has a no-slip condition along the North American continent. Elsewhere, the boundary condition is open (Carmerlengo and O'Brien, 1980). An equatorial model is dynamically connected to a northeast Pacific model at the southern boundary of the eastern-most grid point of the northeast Pacific model domain to allow Kelvin waves to propagate poleward along the coast (Kamachi and O'Brien, 1994). The northeast Pacific model is forced with COADS winds (Slutz et al., 1985). The equatorial model is forced with FSU winds (Stricherz et al., 1992). After spin-up, both wind sets begin forcing the model in 1961.

### Table 2

The first entry in each parenthesis is the percentage of non-deterministic travel time variance in the NRL model given by \( P \) in Eq. (4).

<table>
<thead>
<tr>
<th>Receiver</th>
<th>Full</th>
<th>Non-seasonal</th>
<th>Seasonal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>All Scales</td>
<td>&gt; 500 km</td>
</tr>
<tr>
<td>R1</td>
<td>(3.2, 0.030)</td>
<td>(3.1, 0.030)</td>
<td>(15, 0.012)</td>
</tr>
<tr>
<td>R2</td>
<td>(3.7, 0.033)</td>
<td>(3.4, 0.030)</td>
<td>(23, 0.013)</td>
</tr>
<tr>
<td>R3</td>
<td>(5.6, 0.040)</td>
<td>(5.6, 0.040)</td>
<td>(24, 0.0088)</td>
</tr>
<tr>
<td>R4</td>
<td>(5.5, 0.047)</td>
<td>(5.0, 0.045)</td>
<td>(29, 0.011)</td>
</tr>
<tr>
<td>R5</td>
<td>(3.6, 0.032)</td>
<td>(3.6, 0.032)</td>
<td>(3.1, 0.0069)</td>
</tr>
<tr>
<td>R6</td>
<td>(16, 0.044)</td>
<td>(16, 0.044)</td>
<td>(10, 0.0073)</td>
</tr>
<tr>
<td>R7</td>
<td>(3.3, 0.033)</td>
<td>(3.2, 0.032)</td>
<td>(100, 0.015)</td>
</tr>
</tbody>
</table>

The second entry is the standard deviation of the travel time due to non-deterministic features. Units are seconds. Values are estimated from the 10-year period 1983 through 1992. \( P \) can exceed 100% when the variance of the deterministic component, \( \text{Var}[\delta T(t)] \), is less than the variance of the non-deterministic component, \( \text{Var}[\delta T(t) - \delta T(t)] \), in Eq. (4).
3. Computing acoustic travel times from ocean models

Changes in travel time depend on sound speed and currents as,

\[ \delta \tau(t) = - \int_\Gamma \frac{\delta c(\vec{r},t)}{c^2(\vec{r})} \, ds - \int_\Gamma \frac{\bar{u}(\vec{r},t) \cdot \delta(\vec{r})}{c^2(\vec{r})} \, ds, \]  

where the differential element of ray length is \( ds \), with unit vector, \( \hat{s}(\vec{r}) \), along the ray path, \( \Gamma \), at spatial coordinate \( \vec{r} \). Geophysical time is \( t \), the reference and perturbation fields of sound speed are \( c(\vec{r}) \) and \( \delta c(\vec{r},t) \), respectively, the current field is \( \bar{u}(\vec{r},t) \), and the vector dot product is \( \cdot \).

Comparisons are made between changes in modelled and measured travel times,

\[ \delta T(t) = \delta \tau(t) - \delta \tau(t_0), \]  

where \( t_0 \) is a reference date. The date is 1983.0 for the model. The measurements use different dates. Modelled changes are computed with an accuracy of 1 ms.

Fig. 5. Top: The 1983–1993 time-averaged percentage of non-deterministic variance between the two realizations of the NRL model (Eq. (7)) for the surface and third interface. The depth of the third interface is 320 m before the NRL model is spun up with winds. Bottom: The difference between the two NRL model simulations on 13 April 1985.
Fig. 6. The complete and low-passed anomaly of the third interface in the NRL model is shown at 40°N as a function of longitude and time. A positive displacement indicates downwelling. The black line gives the computed phase speed from Eq. (8) for the first baroclinic mode computed from the climatological data of Levitus (1982). The time at which the 1982–1983 and 1986–1987 El Niño’s (EN) and the 1985–1986 and 1988–1989 La Niña’s (LN) occur along the west coast of the U.S. are indicated. The upwelling (cold) feature originating from the lower right-hand corner is probably fictitious due to the start-up transient associated with using Hellerman–Rosenstein (HR) winds up to 1981 and ECMWF/HR hybrid winds thereafter. Hybrid refers to replacing the 1981–1991 ECMWF mean with the annual mean from HR.

Changes in sound speed are required for computing changes in acoustic travel time, but sound speed is not a state variable in the FSU nor NRL model. We assume that vertical displacements of modelled density interfaces lead to adiabatic changes in the speed of sound (Appendix A). This approximation is commonly used (Flatte et al., 1979). In the main thermocline, displacements affect sound speed significantly (Fig. 3 in Spiesberger et al., 1997). Below this thermocline, the speed increases primarily with pressure, so vertical displacements lead to minimal changes. Currents are state variables in the models. No additional calculations are required to assess their effects.

Modelled travel times are partitioned into seasonal and non-seasonal components. The non-seasonal components are further partitioned into wavelengths greater than 500 km, called the lowpass, and wavelengths less than 500 km, called the highpass (Appendix A, part c).

3.1. Ray paths

The source and receivers are mounted on the seabottom (Fig. 1). The rays corresponding to the four stable arrivals tracked at R3 have been well determined (Spies-
These rays have upper turning depths near 300 m near Oahu and shoal to near 50 m near the receiver. The changes in travel time of the stable arrivals at all the receivers are well approximated by integrating sound speed fluctuations along a frozen ray path (Appendix B in Spiesberger et al., 1992; Spiesberger and Tappert, 1996).

The paths of the stable arrivals have not been identified at the other receivers. These paths are taken to have the same upper turning depths after leaving Oahu as R3, namely about 300 m. This approximation is probably best for R2 and R4 which are adjacent. As

![Diagram](image)

Fig. 7. Same as Fig. 6 except these anomalies at 20, 30, 40, and 50°N are all from the low-passed de-seasoned NRL model.
will be seen in Section 5.2, this approximation yields modelled travel times with characteristics similar to the measurements.

4. Effects of modelled features on travel times

4.1. Travel time changes dominated by temperature not currents

The NRL model is used to estimate the effects from currents because the FSU model has no currents in its lower layer. The effects of modelled currents on modelled acoustic travel times are found to be insignificant at all the receivers (not shown). Instead, modelled temperature is the dominant parameter affecting changes at all the receivers because currents account for only 0.2% of the variance in modelled acoustic travel time, equivalent to a standard deviation of 0.007 s. This is less than the 0.03 s estimated previously, indicating that acoustic thermometers may be more accurate than 0.02°C (Spiesberger and Metzger, 1992). Modelled currents include baroclinic and barotropic components including changes in the large-scale wind driven circulation and quasi-geostrophic flows associated with Rossby waves and other features.
Fig. 9. The running yearly average of sea level at Sitka, Alaska (solid line) compared with the output of the NRL model (dashed line). Comparisons are made in three periods indicating effects from the 1982–1983, 1986–1987, and 1991–1992 El Niño’s.

4.2. NRL model

Between 1981–1993, travel times from the complete NRL model exhibit maximum changes of about 1 s and standard deviations of about 0.2 s (Fig. 2; Table 1). Except at R6, the largest event is a drop and rise of about 0.5 s between 1983–1986. As shown later, this is due to the 1982–1983 El Niño.

### Table 3

<table>
<thead>
<tr>
<th>Sea level station</th>
<th>Location</th>
<th>( \rho ) and fractional variance in sea level, ( F ), accounted for with the NRL model at the twelve indicated integrated global ocean services system stations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seldovia, AK</td>
<td>208.28</td>
<td>0.43</td>
</tr>
<tr>
<td>Seward, AK</td>
<td>210.55</td>
<td>0.51</td>
</tr>
<tr>
<td>Yakutat, AK</td>
<td>220.27</td>
<td>0.55</td>
</tr>
<tr>
<td>Sitka, AK</td>
<td>224.67</td>
<td>0.44</td>
</tr>
<tr>
<td>Prince Ruppert, Canada</td>
<td>229.67</td>
<td>0.65</td>
</tr>
<tr>
<td>Tofino, Canada</td>
<td>234.08</td>
<td>0.69</td>
</tr>
<tr>
<td>Bamfield, Canada</td>
<td>234.87</td>
<td>0.05</td>
</tr>
<tr>
<td>Crescent City, CA</td>
<td>235.80</td>
<td>-0.42</td>
</tr>
<tr>
<td>San Francisco, CA</td>
<td>237.53</td>
<td>-0.10</td>
</tr>
<tr>
<td>San Diego, CA</td>
<td>242.83</td>
<td>0.63</td>
</tr>
<tr>
<td>Buenaventura, Columbia</td>
<td>282.90</td>
<td>0.14</td>
</tr>
<tr>
<td>Santa Cruz, Ecuador</td>
<td>269.68</td>
<td>0.42</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>0.33</td>
</tr>
</tbody>
</table>

The average values are indicated. Note that the model explains a larger fraction of the measured variance for the 1982–1983 and 1991–1992 El Niños than for the 1986–1987 El Niño. See Eq. (10) for the definition of \( F \). The measured and modelled sea levels at Sitka, AL are shown in Fig. 9. A running yearly average has been applied to the data and model to suppress annual cycles. The data are available at monthly resolution.
More than 96% of the variance in modelled travel time is due to non-seasonal components having wavelengths exceeding 500 km (Table 1, Fig. 3). The sensitivity to long wavelengths is a consequence of the red spectrum of modelled fluctuations and the suppression of wavelengths that are small compared with the length of the section (Fig. 10 in Spiesberger et al., 1992; Fig. 12 in Spiesberger et al., 1997). The theoretical sensitivity of travel times to wavelengths at about 25 and 50 km (Cornuelle and Howe, 1987; Spiesberger et al., 1992) has an insignificant contribution.

Non-seasonal components with wavelengths less than 500 km amount to 1% of the variance. Seasonal cycles amount to less than 4% of the variance. Likewise, the data show little evidence of seasonal cycles (Fig. 1). For example, at R1, the ocean begins a warming trend in December that lasts until April; a trend opposite to that obtained from a seasonal cycle. Measurements taken 1 year apart at the same receiver can exhibit trends of opposite sign, e.g., at R3, R5, R6, and R7 (Fig. 1).

The seasonal effect from the model is small for two reasons. First, it does not incorporate thermodynamics that leads to strong seasonal cycles of temperature in the upper 100 m (Kang, 1980; Hurlburt et al., 1992, 1996). Even if modelled thermodynamically, R3’s ray paths do not spend much time in the seasonal thermocline, and would not be much affected (Fig. 2 of Spiesberger and Tappert, 1996). A similar statement cannot be made about the other receivers until their acoustic paths have been located. Second, the seasonal cycles in the winds yield Rossby waves whose effects on travel time are small compared with interannual forcing related to ENSO. This will be explored later.

4.2.1. Deterministic part of model

The NRL model is run twice from 1981–1993 to estimate deterministic and non-deterministic components. The runs are identical except for small differences in the initial state in 1981 and a small difference in the annual mean Hellerman–Rosenstein wind stress component.

Comparisons are made from 1983–1993 because the output from one run, called the tilde simulation, is only saved at 30.5 day intervals up to 1983. At later dates, both runs are saved at 3.05 day intervals. The run saved at 3.05 days since 1981 is referred to as the non-tilde simulation. A tilde is used below to distinguish these runs.

Changes in modelled travel times are not very sensitive to non-deterministic features (Fig. 4). Instead, travel times are sensitive indicators of deterministic and perhaps predictable features. Since changes in modelled travel times are associated with wavelengths exceeding 500 km (Table 1; Fig. 3), the deterministic features are associated with wavelengths exceeding 500 km.

Fig. 10. Comparison of travel time changes from measurements (dark–solid) and the NRL model (light–solid) at R1, R2, and R4. Modelled changes in travel time are set to zero at 1983.0. A constant offset is added to the changes in measured travel times to minimize the sum of the squares of the differences between the modelled and measured travel time changes. Travel times are averaged from the two NRL model runs to suppress non-deterministic variations.
Fig. 1. Same as Fig. 10 except for the indicated receivers.
An estimate for deterministic changes in travel time is,

$$\delta T(t) = \frac{\delta T(t) + \delta T(t)}{2}. \quad (\omega)$$

The percentage of non-deterministic travel time variance is,

$$P = 100 \frac{\text{Var}[\delta T(t) - \delta T(t)]}{\text{Var}[\delta T(t)]}, \quad (4)$$

where \( \text{Var}[\cdot] \) denotes variance. Note that this may be an underestimate because there are only two realizations.

Table 4

<table>
<thead>
<tr>
<th>Receiver</th>
<th>Travel time</th>
<th>Measured</th>
<th>NRL simulations</th>
<th>NRL simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sigma ) (s)</td>
<td>( \sigma ) (s)</td>
<td>( \bar{\sigma} ) (s)</td>
<td>( 2\sigma ) (s)</td>
</tr>
<tr>
<td>R1</td>
<td>0.13</td>
<td>0.12</td>
<td>0.09</td>
<td>0.19</td>
</tr>
<tr>
<td>R2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R3</td>
<td></td>
<td></td>
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<tr>
<td>R4</td>
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<td>R5</td>
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<tr>
<td>R6</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R7</td>
<td></td>
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</tr>
</tbody>
</table>

The third and fourth columns denote outputs from the non-tilde and tilde simulations, respectively. Those columns' standard deviations are estimated using only the dates on which the travel times were measured from the Kaneohe source. Standard deviations from the entire modelled time series, 1981–1993 (Table I), are about twice those shown in column three. The two most right-hand columns give the mean, \( \bar{\sigma} \), and two standard deviation, \( 2\sigma \), of the standard deviations of travel times computed from the non-tilde NRL simulation on the same months during which data were taken, but -on different years. The measured standard deviations are consistent with the NRL model if they fall within \( \bar{\sigma} \pm 2\sigma \).
Except for R6, less than 6% of the variance in travel time stems from non-deterministic features. This is equivalent to a standard deviation of less than 0.03 s (Table 2). At R6, the non-deterministic variance is 16%, with an associated standard deviation of 0.04 s. Model wavelengths less than 500 km and some seasonal cycles are sensitive to model realization (Table 2).

Non-deterministic components of the model are quantified to verify that they usually occur at small scales. The vertical coordinate, with positive sign upwards, of the ith interface between constant-density layers is $D_i(x, y, t)$ where $(x, y)$ denotes horizontal position and $t$ time. Its anomaly is,

$$\eta_i(x, y, t) = D_i(x, y, t) - m_i(x, y) \quad i = 1, 2, \ldots 6$$

where $m_i(x, y)$ is the temporal mean of $D_i(x, y, t)$. The average anomaly is,

$$a_i(x, y, t) = \frac{\eta_i(x, y, t) + \bar{\eta}_i(x, y, t)}{2}$$

The ratio of non-deterministic to deterministic variance of an interface, expressed as a percentage, is,

$$P_i(x, y) = 100 \frac{\text{Var}[\eta_i(x, y, t)]}{\text{Var}[a_i(x, y, t)]}$$

where the variance is taken over time. The non-deterministic variance may be underestimated from only two realizations.

Non-deterministic features occur primarily at scales less than 500 km. (bottom row, Fig. 5). The percentage of non-deterministic variance is greater in the interior than at the surface because higher order modes are less deterministic than mode one which is responsible for most of the surface variations (top row, Fig. 5). Some of the non-determinism stems from baroclinic instabilities.

Fig. 13. Histograms of the measured and modelled absolute values of the rates of changes of acoustic travel time at the seven receivers. The NRL model is used to obtain values in the right-hand panel. Rates of change are evaluated at 2-month intervals.
4.3. Rossby waves in the NRL model

Depth anomalies of the third modelled interface are shown in Figs. 6 and 7 for 1981–1993 as a function of longitude at a fixed latitude. This interface is near the depths where sound speed anomalies are large (Fig. 3 in Spiesberger et al., 1997). Its depth is 320 m before the model is spun up. After spin up, it slopes downward from the east to the western subtropical gyre.

The non-seasonal low-passed anomaly is the principal contributor to the modelled changes in acoustic travel time (Section 4.2). This anomaly propagates at speeds similar to those computed from the linear phase speeds for long Rossby waves of first baroclinic mode (black curves, Figs. 6 and 7),

\[
\frac{\omega}{k} = \frac{c_e^2 \cos \phi_0}{2 \Omega R \sin^2 \phi_0}
\]

(8)

where the wave's frequency and wavenumber are \(\omega\) and \(k\), respectively, the latitude is \(\phi_0\), the Earth's radius is \(R\), and the rotational frequency of the Earth is \(\Omega\) (p. 503, Gill, 1982). \(c_e\) is defined as,

\[
c_e = \sqrt{g \bar{H}_e}
\]

(9)

where the equivalent depth, \(\bar{H}_e\), is obtained by solving the Sturm–Liouville problem in Eq. (6.11.17) of Gill (1982). Equivalent depths are estimated at a grid spacing of 100 km to account for geographic variability of the ocean's depths and profiles of buoyancy frequency. Buoyancy frequency is estimated from the data of Levitus (1982).


<table>
<thead>
<tr>
<th>Receiver</th>
<th>Correlation coefficient</th>
<th>Model consistent with data?</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>+0.97</td>
<td>Yes</td>
</tr>
<tr>
<td>R2</td>
<td>-0.0055</td>
<td>Yes</td>
</tr>
<tr>
<td>R3</td>
<td>+0.23</td>
<td>No</td>
</tr>
<tr>
<td>R4</td>
<td>+0.089</td>
<td>Yes</td>
</tr>
<tr>
<td>R5</td>
<td>+0.30</td>
<td>Yes</td>
</tr>
<tr>
<td>R6</td>
<td>+0.77</td>
<td>Yes</td>
</tr>
<tr>
<td>R7</td>
<td>+0.14</td>
<td>No</td>
</tr>
<tr>
<td>Average</td>
<td>+0.36</td>
<td>--</td>
</tr>
</tbody>
</table>

The average is shown for each column. If the measured correlation coefficient is greater than given in the right-hand column in Table 7, then the model is consistent with the data. Consistency means that the differences between model and data can be accounted for with the effects of measurement noise and non-deterministic components of the model (Appendix B).
1994). The third interface in the model exhibits displacements up to 100 m. A downwelling wave of between 50-100 m causes sound speeds to increase by about 1.5 to 3 m s\(^{-1}\) and temperatures to increase by about 0.3 to 0.7°C near 300 m depth (Fig. 3 in Spiesberger et al., 1997). These intense waves are not observed below the black lines in Fig. 7 which indicate the phase speeds of the Rossby waves. Before 1981, the model was spun up with Hellerman–Rosenstein winds which do not contain a Southern Oscillation. The intense features propagating westward thousands of kilometers in Figs. 6 and 7 are dispersionless Rossby waves linked to ENSO (Introduction). The smaller Rossby wave signals below the black lines are forced by local winds.

The modelled annual signals from downwelling Rossby waves (left panel, Fig. 6) do not propagate more than about 500 km west at 40°N. They are not evident in the low-passed output of the model and they contribute little to the changes in modelled travel times.

There is a critical latitude, poleward of which flows less than a critical period cannot radiate into the interior of the ocean (Clarke and Shi, 1991). The critical periods along the west coast of North America at 20, 30, 40, and 50°N are 0.6, 0.6–0.9, 0.9, and 3–4 years, respectively. Model anomalies tend to exhibit longer periods at more northerly latitudes (Fig. 7). As a result, travel time changes at the southern receivers such as R5 should and do exhibit higher frequency oscillations in travel time than northern receivers (Fig. 2). The modelled Rossby wave from the 1982–1983 El Niño tapers out between 45 and 50°N (Fig. 10 in Spiesberger et al., 1997).

4.4. FSU model

Outputs of the FSU and NRL models are compared because they are forced by different wind-stress fields and they implement the primitive equations in different ways. Nonetheless, they both propagate Rossby waves at mid-latitudes in response to major ENSO events. Their modelled travel time changes have similar magnitudes and both exhibit a decrease and subsequent increase of about 0.5 s between 1984–1986 at R1–R5 in response to the 1982–1983 El Niño (Figs. 2 and 8). The warming trend in the FSU model is not evident in the NRL model because this FSU model does not conserve mass.

---

### Table 6

Cross-correlation coefficients between series of travel time change at neighboring receivers.

<table>
<thead>
<tr>
<th>Receiver pair</th>
<th>Cross-correlation coefficient</th>
<th>Cross-correlation coefficient</th>
<th>(Mean, Std) Min</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>NRL non-tilde simulation</td>
<td>NRL tilde simulation</td>
</tr>
<tr>
<td>R1–R2</td>
<td>−0.17</td>
<td>+0.97</td>
<td>+0.71</td>
</tr>
<tr>
<td>R2–R3</td>
<td>−0.26</td>
<td>+0.84</td>
<td>+0.66</td>
</tr>
<tr>
<td>R3–R4</td>
<td>+0.76</td>
<td>+0.72</td>
<td>+0.85</td>
</tr>
<tr>
<td>R4–R5</td>
<td>+0.80</td>
<td>−0.49</td>
<td>+0.58</td>
</tr>
</tbody>
</table>

The values from the models in columns three and four are computed using the same geophysical times at which the data are measured. The values in the last column give the mean, standard deviation, and minimum value of the cross-correlation coefficient from the 12-years of the non-tilde NRL simulation analyzed in 5 mo sections.
due to its open boundary conditions. The NRL model has closed boundary conditions and conserves mass.

Because the FSU model is simpler and more efficient to run than the NRL model, it is desirable to estimate the similarity of modelled travel times. The modelled propagation speeds of the waves are different because the modelled density fields differ. When plausible differences are accounted for in the propagation speeds of the Kelvin and Rossby waves, and when a linear trend is removed from both model's travel times at each receiver, the correlation coefficient of travel times between 1981–1990 is 0.33 with

![SOUND SPEED ANOMALY (m/s)]

Fig. 14. The top panels show the sound speed anomaly from the NRL model between the Kaneohe source (left) and R3 at two dates (3 April 1985 and 24 January 1990) corresponding to a warm period (left) and colder period (right). The lower panels show the sound speed anomalies from the third interface in the model on those same dates. The fast/warm anomaly in the left hand column is the Rossby wave which is linked to the 1982–1983 El Niño. This Rossby wave is what causes modelled travel times to decrease by up to 0.8 s at most of the receivers between 1984–1986 (Fig. 2). The sound speed anomalies are taken with respect to the time-averaged interface depth, \( m_i(x, y) \).
a standard deviation of 0.09. It is beyond the scope of this paper to quantify this difference.

5. Comparison with data

5.1. Sea level

Following El Niños, poleward traveling Kelvin waves change sea level at Sitka, Alaska near 57.05°N and 224.67°E, where data provided by the Integrated Global Ocean Services System are smoothed with a 1-year running mean to suppress annual cycles (Fig. 9). The output of the NRL model fits the events in 1982–1983 and 1991–1992 better than that in 1986–1987. The fraction of sea level variance,

\[ F = 1 - \frac{\text{Var}[\text{residual}]}{\text{Var}[\text{data}]} \]  

accounted for with the model in each of the three periods indicated in Fig. 9 is 0.85, 0.44, and 0.82, respectively. The residual is the difference between the model and data following a least-squares fit. Similar results are obtained at eleven other tide gauge stations along the east coast of the Pacific from Alaska to Ecuador (Table 3). The mean fractional variances accounted for with the model are 0.76, 0.33, and 0.68, respectively for the 1982–1983, 1986–1987, and 1991–1992 events. The comparison can be made with a normalized correlation coefficient if one is not interested in a constant scaling factor. The mean correlation coefficients for these three events are 0.90, 0.63, and 0.85, respectively.

The unrealistically weak poleward travelling Kelvin wave following the 1986–1987 El Niño is probably unrelated to the model’s resolution, topography, and coastline since the wave is accurately portrayed following the other two El Niño’s. It is difficult to blame the mismatch on unmodelled thermodynamic effects, as Kelvin waves are hydrodynamic. The mismatch between modelled and measured sea level may be related to the model’s forcing field (Section 1). The 1982–1983 El Niño was the strongest of the century, so it is more difficult to miss the associated wind events in the tropical Pacific than for the weaker 1986–1987 El Niño. The ECMWF wind products improved significantly in the early 1990’s due to a new data source, i.e., the TOGA–TAO array and associated improvements in the ECMWF prediction model. The agreement between measured and modelled sea level in the 1990’s may be due in part to this improved wind product. Satellite altimeters pick up Rossby waves associated with both the 1982–1983 and 1986–1987 El Niño’s in the N. Pacific. However, there is a weak response of the NRL model to the ECMWF wind product during and following the 1986–1987 El Niño.

Fig. 15. Same as Fig. 1 except these are travel times from the NRL model. The complete set of modelled travel times is shown in Fig. 2. Note the striking differences in travel times at adjacent receivers, especially at R1–R3. The starting and ending months of the modelled travel times are similar to the measurements in Fig. 1, except these results are 2 years later.
The modelled Rossby wave associated with the 1986–1987 event is much weaker than observed.

5.2. Acoustic travel times

Acoustic arrivals have been identified with rays at R3 (Section 3.1). The modelled travel times at the other receivers are approximated by assuming that the upper turning depths of the rays are similar to those from R3 just after leaving Oahu. These paths are used in Sections 5.2.1, 5.2.2 and 5.2.4 below and in Figs. 10, 11 and 15. Additionally, the results in Section 5.2.3 below do not depend on which ray paths are used for the calculations.

The changes in modelled travel times are not out of character with the data (Figs. 10–12). Unlike the other receivers, the discrepancies at R3 cannot be blamed on the uncertainties in the ray paths (Fig. 12). At R3 in 1983, modelled times are 0.4 s longer than measured and this cannot be explained by non-deterministic features of the model, 0.04 s (Table 2), or by noise in the daily measurements, 0.03 s. The NRL model has not had time to propagate Rossby waves from ENSO events along the entire section by 1983, as the transition to ECMWF winds is made in 1981. Interannual winds are applied to the FSU model in 1961 leaving sufficient time for spin-up along this section. In both the FSU and NRL models, acoustic travel times are longest in the middle to last part of 1983 at R3 (Figs. 2 and 8). If travel times from the FSU model are matched to the NRL model in early 1984, we see that travel times from the FSU model are not 0.4 s less in late 1983 as would be needed to fit the data. Indeed the FSU travel times are longer in late 1983 than early 1984. Consequently, the FSU model does not indicate that the 0.4 s discrepancy between the NRL model and the data is due to the onset of ECMWF winds in 1981.

5.2.1. Standard deviations

At R1–R7, the standard deviations of the measured travel times are between 0.025 and 0.12 s. These are similar to both realizations of the NRL model (columns 3 and 4, Table 4). Some mismatch is expected since there are non-deterministic components in the model and there are inaccurate Kelvin/Rossby waves following the 1986–1987 El Niño. Sub-sampling modelled travel times during the same months as the measurements, but on different years, yields modelled standard deviations that are consistent with measurements (columns 5 and 6, Table 4). The modelled variance comes from Rossby waves linked to ENSO (Table 1; Fig. 7). Contributions from other modelled features contribute less than 15% of the variance. They are too small to account for the measurements.

5.2.2. Rates of changes of travel times

The absolute values of the rates of changes of the acoustic travel times are similar to those predicted with the NRL model (Fig. 13). The data undergo changes as much as 0.1 s mo⁻¹ that may be maintained for several months. For example, at R3 they decrease at 0.1 s mo⁻¹ for 3 to 4 months in early 1988 (Fig. 1). The change of −0.4 s can only be caused by deterministic features in the model, i.e., Rossby waves linked to ENSO, since
the modelled standard deviations of the non-deterministic components are 0.04 s (Table 2).

5.2.3. Correlation coefficients

For each receiver, travel times are computed from the NRL model for ray paths with upper turning depths between 50 and 550 m near Oahu. Correlation coefficients are computed between modelled and measured travel times. The values of the correlation coefficients given in Table 5 change less than 10% except for R7, where coefficients range from $-0.06$ to $0.27$. Thus, the correlation coefficients are insensitive to the geometry of a ray path.

This insensitivity comes about because of the dominant role played by Rossby waves. Steeper (flatter) rays spend more (less) time in the sound speed perturbations associated with these waves. Changing the ray’s steepness and associated upper turning depth changes the magnitude of the change in travel time, but not the shape. Thus the travel time is given by $\alpha(\hat{z})\delta T(t)$ where $\delta T(t)$ is the travel time shown in Fig. 2 and $\alpha(\hat{z})$ is a constant depending on the upper turning depth, $\hat{z}$, of the ray near Oahu. Since the correlation coefficient is independent of the variance of a time series, little change in correlation coefficient is found for different rays.

Non-deterministic features of the model and errors in the measurements are too small to account for the differences with the data at R3 and R7 (Table 5; Appendix B). However, non-deterministic features and measurement errors are sufficient to account for the differences at the other five receivers. For the non-tilde NRL simulation, the correlation coefficients at R1–R7 have a mean and standard deviation of 0.36 and 0.37, respectively. For the tilde NRL simulation, the mean and standard deviation are 0.39 and 0.37, respectively.

5.2.4. Cross-correlations between neighboring receivers

We see if the NRL model can yield travel times with distinctly different changes at neighboring receivers, as observed (Fig. 1). These differences are reflected in some of the cross-correlation coefficients between neighboring receivers. Values vary from $-0.26$ to $+0.80$ (Table 6). During the measurements, the modelled cross-correlation coefficients, from the two NRL model runs, are typically not similar to those computed from the data (Table 6).

Other dates are examined to see if the NRL model ever generates travel times that resemble the measurements. The modelled travel times from 1981–1993 are divided into 5 mo segments, similar in duration to the data. These often yield low values for cross-correlation coefficients (column five, Table 6). This might seem surprising since modelled Rossby waves have large scales. But modelled Rossby waves have phase crests nearly parallel to the acoustic sections by the time they have propagated about half-way between California and Hawaii (red region in lower left-hand panel, Fig. 14). One section can pass across the Rossby wave’s phase crest, while the section at an adjacent receiver passes through a significantly different region of the wave. Also note that modelled Rossby waves are modulated at smaller scales (lower left-hand panel, Fig. 14). This structure further contributes to differences in modelled travel times.

Modelled travel times are plotted during the same months as measured but 2 years later (Fig. 15). The changes at R1–R3 are very different, as observed (Fig. 1). In Fig.
15, the cross-correlation coefficients between R1–R2, R2–R3, R3–R4, and R4–R5 are −0.36, −0.68, +0.84, and +0.64.

6. Summary and discussion

The FSU and NRL models are quite different, and they are forced by different interannual wind-stress fields. They both predict that Rossby waves, linked to ENSO, are the dominant features which affect modelled acoustic travel times over basin-scales in the eastern North Pacific. The measured travel times do not overlie those from the NRL model at some receivers. The measured and modelled standard deviations, rates of changes, and the structures of trends between adjacent receivers are similar.

Acoustic transmissions over basin-scales are particularly well suited for checking large scale anomalies in ocean model simulations because the data suppress the mesoscale and small wavelengths (Figs. 10 and 11 in Spiesberger et al., 1992). Calculations with the NRL model indicate that the variances of acoustic travel times over basin-scales in the eastern North Pacific depend on the horizontal wavenumber of temperature as $k^{-5.5}$ where the horizontal wavenumber is $k$ (Spiesberger et al., 1997). The discrepancies between the NRL model and the data suggest improvements need to be made to either the forcing functions of the NRL model and/or to the NRL model itself.

Clarke and Levedev (1996) show that equatorial wind-stress decreases from the 1970’s to the 1980’s with a magnitude similar to that from a moderate El Niño. To the extent that the ocean reacts linearly, this anomaly would lead to poleward travelling Kelvin waves and westward propagating Rossby waves at mid-latitudes. Since the Rossby wave is dispersionless, its scale would be several times greater than that from the 1982–1983 El Niño, and thus affect acoustic travel times more homogeneously than the 1000 km Rossby waves modelled in this paper. It should take many decades to understand the origin of natural variations in oceanic temperature.

Acoustic data may estimate changes in the sea-surface height more accurately than satellite altimeters. If the average temperature in the upper kilometer changes by 0.02°C, the elevation of the surface changes by about 0.3 cm (Gill, 1982). The standard deviation of the time-dependent error of the TOPEX–POSEIDON altimeter is about 3.5 cm, with an associated covariance scale of 1500 km (Tsaoussi and Koblinsky, 1994). The change in section-averaged sea-surface height is $\Delta H(t) = \frac{1}{S} \int_0^S h(s,t) ds - \frac{1}{S} \int_0^S h(s,t_0) ds$ where sea-level along the section is $h(s,t)$, $s$ is distance along a section of length $S$, $t$ is time, and $t_0$ is a reference time. Assuming altimeter errors are uncorrelated at intervals $t-t_0$, the standard deviation of $\frac{1}{S} \int_0^S h(s,t) ds$ over a 4000 km section is about $3.5\text{cm}/\sqrt{4000/1500} \approx 2\text{cm}$. The standard deviation of $\Delta H(t)$ is $\sqrt{2^2 + 2^2} \approx 3\text{cm}$. Consequently, acoustic data over the same distance may be ten times more accurate, and may provide validation of satellite data and accurate estimates for the integral changes in geostrophic currents at the surface. Acoustic travel times are less sensitive than altimetry measurements to changes in height due to convergence of mass. How acoustic tomography and satellites best complement one another requires further exploration.
Acknowledgements

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Appendix A. Computing acoustic travel times with hydrodynamic models

A.1. Estimating sound speed from the NRL model

Vertical displacement between interfaces is estimated using linear interpolation. Adiabatic changes in sound speed are computed from the second term on the right-hand side of Eq. (1) of Spiesberger et al. (1997), where ζ there denotes depth, positive down. The speed of sound is obtained from Eq. (1) of this same reference. Sound speed, c(x, y, z), is estimated from the algorithm of Del Grosso (1974) using the climatological values of temperature, salinity, and depth of Levitus (1982). Bilinear interpolation is used to compute sound speed perturbations between the grid points of the model. Six layers, accommodating five baroclinic modes, are an acceptable representation of the sound speed field since more than 90% of the variance is typically accounted for with the first three or four vertical empirical orthogonal functions (Cornuelle et al., 1985; Malanotte-Rizzoli et al., 1985).

A.2. Estimating sound speed from the FSU model

The vertical displacement of the single FSU interface from its time-averaged depth is η̃(x, y, t) in analogy with Eq. (5). Before spinning up the model with wind, the interfacial depth is 175 m. The vertical displacement of water is approximated using,

η(x,y,z,t) = η̃(x,y,t) \frac{\hat{x}(x,y,z)}{\hat{x}(x,y,z = m(x,y))} \tag{A1}

where \hat{x}(x,y,z), is the eigenfunction of the Sturm–Liouville problem given by Eq. (6.11.17) in Gill (1982). The maximum value of \hat{x}(x,y,z) is normalized to one. Levitus’s data of temperature and salinity are used to estimate the density and buoyancy frequency. Gill’s equation is solved along the acoustic path at 100 km intervals to
account for the geographic variability of the density and buoyancy profiles. The perturbation in the speed of sound is obtained by substituting \( \eta(x, y, z, t) \) from Eq. (A1) into Eq. (1) of Spiesberger et al. (1997). Flierl (1978) has considered other methods which relate reduced-gravity models to vertical modes.

A.3. Decomposition of travel time change.

Changes in the vertical interfaces of the models are decomposed as,

\[ \eta_i(x, y, t) = m_i(x, y) + \delta s_i(x, y, j) + LP[\delta a_i(x, y, t)] + HP[\delta a_i(x, y, t)], \]

where the time-averaged part is \( m_i(x, y) \), the seasonal variations for month \( j \) are \( \delta s_i(x, y, j) \), and the non-seasonal variations, \( \delta a_i(x, y, t) \), are spatially filtered into wavelengths greater than \( LP[\delta a_i(x, y, t)] \), and less than \( HP[\delta a_i(x, y, t)] \), 500 km, respectively using a Chebyshev filter (Parks and Burrus, 1987). The low-pass filter’s response is \(-1\) dB at 1000 km wavelength, \(-1.8\) dB at 750 km, \(-4.2\) dB at 500 km, \(-12\) dB at 250 km, and \(\geq 30\) dB for wavelengths \(\leq 200\) km.

Appendix B. Significance of correlation coefficient in Table 5

At each receiver, the travel times changes from 1983.0 to 1993.0 from the tilde and non-tilde simulation of the NRL model are divided into \( i = 1, 2, 3, \ldots, I \) segments of equal duration. The durations are the same as measured. The \( i \)th segment has deterministic component, \( \delta T_i(t) \), and non-deterministic component,

\[ \epsilon_i(t) = \delta T_i(t) - \overline{\delta T_i(t)}, \]  

(Eqs. (2) and (3)). For segment \( p \), there are \( k - 1 \) mimics of modelled travel time change,

\[ \delta T_{p}^j(t) = \overline{\delta T_p(t)} + \epsilon_j(t); \quad j = 1, 2, 3, \ldots, k - 1, \]  

where variations between each mimic are due to non-deterministic components only. Mimics for the \( p \)th measured segment are,

\[ \delta T_{p}^k(t) = \overline{\delta T_p(t)} + \epsilon_k(t) + n_p(t); \quad k = 2, 3, \ldots, I \text{ and } j < k, \]  

where the noise, \( n_p(t) \), is added to imitate measurement noise. \( j < k \) to ensure non-deterministic components are almost uncorrelated in the mimics for the modelled and measured segments. For segment \( p \), there are \( R = I(I - 1)/2 \) realizations, \( \text{Corr}[\delta T_{p}^j(t)\delta T_{p}^k(t)], 1 \leq j < k \leq I, \) for the correlation coefficient. It has sample mean, \( \rho_p \), and sample standard deviation, \( \sigma_{\rho_p} \). These are strong functions of the standard-deviation of the deterministic component of the travel time change (Fig. 16), since the expected value of the correlation coefficient is,

\[ E[\rho_i] = \frac{\sigma_{\delta T_i}}{\left[\sigma_{\delta T_i}^2 + 2\sigma_{\epsilon_i}^2\sigma_{\delta T_i}^2 + \sigma_{\epsilon_i}^2 + \sigma_{\epsilon_i}^2 + \sigma_{\epsilon_i}^2\right]^{1/2}}, \]
assuming the covariance, Cov\[n_i(t), \epsilon_j(t)]\], equals zero, an excellent approximation. The variances of the deterministic and non-deterministic travel time changes are $\sigma_{\tau T_i}^2$ and $\sigma_\epsilon^2$, respectively, and the variance of the noise is $\sigma_n^2$, taken to equal $(0.03s)^2$, the same

Table 7
The right-hand column is an estimate of the smallest value that a correlation coefficient can have between model and data and support the hypothesis that the modelled and measured travel times are consistent

<table>
<thead>
<tr>
<th>Receiver</th>
<th>$\sigma_{\tau T_i}$ (s)</th>
<th>$\sigma_n$ (s)</th>
<th>$\sigma_\epsilon$ (s)</th>
<th>$\sigma_{\tau T_i}$ (s)</th>
<th>$\rho_i$</th>
<th>$2\sigma_{\rho_i}$</th>
<th>$\bar{\rho}<em>i - 2\sigma</em>{\rho_i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>0.12</td>
<td>0.03</td>
<td>0.030</td>
<td>0.11</td>
<td>0.95</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>0.025</td>
<td>0.03</td>
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<td>0</td>
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<td>1</td>
<td></td>
</tr>
<tr>
<td>R3</td>
<td>0.011</td>
<td>0.03</td>
<td>0.040</td>
<td>0.098</td>
<td>0.90</td>
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<td></td>
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<tr>
<td>R4</td>
<td>0.050</td>
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<td>0.047</td>
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<td>0</td>
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<td></td>
</tr>
<tr>
<td>R5</td>
<td>0.046</td>
<td>0.03</td>
<td>0.032</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>R6</td>
<td>0.089</td>
<td>0.03</td>
<td>0.044</td>
<td>0.071</td>
<td>0.85</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>R7</td>
<td>0.095</td>
<td>0.03</td>
<td>0.033</td>
<td>0.084</td>
<td>0.9</td>
<td>0.08</td>
<td></td>
</tr>
</tbody>
</table>

If the correlation coefficients in Table 5 are less than given in the right-hand column, then the NRL model is inconsistent with the data at the two-standard deviation confidence level. The second column is the standard deviation of measured travel times (column 2 in Table 4). The standard deviation of the non-deterministic components of modelled travel time, given by Eq. (B1), is $\sigma_\epsilon$. The estimate of the deterministic standard deviation of the measured travel time, obtained from Eq. (B5), is $\sigma_{\tau T_i}$. The expected value and standard deviation of the correlation coefficient are $\bar{\rho}_i$ and $\sigma_{\rho_i}$, respectively.
as measured (Spiesberger et al., 1992). The number of realizations going into these samples can be large, since \( R \) varies between 190 to 66 for data-length segments of 6 and 10 months, respectively.

At each receiver, a figure like Fig. 16 is used to estimate the expected value of the correlation coefficient between data and model due to measurement noise and non-deterministic components of the model. To do this, one requires an estimate of the standard deviation of the deterministic component of the data which can be obtained from,

\[
\sigma_{0T} = \sqrt{\sigma_{\text{data}}^2 - \sigma_n^2}
\]

where the variance of the data is \( \sigma_{\text{data}}^2 \). If the measured correlation coefficient, shown in Table 5, is less than \( \rho_i - 2 \sigma_{\rho_i} \), as given by the right-hand column in Table 7, then the modelled travel times are inconsistent with the measurements.

References


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