MET3220C & MET6480 Computational Statistics

Error and Error Propagation

Systematic and Random errors

Key Points:
1) Random errors can be mistaken for biases when examining paired data.
2) Random errors cause uncertainty in the answers to questions.
3) Sometimes these questions are best answered with probabilities.

Types of Error

- There are two general types of errors: systematic errors and random errors.
- Systematic errors follow a mathematical pattern
- Examples:
  - Bias: a uniform error. E.g., the temperature is always 3°C too high.
  - Gain: An bias in slope. E.g., y = 3.1 x rather than y = 2.9 x.
  - Complex function. E.g., error in sheltered thermometer's temperature is equal to
    constant1 * (solar radiation + constant2) / (wind speed + constant3)
- Random errors are NOT systematic.
  - They appear to be random.
  - The often have a Gaussian distribution.
  - Example: estimating the decimal place in temperature, when the thermometer only indicates integers.

Why Do We Care About Errors?

- It is hard to make conclusions about physics (e.g., climate) if we can't tell if the differences are due to something physical or due to errors.
- Example: Global averaged temperatures are increasing. Is it due to an actual increase in temperature, or a bias due to changes in the observing system (e.g., urban heat islands).
- Example: less rain falls in Florida during an El Nino year. Is this finding due to a real change in rain totals, or due to random errors in the very limited and noisy observations?
- Ideally, biases are determined through comparison to independent data, and then removed from the data set.
  - This ideal is great for laboratory data, but is hard to work with in the real world. Why?
    - It is hard to get independent high quality data, that is physically similar to the data in question.
- We want to be able to say how likely it is that a difference is physical, rather than an artifact of random error.

Is a Difference Due to Random Error?

- Our ability to answer this question depend on how we can characterize the random error.
  - A measure of the largest possible error (called absolute error).
  - A measure of spread (the common approach)
- Absolute error example: a mean of a population of 10 items.
  - Take, for example, the lengths of the 10 hairs remaining on a professor's head. Is the total hair length greater than 1m?
  - Assume that our measuring tool has a scale in millimeters.
    - Assume that we can be accurate to 0.5mm
  - Assume that we have sufficient attention span that our accuracy will not suffer from boredom.
  - The largest possible error assumes that none of the errors will cancel out.
  - For the total length, the absolute error is the sum of the individual errors (10*0.5mm). For the mean, we would then divide this error by 10, resulting in 0.5mm absolute uncertainty.
  - If the total length is greater than 1m plus the absolute error, then we can be sure the prof's total hair length is greater than 1m.

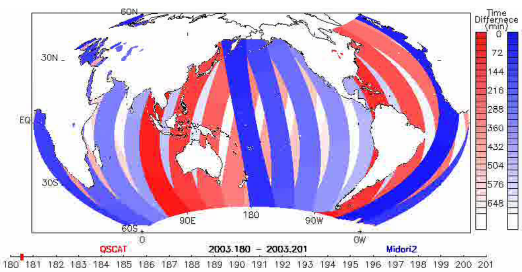
More Complex Example of Absolute Error

- Note that in cases with many samples, absolute error can get rather large.
- Consider the ideal case law (meteorology version of it):
  - Pressure = density * gas constant for dry air * absolute temperature
  - Assume that we are determining density, from observations of pressure and temperature: $\rho = R_d T / P$
  - The worst case interpretation is that P is underestimated, and that T is overestimated. We will assume that R_d is known accurately enough that considering error in R_d has negligible influence on the absolute error in ρ.
  - For one sample, the absolute error (worst case) in ρ is
    $AE(\rho) = R_d (T + AE(T)) / (P - AE(P)) - R_d T / P$
  - This is equal to the most changed value minus the original value.
  - Consider adding up absolute errors for all the partial pressures of the atmospheric constituents.

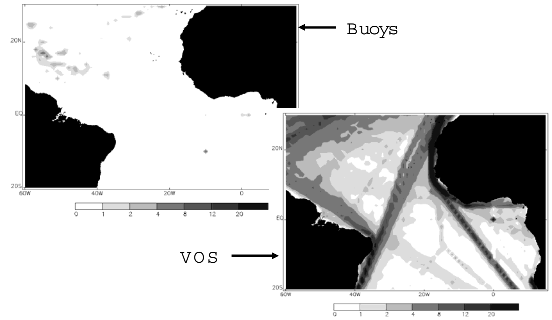
Uncertainty Measured as a Spread

- The big difference from absolute error is that some random errors are assumed to cancel out. All random errors are not in the same direction!
- There are two types of uncertainty to be considered.
  - Observational (or recording) error, and
  - Sampling error.
- Observational error
  - Errors refer to uncertainty in observations.
    - Example: random errors in pressure might have a standard deviation of 0.01kPa.
    - Example: weather station temperatures are recorded with a precision of 1°F, resulting in a standard deviation of about 0.4°F.
- Sampling error is due to insufficient sampling of a population.
  - Example: mean height of meteorology students, based on heights of students in MET3220C-02. The uncertainty in the mean due to sampling is equal to the standard deviation divided by squareroot N.

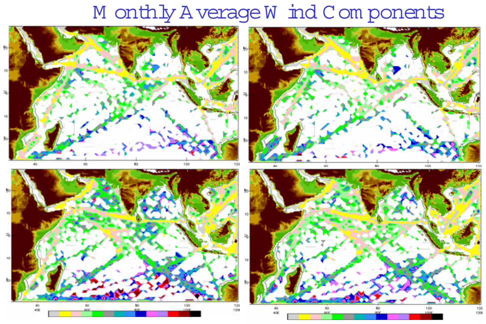
Coverage by Two SeaWinds Scatterometers



Example VOS and Buoy Observations Dec. Average from 1988-1997



Observational Error + Sampling Error Monthly Average Wind Components



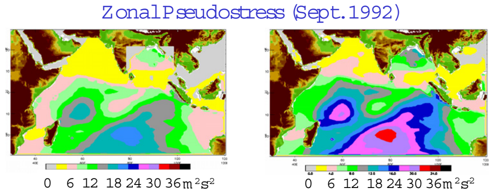
Combining Sampling And Observational Errors

- In practice, random errors due to observational error and sampling error both contribute to random uncertainty.
- Key (good) assumptions:
  - Observational errors are independent from sampling errors.
    - This is a great assumption for random error
    - Not so good for complex biases.
  - Biases have been removed (or are small compared to random errors).
    - Sometimes this ideal is hard to achieve.
- If the above assumptions are met, then the variances associated with each type of random error are additive.
  - Recall that variance is the square of the standard deviation.
- In other words, the standard deviations are additive in a root-mean-square sense: total uncertainty = $[(\text{obs uncert})^2 + (\text{samp uncert})^2]^{1/2}$
  - This equation applies to the uncertainty in one term.

Representation Error

- The 'total random error' on the previous slide is based on the assumption that the proverbial apple is being compared to another proverbial apple.
  - Example: wind speeds from one type of anemometer being compared to wind speeds at a nearby location, and measured with the same type of anemometer (calibrated identically to the first anemometer).
- In the field (AKA the real world), this ideal is rarely achieved. Why?
- We rarely have two of the same instruments in the same location, unless they are part of a planned exercise in validation.
  - Usually we are working with different types of instruments, measuring at different times over different periods, and usually in different locations.
  - Example: comparing satellite footprints to in observations from ships or buoys.
- Nevertheless, representation error is often ignored – sometimes safely

Example of Various Errors Zonal Pseudostress (Sept. 1992)



- Uncertainty including observational and representation errors (upper left)
- Total uncertainty in background: observational, representation, and sampling (upper right).
  - Fields are monthly averaged and smoothed over a large spatial domain.
  - The smoothing results in uncertainty related to representation errors.

Error Propagation: How Do Errors Combine in Equations?

- The previous pages described how to combine different types of errors contributing to uncertainty in a single observation.
- How do we combine uncertainties in different terms in an equation?
  - Example: consider the zonal wind component (u), determined from observations of wind speed (w) and wind direction (θ)
    - u = w cos(DTOR * (90 – θ))
    - Where DTOR is a constant converting from degrees to radians, and there is uncertainty in θ and q.
- Fortunately, there is a single equation that explains how to handle error propagation.

$$\text{Given } y = f(x_1, x_2, x_3, \ldots, x_N), \quad \sigma_y^2 = \sum_i^N \left[\left(\frac{\partial f}{\partial x} \right) \sigma_x \right]^2$$

$$\sigma_u^2 = \left[\cos\left(DTOR * (90.0 - \theta)\right) \sigma_w \right]^2 + \left[-w\,DTOR \sin\left(DTOR * (90.0 - \theta)\right) \sigma_\theta \right]^2$$

- Which is likely to be the bigger cause of error: speed errors or direction errors?

Example

- Consider temperature data, distributed messily in two dimensions. For example temperatures from surface stations.
  - Pretend these are from an area without changes in altitude
- Smoothing is sometimes applied with a Gaussian filter. This filter weights the data, based on a Gaussian function, with the weight decreasing as distance increases away from the point of interest.

$$\overline{T} = \frac{1}{N} \sum_i^N G(\Delta r_i) T_i$$

$$\sigma_{\overline{T}}^2 = \frac{1}{N} \sum_i^N G_i^2(\Delta r_i) \sigma_{T_i}^2$$