A Selection of Ocean Model Fundamentals

Stephen Griffies (NOAA/GFDL)

Lectures given at the GODAE School for Operational Oceanography (Ecole d’Ete Oceanographie Operationnelle GODAE) September 2004
La Londe Les Maures, France

9/23/2004
Goals of these lectures

• To pedagogically introduce elements of ocean models, their uses, and their fundamentals.
• To motivate learning model fundamentals—they are actually quite interesting, fun, and critical to the evolution of ocean modeling into a robust science from a somewhat ad hoc art.
• To explore some model formulation issues (kinematics, dynamics, algorithms) to whet the student’s appetite.
• Expose some issues of generalized vertical coordinates. These are the basis for new model codes in use today (e.g., HYCOM and the MITgcm), and actively being developed for future research and operational uses (e.g., HOME=Hybrid Ocean Model Environment).
Three Main References

• A Selection of Ocean Model Fundamentals, Lectures from the 2004 GODAE School on Operational Oceanography, to be published in by Kluwer in 2005.

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Fundamentals of Ocean Climate Models

A monograph that describes some physical, mathematical, and numerical foundations for ocean climate models. Much material is general, and so of relevance to operational oceanography and arbitrary vertical coordinates (though the author’s expertise is z-modeling for global climate, and so prejudices are apparent).
Developments in Ocean Climate Modelling

Review paper summarizing developments up through 2001 important for ocean climate modeling, much of which are also relevant for operational ocean modeling.

Developments in ocean climate modelling

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Abstract

This paper presents some research developments in primitive equation ocean models which could impact the ocean component of realistic global coupled climate models aimed at large-scale, low frequency climate simulations and predictions. It is written primarily to an audience of modellers concerned with the ocean component of climate models, although not necessarily experts in the design and implementation of ocean model algorithms. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

The purpose of this paper is to present some developments in primitive equation ocean models which could impact the ocean component of realistic global climate models aimed at large-scale, low frequency climate simulations and predictions. It is written primarily to an audience of

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Some purposes/uses of ocean models

• Scientifically rationalize the observed ocean. Examples:
  – To assimilate WOCE data sets to provide a mechanistic interpretation of observations
  – To test hypotheses for physical, chemical, or biological mechanisms underlying observations.

• Predict future changes in the ocean. Examples:
  – To forecast mesoscale features (e.g., Gulf Stream)
  – To determine scenarios for large scale trends arising from changes in anthropogenic forcing (e.g., changes/collapse in Atlantic meridional circulation in a warmer world).

• Provide scientifically based advice to policy makers for managing coastal related commerce
  – fisheries and other resources
  – shipping and recreation
  – energy use policy
  – waste disposal
  – coastal development
  – coastal impacts of climate change
Ocean Observations: growing in space-time
ARGO Floats: models help rationalize via assimilation
Multidecadal Ocean Variability

- Proxy records exhibit this variability back hundreds of years.
- The temporal agreement in this ensemble member is fortuitous, but it suggests variability is natural.
North Atlantic Ocean Decadal Predictability

- Coupled simulations suggest predictability on timescales of years
- Predictability depends on initial conditions
- Examples at left are predictable for 13, 9, and 8 years.

An R&D project funded by the European Union under Framework 5

Coordinator
Rowan Sutton
Centre for Global Atmospheric Modelling
University of Reading

A more serious project to study possibilities for Atlantic decadal predictability
Oceanic Heat Storage
Delays warming to changes in forcing

◆ Current climate would warm about 1°C assuming no further increase in GHG (near zero emissions).

◆ Modeled ocean heat storage trends similar to observed
Can Human Induced Climate Change Alter the Overturning Circulation?

- The overturning circulations in most models weaken as climate warms.
- Models and data indicate the presence of multi-decadal oscillations.
- Is this happening in Nature’s ocean?

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NOAA/GFDL coupled model circa 2000
Possible effects from North Atlantic changes??

Fictional depiction from Hollywood. 
Climate change has made it to the Big Time!

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Coastal and climate interactions

An area of increasing importance and from many applied sectors, and an interesting research problem as well.

Large-scale coupled climate dynamics

Coastal influences

Schematic from Hans Von Storch

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Two examples of ocean model simulations

• Modeling Eddies in the Southern Ocean (MESO). A NOAA/GFDL project using the Hallberg Isopycnal Model (HIM) to study the impacts of eddies on the Southern Ocean circulation.

• High resolution global modeling from the MITgcm on the cubed sphere. An example of how non-traditional horizontal grids resolve some problems with spherical grids on the sphere related to coordinate singularities.
Studies of the role of resolution and eddies in climate variability

Courtesy of Bob Hallberg, NOAA/GFDL

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Cubed Sphere Ocean with MITgcm

\[ C_{512} = 512^2 \cdot 6 \]

7 km \( \leq \Delta x \leq \) 19 km

10 years/day

Includes Arctic – has sea-ice

\[ \text{Simulated ocean current speed at 15m} \]

\[ \text{Jan} \quad 1993 \]

• 480 SGI Altix processors, NASA

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Courtesy of Alistair Adcroft, Princeton/GFDL
Some Challenges for Ocean Modeling

• Role of mesoscale eddies
• Influence of marginal seas and topographic control on the open ocean
• Influence of open ocean circulation on shelf circulation and ecosystems
• Representing the ocean carbon cycle
• Improved understanding of modes of large-scale ocean variability

• Interactions between the mixed layer, atmosphere and subsurface ocean
• Estimation of ocean state
• Rapid ocean initialization
• Physically consistent estimates and parameterizations of diapycnal mixing
• Replacing numerical closures with physical parameterizations
• Reducing bias induced by model numerics
Strategy for Improved Ocean Models

High Res Regional Models

Ocean Obs

Global Climate Models

High Res Global Models

Parameterizations

Observing System Design

Assess Robustness of Findings

Courtesy Bob Hallberg, NOAA/GFDL
Models are key to understanding ocean

- **Models are not reality:** Egregious problems with their representation of Nature’s ocean due to
  - limitations in computational power
  - incomplete understanding of model fundamentals such as subgrid scale parameterizations
  - poorly known forcing fields
  - inaccurate interactions with other components of the climate system such as the atmosphere and sea ice

- **Observations are not reality.** Many holes in space-time that preclude full information about ocean’s state, its variability, trends, and possible instabilities and regime shifts.

- **Models provide an apparatus to scientifically understand the ocean via hypothesis testing and experimentation.**
  - There is only one ocean, whereas there are many realizations of ocean simulations.
  - Judicious use of ocean observations, theories, model hierarchies, assimilation, hindcasts, and forecasts, can help to deduce and infer elements of Nature’s ocean.
What are ocean model fundamentals? (1)

• Nearly every question about ocean modeling boils down to three issues
  – Fundamentals
  – Boundary forcing
  – Analysis methods

• Fundamentals are concerned with underlying physical, mathematical, and numerical aspects of an ocean model.
  – Geophysical and computational fluid mechanics
  – Oceanography—descriptive and theoretical
  – statistical physics—for subgrid scale parameterizations
  – algorithm design—methods to solve the equations on a computer

• Equations:
  – hydrostatic or non-hydrostatic?
  – Boussinesq or non-Boussinesq?
  – Rigid lid or free surface?
  – Virtual tracer fluxes or real water fluxes?
  – Advective form of momentum equations or vector invariant form?
What are ocean model fundamentals? (2)

• **Formulation:**
  - Vertical coordinates—geopotential, pressure, terrain, isopycnal, generalized hybrid?
  - Horizontal grid: Arakawa A,B,C,D,E, spectral, finite element?
  - Horizontal grid structure: regular spherical coordinates, regular generalized, tripolar, cubed sphere, icosahedra, nested, unstructured finite elements, time dependent adaptive?
  - Finite volume foundation?

• **Algorithms:**
  - time stepping
  - discrete advection operators
  - Coriolis force
  - implicit vertical physical processes
  - pressure gradient force
  - equation of state

• **Subgrid scale closure:** Unresolved processes, both physical and numerical, are ubiquitous and often of first order importance.

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Three main vertical coordinates in use, none of which provides universal utility. This motivates research and development of hybrid approaches.
What’s the big deal about vertical coordinates?

• Properties of the ocean:
  – Hydrostatic (at scales > 1km)
  – Quasi-Adiabatic and density stratified (away from boundaries)
  – Rotating
  – Surface forced from atmosphere, ice, rivers
  – Constrained by bathymetry

• Consequences: quasi-conservative of PV, tracers, momentum, etc.

• The vertical coordinate strongly affects ability of a numerical model to respect these properties, and to parameterize unresolved physical processes.
Contradictory Considerations in Choosing a Vertical Coordinate

1. The vertical coordinate must be monotonic with depth for any stably stratified density profile.

2. The solenoidal pressure gradient term should be absent or relatively small compared to the non-solenoidal pressure gradient term with an accurate equation of state.

3. Material changes in density due to numerics should be much smaller than changes due to physical processes.

4. Coordinate surfaces should coincide with locally-referenced neutral surfaces to permit a nearly two-dimensional representation of advection and isoneutral mixing.

5. It should be possible to concentrate resolution wherever important processes occur, including boundary layers and interior regions of large gradients.

6. Consistency is much easier to establish with a single vertical coordinate.

7. The coordinate should make the top and bottom boundary conditions easy to implement exactly.

8. The coordinate should facilitate analysis of simulations.

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Courtesy Bob Hallberg, NOAA/GFDL
Community supported z-models circa 2001

Table 1
Summary of z-coordinate ocean models currently developed and supported with applications to climate related studies a

<table>
<thead>
<tr>
<th>Model/Institute/Language</th>
<th>Documentation</th>
<th>Web site</th>
</tr>
</thead>
<tbody>
<tr>
<td>CANDIE/Dalhousie/F77</td>
<td>Sheng et al. (1998), Lu et al. (2000)</td>
<td><a href="http://www.phys.ocean.dal.ca/programs/CANDIE">www.phys.ocean.dal.ca/programs/CANDIE</a></td>
</tr>
<tr>
<td>COCO/CCSR/F77</td>
<td>Hasumi (2000)</td>
<td><a href="http://www.ccsr.u-tokyo.ac.jp/~hasumi/COCO/">www.ccsr.u-tokyo.ac.jp/~hasumi/COCO/</a></td>
</tr>
<tr>
<td>Hadley/Hadley/F77</td>
<td>Gordon et al. (2000)</td>
<td><a href="http://www.dkrz.de/forschung/reports.html">www.dkrz.de/forschung/reports.html</a></td>
</tr>
<tr>
<td>HOPE/DKRZ/F77</td>
<td>Wolff et al. (1997)</td>
<td>mitgcm.lcs.mit.edu</td>
</tr>
<tr>
<td>MIT/MIT/F77</td>
<td>Marshall et al. (1997a,b)</td>
<td><a href="http://www.gfdl.gov/MOM.html">www.gfdl.gov/MOM.html</a></td>
</tr>
<tr>
<td>MOM/GFDL/F77-F90</td>
<td>Pacanowski and Griffies (1999)</td>
<td><a href="http://www.mth.uea.ac.uk/ocean/SEA">www.mth.uea.ac.uk/ocean/SEA</a></td>
</tr>
<tr>
<td>MOMA/SOC/F77</td>
<td>Webb et al. (1997, 1998a)</td>
<td><a href="http://www.mth.uea.ac.uk/ocean/SEA">www.mth.uea.ac.uk/ocean/SEA</a></td>
</tr>
<tr>
<td>SEA/EA/F77</td>
<td>Webb (1996)</td>
<td><a href="http://www.soc.soton.ac.uk/JRD/OCCAM/">www.soc.soton.ac.uk/JRD/OCCAM/</a></td>
</tr>
<tr>
<td>OCCAM/SOC/F77</td>
<td>Webb et al. (1997, 1998a)</td>
<td><a href="http://www.ipsl.jussieu.fr/~gmlod/OPA_web">www.ipsl.jussieu.fr/~gmlod/OPA_web</a></td>
</tr>
<tr>
<td>OPA/LODYC/F77</td>
<td>Madec et al. (1998)</td>
<td><a href="http://www.acl.lanl.gov/climate/models/pop">www.acl.lanl.gov/climate/models/pop</a></td>
</tr>
<tr>
<td>POP/LANL/F90</td>
<td>Smith et al. (1992)</td>
<td></td>
</tr>
</tbody>
</table>

Many models and many years of experience, mostly for global climate and regional modeling

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Table 2
Summary of $\rho$-coordinate ocean models currently developed and supported\textsuperscript{a}

<table>
<thead>
<tr>
<th>Model/Institute/Language</th>
<th>Documentation</th>
<th>Web site</th>
</tr>
</thead>
<tbody>
<tr>
<td>MICOM (HYCOM)/Miami, LNL, Stennis/F77</td>
<td>Bleck et al. (1992)</td>
<td>panoramix.rsmas.miami.edu/micom/</td>
</tr>
<tr>
<td>POSEIDON/cola, George Mason/F90-F95</td>
<td>Schopf and Loughe (1995)</td>
<td>grads.iges.org/poseidon</td>
</tr>
<tr>
<td>POSUM/Oregon State/F77</td>
<td>de Szoeke &amp; Springer</td>
<td>posum.oe.orst.edu/</td>
</tr>
</tbody>
</table>

Fewer than z-models, but more cohesion between the models
Community supported sigma models circa 2001

Relatively few. Used by thousands of scientists and engineers, especially for coastal applications.

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<th>Web site</th>
</tr>
</thead>
<tbody>
<tr>
<td>POM/Princeton, GFDL/F77</td>
<td>Blumberg and Mellor (1987)</td>
<td><a href="http://www.aos.princeton.edu/WWW-PUBLIC/htdocs.pom">www.aos.princeton.edu/WWW-PUBLIC/htdocs.pom</a></td>
</tr>
<tr>
<td>SPEM (ROMS)/Rutgers, UCLA/F77</td>
<td>Haidvogel et al.</td>
<td>marine.rutgers.edu/po/</td>
</tr>
</tbody>
</table>
2.3 Z-models

Geopotential, or z-models, discretize the vertical according to the distance from a resting ocean surface, where the resting ocean approximates the geoid. Some key advantages of z-models are the following:

- Simple numerical methods have been used, to some success, in this framework.
- For a Boussinesq fluid, the horizontal pressure gradient can be easily represented.
- The equation of state for ocean water is accurately represented.
- The upper ocean mixed layer is well parameterized using a z-coordinate.

Some of the disadvantages include:

- The representation of tracer advection and diffusion along inclined density surfaces in the ocean interior is cumbersome.
- Representation and parameterization of the bottom boundary is unnatural.
2.4 Isopycnal models

Isopycnal models discretize the vertical into potential density classes. Some key advantages of isopycnal models are the following:

- Tracer transport in the ocean interior is well represented due to the natural ability of these models to respect water mass properties.
- The bottom topography is represented in a piecewise linear fashion, hence avoiding the need to distinguish bottom from side as traditionally done with z-models.
- Flows near topographically critical regions, such as overflows, are well resolved by isopycnal models due to the natural tendency of the coordinate surfaces to become refined in these regions.
- For an adiabatic fluid, the horizontal pressure gradient can be easily represented.
- For an adiabatic fluid, the volume (for a Boussinesq fluid) or mass (for a non-Boussinesq fluid) between isopycnals is conserved.

Some of the disadvantages are the following:

- Representing the effects of a realistic (nonlinear) equation of state is cumbersome.
- An isopycnal coordinate is inappropriate for the mixed layers, where density becomes vertically unstratified.
Pros and cons of sigma models

- They provide a natural framework to represent bottom influenced flow and to parameterize bottom boundary layer processes.
- Thermodynamic effects associated with the equation of state are well represented.

Some of the disadvantages are the following:

- As with the z-models, the representation of the quasi-adiabatic interior is cumbersome due to numerical truncation errors inducing unphysically large levels of spurious mixing, especially in the presence of vigorous mesoscale eddies.
- $\sigma$-models have difficulty accurately representing the horizontal pressure gradient. We discuss this point more fully in Section 10.
Community supported hybrid models circa 2004

- HYCOM Bleck (2002).
  - Emphasis on pressure, isopycnal, and sigma hybrid.
  - Does not include best of each individual model class, such as sophisticated pressure gradient algorithms or rotated physical parameterizations.
  - Nonetheless, the most advanced hybrid model of today.

- MITgcm Adcroft and collaborators
  - Emphasis on z-like coordinates, such as zstar and pressure
  - No isopycnal option.

- POM and ROMs: Rutgers and UCLA
  - Emphasis on sigma and depth options
  - No isopycnal option.

- Overall, hybrid models are immature and undergoing a rapid research and development trajectory. Compelling reasons to move forward with various hybrid ideas.

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A global 1 degree isopycnal climate model

5-Day Average Sea Surface Height

Isopycnal Surfaces at 140°W

Pacific section

Courtesy Bob Hallberg, NOAA/GFDL

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The z-p Isomorphism

- **Atmospheric equations**
  \[ \begin{align*}
  D_t \vec{v}_h + 2\Omega \times \vec{v}_h + \nabla_p \Phi &= \vec{F} \\
  \alpha + \partial_p \Phi &= 0 \\
  \nabla_p \cdot \vec{v}_h + \partial_p \omega &= 0 \\
  \partial_t p_s + \nabla \cdot p_s \langle \vec{v}_h \rangle &= 0 \\
  D_t \theta &= Q_\theta \\
  \alpha &= \theta \partial_p \Pi
  \end{align*} \]
  - non-Boussinesq

- **Oceanic equations**
  \[ \begin{align*}
  D_t \vec{v}_h + 2\Omega \times \vec{v}_h + \frac{1}{\rho_o} \nabla_h p &= \vec{F} \\
  g\rho + \partial_z p &= 0 \\
  \nabla_z \cdot \vec{v}_h + \partial_z w &= 0 \\
  \partial_t \eta + \nabla \cdot (H + \eta) \langle \vec{v}_h \rangle &= P - E \\
  D_t \theta &= Q_\theta
  \end{align*} \]
  - Boussinesq

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Menagerie of z-like and p-like vertical coordinates

FV = finite volume

FV z coordinate

z = η

FV z* coordinate

z* = 0

η coordinate

z* = -H*

p coordinate (FV)

p = p_s

p* = p_s^o

Courtesy Alistair Adcroft, Princeton/GFDL
Motivation for $z^*$ coordinate

- **Free surface height ($z$) coordinate models**
  - Accurate FV topography
  - No pressure gradient errors
  - Irreg./variab. comp. domain
  - Vanishing surface layer

- **Terrain following coordinate ($\sigma$) models**
  - Smooth topography(?)
  - Pressure gradient errors
  - Regular comp. domain
  - Fixed comp. domain

- **$z^*$ coordinate**
  - Best of both worlds?
  - Irregular comp. domain
  - Fixed comp. domain
  - Accurate external mode

*Courtesy Alistair Adcroft, Princeton/GFDL*
Stacey’s $z^*$ coordinate

- Vertical motion due to external mode is absorbed into coord. system
  - more stable
  - reduced spurious fluxes associated with vert. motion
- Easier conservation than varying top layer
- There is a pressure gradient error
  - **BUT** it is small!

$$z^* = \sigma H = \frac{z - \eta}{H + \eta} H$$
$$\partial_{z^*} z = \frac{H + \eta}{H} \sim 1$$

Internal Wave Generation
- Stratified fluid
- Barotropic forcing
- $NH = 20 \text{ cm/s}$
- $U_{\text{baro}} = \pm 10 \text{ cm/s}$

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Courtesy Alistair Adcroft, Princeton/GFDL
What are hybrid coordinates?

HYCOM 2d simulations

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Courtesy of Eric Chassignet, U of Miami
Some trends in ocean modeling (1)

- Fewer approximations and more applications over growing space-time spectrum
  - coastal impacts of climate change
  - paleoclimate modeling with fully coupled climate models
  - climate change predictions with interactive ocean biology
  - operational oceanography over basin and global scales.

- Less models, more environments
  - Aim to incorporate algorithms from multiple models.
  - An outgrowth of need to coordinate diverse efforts to tackle growing needs of models to be evaluated under increasingly complex and critical areas (e.g., science policy and operational forecasts)
  - Scientifically sensible desire to focus many experts towards the development of more sophisticated, and less cumbersome, models with
    - generalized vertical and horizontal coordinates
    - state of the art parameterizations
    - nesting
    - multiple assimilation methods
    - biogeochemistry
    - etc.
Some trends in ocean modeling (2)

• Computational platforms
  – Increasingly powerful yet very complex
  – Abilities of a single group or lab to support codes on various platforms is onerous.
  – Modelers must collaborate to fully exploit power of the various machines.

• Software evolution
  – Infrastructure enhancements provide more flexibility to run efficiently on various platforms.
  – A common set of tools various models can each employ rather than write their own.
What is a “Modeling Environment”? 

A “Model”:
- A specific collection of algorithms – e.g. MICOM v2.8
- or -
- A specific configuration, including parameter settings, geometry, forcing fields, etc. – e.g. The 1/12° North Atlantic MICOM model

A “Modeling Environment”:
- Uniform code comprising a diverse collection of interchangeable algorithms and supporting software from which a model can be selected.

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Courtesy Bob Hallberg NOAA/GFDL
Ocean Modeling Environment Efforts

• Hybrid Ocean Modeling Environment (HOME)
  – HIM, HYCOM, HYPOP, Poseidon, POSUM with MOM, MIT, ROMS, …

• Terrain-following Ocean Modeling System (TOMS)
  – POM, ROMS, …

• Nucleus for European Modeling of the Ocean (NEMO)
  – OPA (IPSL, IFREMER, Grenoble, Kiel, Hadley Centre, Mercator, …)

• Standard Ocean Model Environment (SOME)
  MOM/MITgcm collaboration

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Courtesy Bob Hallberg NOAA/GFDL
There is broad agreement among ocean modelers that generalized vertical coordinates are desirable.

A large fraction of the U.S. ocean model development community will therefore participate in HOME development.

- HOME Predecessor Models:
  - HIM (NOAA/GFDL)
  - HYCOM (U. Miami, Navy NRL, & DOE LANL)
  - HYPOP (DOE LANL)
  - Poseidon (NASA/GMAO & George Mason U.)
  - POSUM (Oregon State U.)

- Contributing Models:
  - MITgcm (MIT and Princeton) – A. Adcroft
  - MOM4 (NOAA/GFDL) – S. Griffies
  - ROMS (Rutgers U. & UCLA) – D. Haidvogel, J. McWilliams, A. Shchepetkin

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HOME=Hybrid Ocean Model Environment
Equations & Approximations

- Non-Boussinesq
- Hydrostatic, initially
- The vertical coordinate is assumed to be Lagrangian.
  - Non-material coordinates (e.g. Z- or sigma- or hybrid) achieved by vertically remapping.
- Equations use generalized orthogonal coordinates.

- C-grid horizontal discretization will be the initial emphasis, but other discretizations will be accommodated if possible.
- A variety of time-stepping schemes will be accommodated.
  - Split explicit
  - Unsplit
  - Reduced gravity
- Many other approximations need to be evaluated - e.g. Constant gravitational acceleration? Traditional approximation? Thin shell? Sphere or Oblate Spheroid?
Remainder of Lectures

• From lectures notes to this school
  – Formulate kinematic and dynamic equations of an ocean model using finite volume methods
  – Pressure gradient force in generalized vertical coordinates (time permitting)

• From “Fundamentals of Ocean Climate Models”
  – Time stepping algorithms for hydrostatic models (vertically integrated and vertically dependent equations) (time permitting)
  – Introduce subgrid scale parameterizations (neutral physics if time allows) (time permitting)