

# IMPROVING THE PARAMETERIZATION OF ERRORS STATISTICS FOR DATA ASSIMILATION IN A HYCOM BAY OF BISCAY REGIONAL CONFIGURATION

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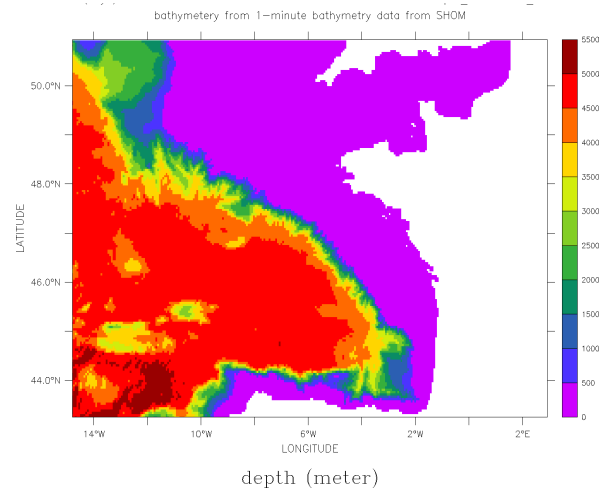
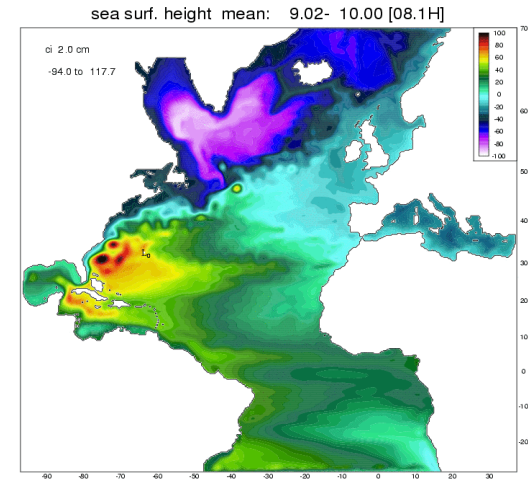
# Context

- Development of the **SEEK** (Singular Evolutive Extended **Kalman filter**) Data Assimilation system for both scientific and operational (MERCATOR / GODAE / HYCOM) mesoscale applications : *Pham et al., 1998; Brasseur and Verron, 2006*

- The SEEK in **MICOM/HYCOM** : experiments in **North Atlantic configurations** within the DIADEM/TOPAZ European project : *Brankart et al., 2003; Birol et al., 2004; Rozier et al., 2007* and at the NRL : *Parent et al., 2007*

- Development of a HYCOM Bay of Biscay configuration at the SHOM

→ application of the SEEK filter to a HYCOM regional configuration in a coastal zone : the **Bay of Biscay model**



Previous issues of the SEEK applications (in OPA / MICOM / HYCOM basin configurations) made more critical because of a higher complexity near the coasts at high resolution : difficulties with the usual crude assumptions of Data Assimilation systems.

# Issues on KF/DA errors statistics

## KF Forecast step

$$\mathbf{x}_{i+1}^f = m(\mathbf{x}_i^a)$$
$$\mathbf{P}_{i+1}^f = \mathbf{M}\mathbf{P}_i^a \mathbf{M}^T + \mathbf{Q}$$

## KF Analysis step

$$\mathbf{K}_{i+1} = \mathbf{P}_{i+1}^f \mathbf{H}^T (\mathbf{H}\mathbf{P}_{i+1}^f \mathbf{H}^T + \mathbf{R})^{-1}$$
$$\mathbf{x}_{i+1}^a = \mathbf{x}_{i+1}^f + \mathbf{K}_{i+1} \left( \mathbf{y}_{i+1} - h(\mathbf{x}_{i+1}^f) \right)$$
$$\mathbf{P}_{i+1}^a = (\mathbf{I} - \mathbf{K}_{i+1} \mathbf{H}) \mathbf{P}_{i+1}^f$$

Errors in the KF (DA in general):

- **Gaussian / not constrained** despite constrained model variables
- $\mathbf{P}_0$ ,  $\mathbf{Q}$  and  $\mathbf{R}$  a priori described with simple characteristics for calculations

$\mathbf{P}^f$  usually stationary  $\mathbf{P}^f = \mathbf{P}_0$ ;  $\mathbf{Q} = 0$  (see also in SEEK  $\mathbf{Q} = \alpha \mathbf{M}\mathbf{P}_a \mathbf{M}^T$ ) with  $\mathbf{P}_0$  estimated:

- analytically (OI, variational systems) : problem of isotropy/homogeneity, stronger in coastal zones
- with the model variability (+ order reduction: through EOF decomposition in the SEEK): supposes that the model main modes have well reproduced real dynamics
- **no true description of a model error Q**, whose weight is higher in coastal zones

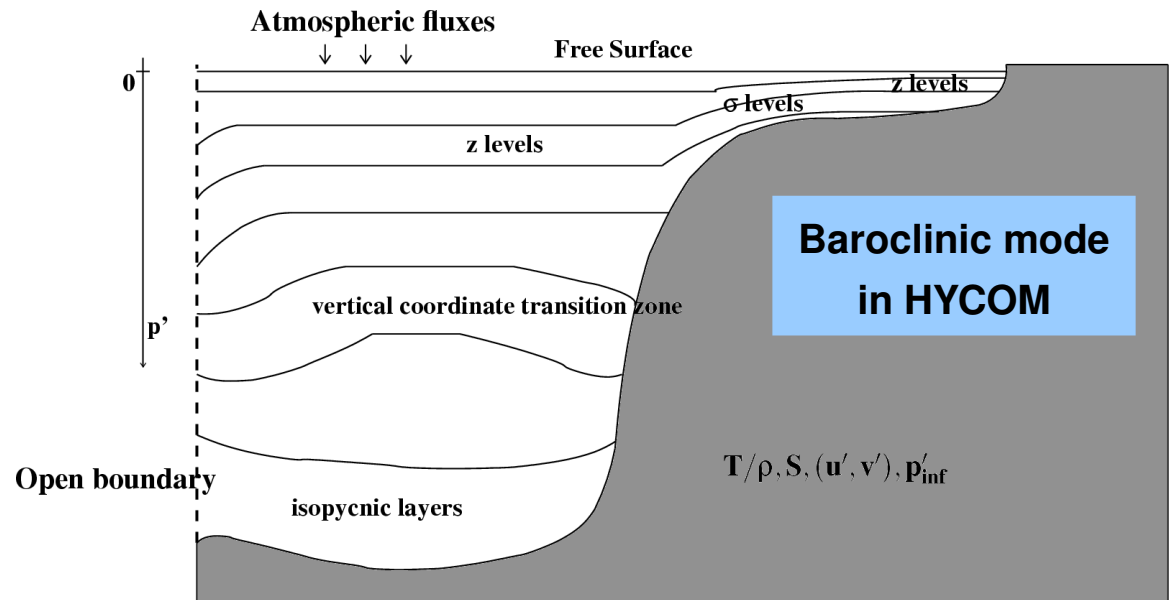
Incorrect representation of error spatial and multivariate covariances:

- **unbalanced new initial conditions**, violating some laws of the model
- adjustment in the observed sub-space, but **lack of sensible extrapolation of the obs information**

# Applying the KF in HYCOM

Pb of bad representation of error highly sensitive in HYCOM: constrained variables, changes in  $T, S$  and  $\rho$  strongly dependent in the hybrid space. Some usual answers:

→ DA in  $z$  space (strong loss of background and obs information from regridding)



→ **In the SEEK:** extrapolation of data is strongly limited (cancellation of many covariances)  
+ **adjustment operator** (loss of statistical optimality): correction after analysis of densities ( $T/S$ ) and of  $\rho$  for **hydrostatic stability** and insuring the **minimum layer thicknesses**

**Objective :** describing more realistic/adapted errors in the HYCOM state space to

- generate more balanced new initial conditions during the KF analysis step
- improve statistical results on both observed and non observed variables

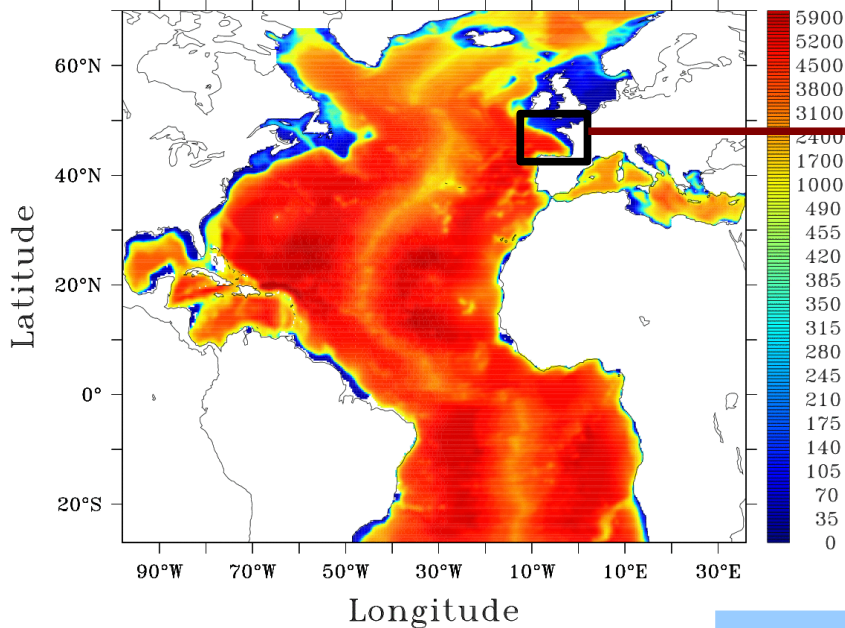
Two approaches

(1) generating/using a better estimation of  $Q$

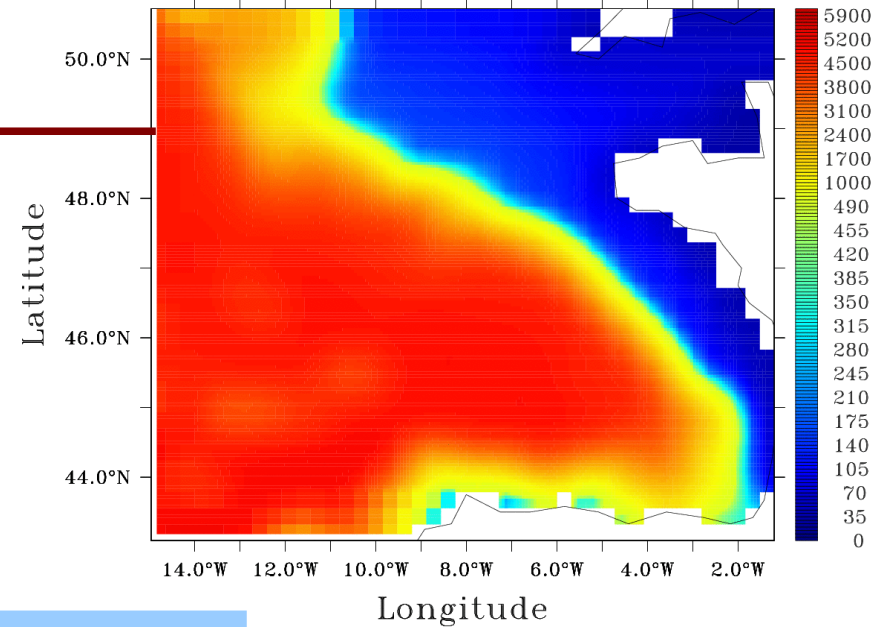
(2) including model inequality constraints on errors/in the KF process

# The Bay of Biscay model

NA3, 1/3° resolution, 26 layers



BB15, 1/15° resolution, 26 layers



Bathymetries (from ETOPO5)

**Forcings at open boundaries:** method of characteristics for barotropic variables:  $\rho_b, (u_b, v_b)$ ;  
relaxation for baroclinic variables:  $S, T, p', (u', v')$

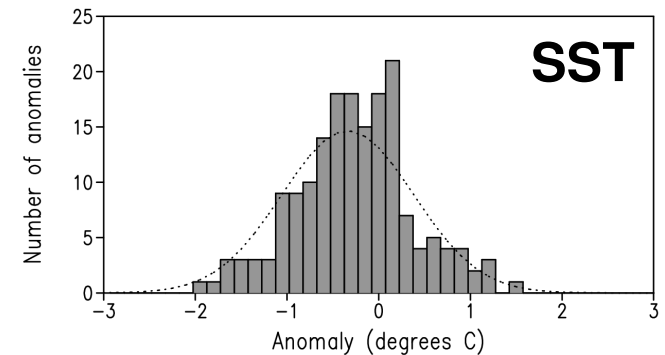
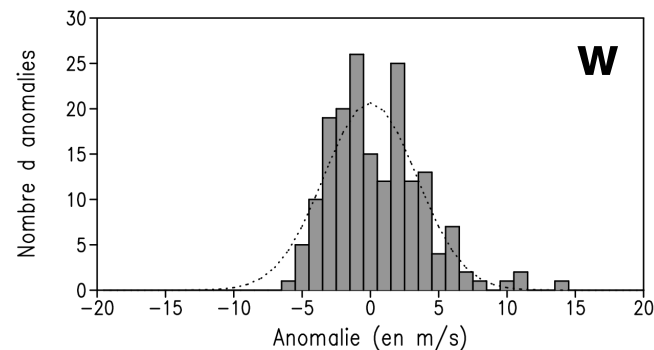
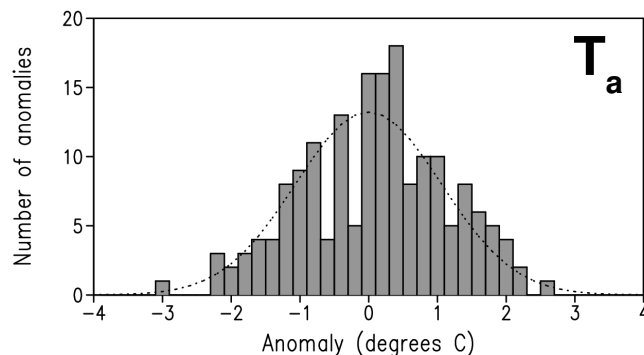
**Forcings at the surface:** Bulk formulation for heat flux, wind stress imposed, no fresh water flux  
(atmospheric parameters:  $w, T_a, q_{lw}, q_{shw}, E_v$ )

# An ensemble method for model error statistics estimation

*Broquet et al. 2007*

**Q** build from a sample of the model response to perturbations in the main source of error (realism of the estimation and allowance for error non Gaussian/linear propagation)

- 1) Identification of **main sources of model error**: increased role and weak representation of **surface forcings** in regional case
- 2) Generation of an ensemble of “perturbed” forcings: use of an **ensemble of realistic forcings**, assuming that the temporal variability (weekly and inter-annual) of data is representative of the forcing error statistics over a given season
- 3) Generation of the related **ensemble of simulation representative of the model error**



**Distribution of 176 anomalies on  $T_a$ ,  $w$  and SST at  $(11.7^\circ W, 45.4^\circ N)$  and  $t=16d$**

**Gaussian pdfs generated on not strongly constrained atmospheric parameters/model variables**

→ optimal trade off between KF assumptions and realism of error distributions

# Imposing inequality constraints in the KF

Lauvernet et al. 2008

Development of a **Truncated Gaussian filter**: an adaptation of the KF to deal with errors of TG pdfs

1) Description of the **“true state” distribution as the TG distribution  $N(\mathbf{x}^f, \mathbf{P}^f, I)$**  associated with an usual “true state” Gaussian distribution  $N(\mathbf{x}^f, \mathbf{P}^f)$  and a natural set of inequality constraints  $I$ : **the hydrostatic stability and the minimum layer thicknesses condition**

$$\rho_k - \rho_{k+1} \leq \epsilon$$

$$p_k - p_{k+1} \leq 0$$

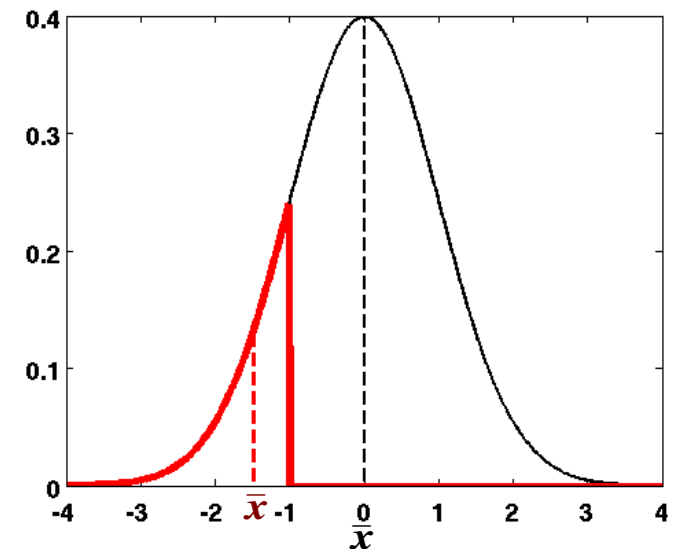
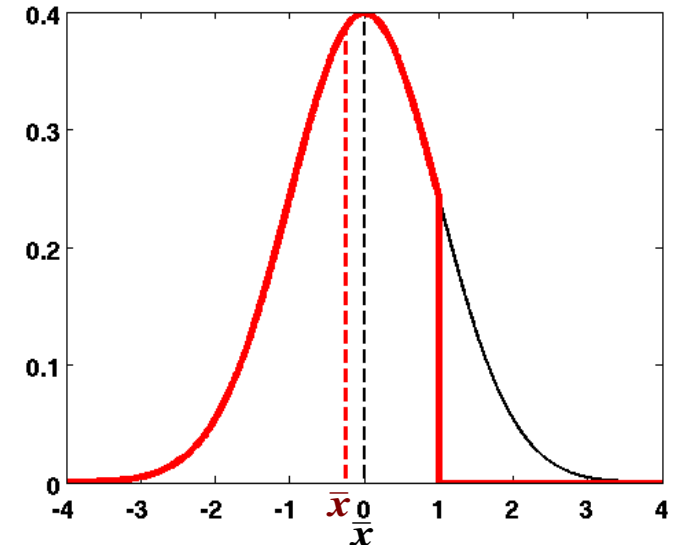
$$P_k \geq P_k^{\min}(H)$$

2) Application of **traditional KF update**  $(\mathbf{x}^f, \mathbf{P}^f) \rightarrow (\mathbf{x}^a, \mathbf{P}^a)$  giving the updated optimal error distribution  $N(\mathbf{x}^a, \mathbf{P}^a, I)$

3) Calculation of the **error variance minimizing estimator from  $N(\mathbf{x}^a, \mathbf{P}^a, I)$**  to forecast  $\mathbf{x}^f$ : **use of a Gibbs sampler.**

Quasi-normal approximation:  $\mathbf{P}^f$  is obtained with classical KF forecast step from  $\mathbf{P}^a$ . A true TG ensemble forecasting on the sample of  $N(\mathbf{x}^a, \mathbf{P}^a, I) \rightarrow N(\mathbf{x}^f, \mathbf{P}^f, I)$  has been tested on a z-coordinate ML model for conservation of hydrostatic stability

Schematic one-dimensional TG pdfs associated to  $N(0,1)$  with constraint  $I : x \leq 1$  or  $x \geq -1$





# Representers of error covariances

*Bennett, 1992 ; Echevin et al. 2000*

KF analysis  
state update

$$K = P^f H^T [H P^f H^T + R]^{-1}$$

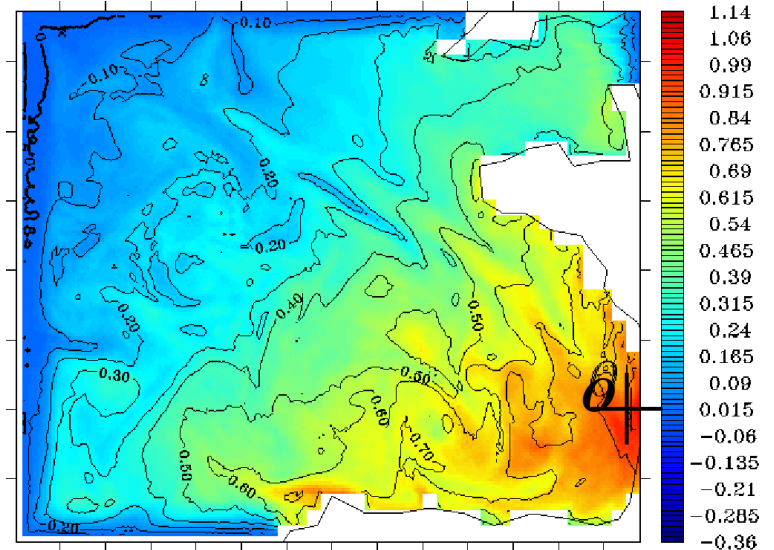
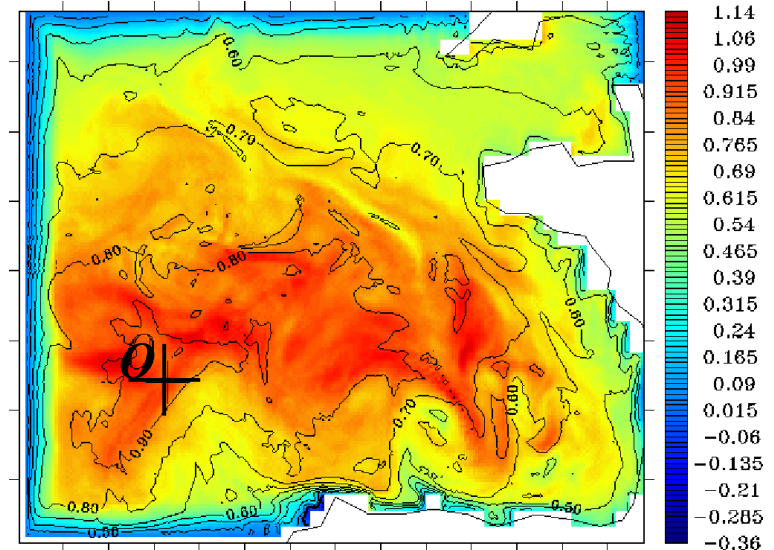
$$x^a = x^f + K [y - H x^f]$$

If  $y$  scalar (1 obs) :  $x^a = x^f + \lambda r$

Representer :  $r = (P^f H^T) / (H P^f H^T)$

**Representer for Q :**  $r_Q = (Q H^T) / (H Q H^T)$

- Their convergence with the sample size is used to check the ensemble **Q statistics convergence**
- They confirm the strong **anisotropy / inhomogeneity** of the dynamics
- They show through the use of **Q** in the KF a **limited influence of surface observations at depth** and a large spread of their influence in the mixed layer



SST field of representers of SST obs



# Impact of the error parameterization for DA

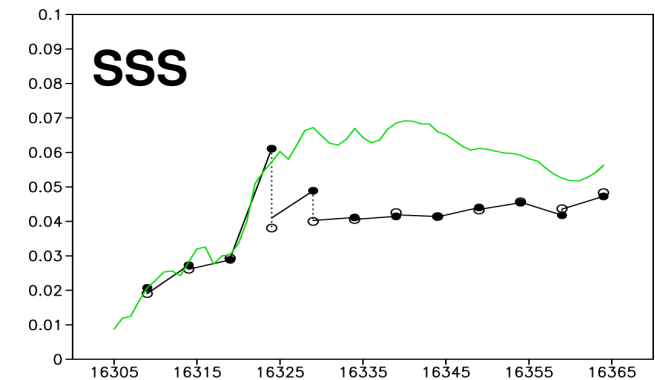
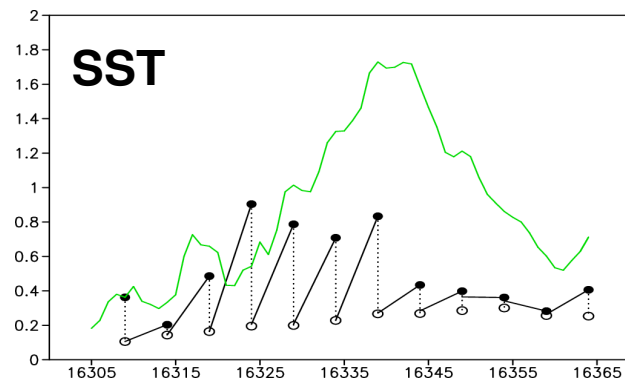
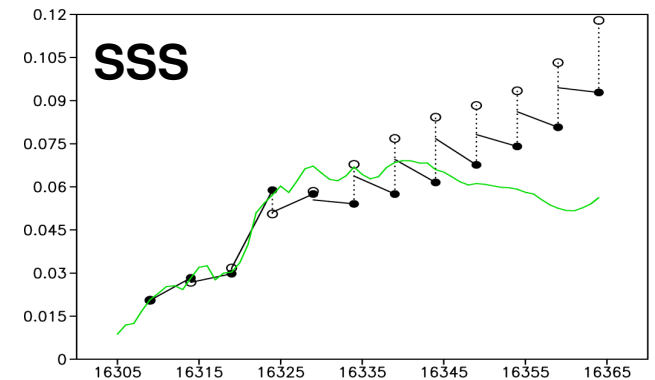
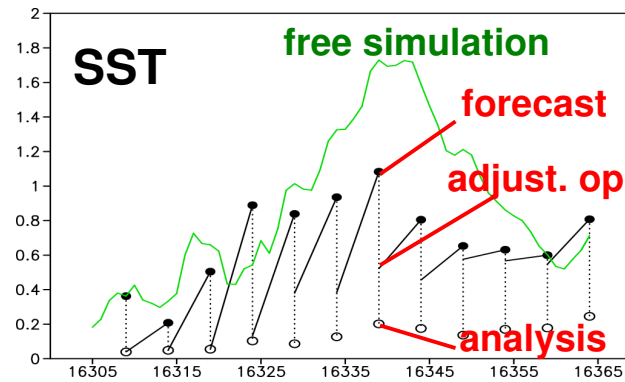
Twin experiments on 60 days, with atmospheric forcing perturbation: the **SEEK filter is applied to assimilate SST** on the perturbed simulation with different static  $\mathbf{P}^f$ :

- $\mathbf{P}^f$  = the control run (REF) **daily variability** on the 60 days (no order reduction): **EXP1**
- $\mathbf{P}^f$  = the 59 main EOFs of **Q estimated through the ensemble method**: **EXP2**

- In **EXP1**, the state is degraded in the non observed sub space / great impact of adjustment operator

- In **EXP2**, improvements on the **whole state space** / dynamically balanced adjustments: **weak impact of the adjustment operator** (both objectives are reached)

→ the system demonstrates its sensitivity to the model error parameterization

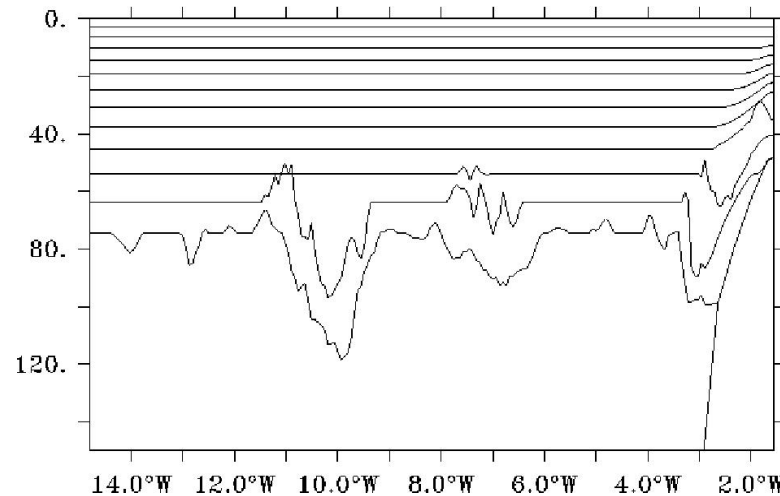


RMS error between REF and EXP1 (top) / EXP2 (bot.)

# Gaussian vs TG observational update (1)

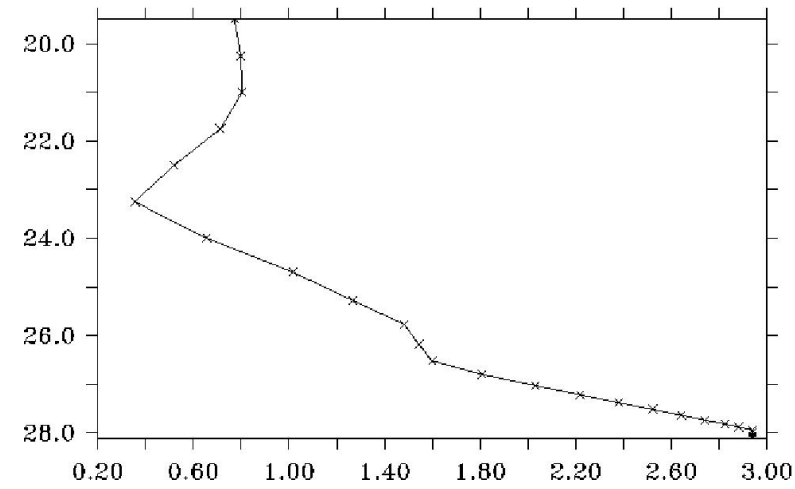
Identical twin experiments: in **EXP3**, the SEEK is applied to assimilate SST with the same  $P^f$  parameterization as in EXP1 and the **TG observational update** instead of the classical Gaussian update

Zonal section (at 45.9N) of interface pressure

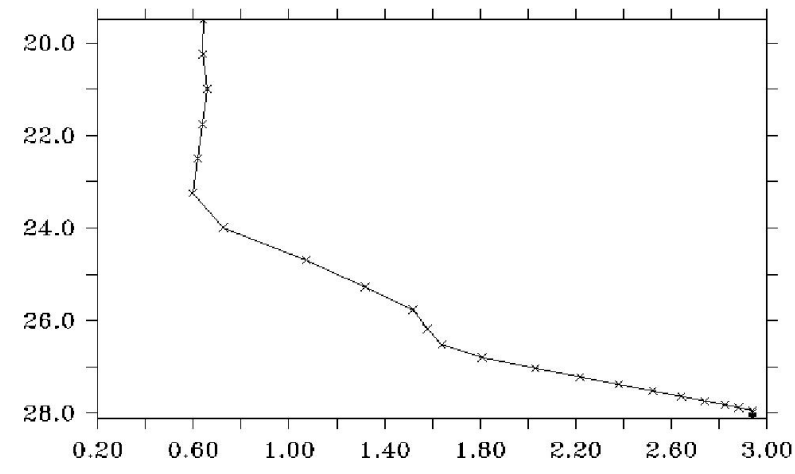
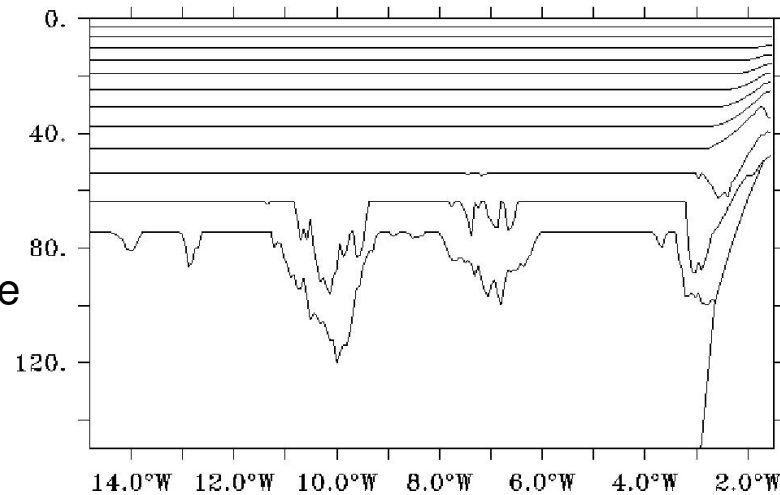


In **EXP1**, all constraints are violated: density inversions, negative layer thicknesses,  $\rho$  below minimum value

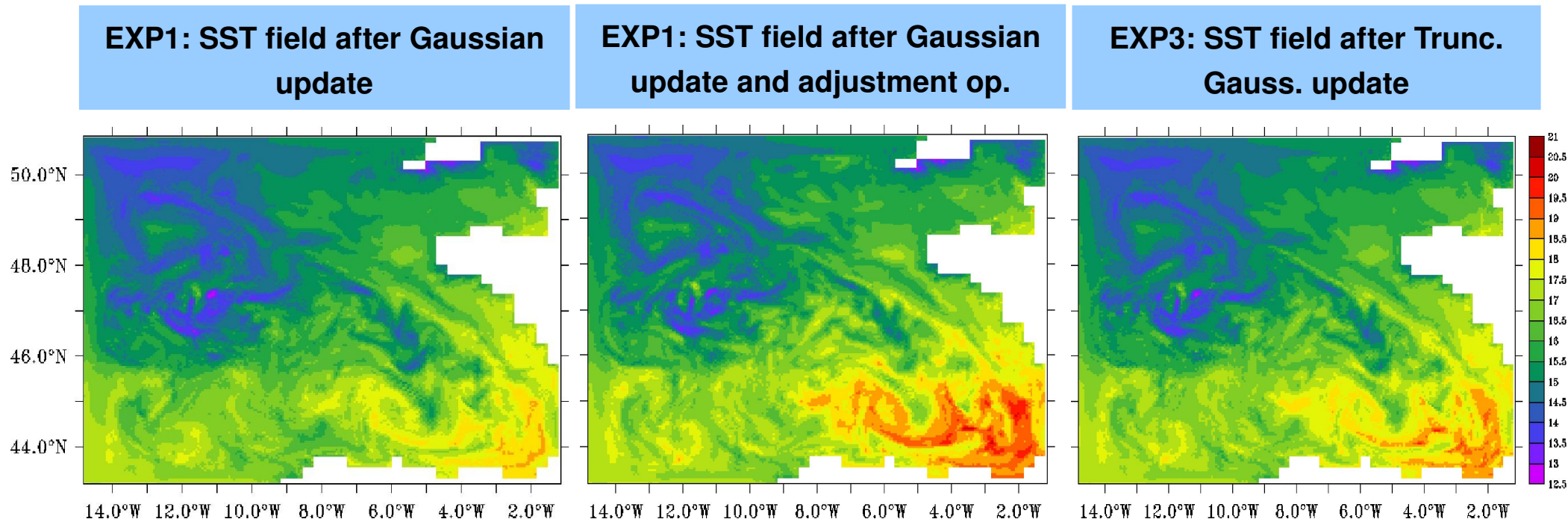
Density profile belonging to that section (at 6.4W)



In **EXP3**, no more density inversion, all layers have retrieved their minimum depth (even in the s-coordinate region), and positive thickness.



# Gaussian vs TG observational update (2)



- The warming relative to the Gaussian update with adjustment operator or with TG update occurs above the first density inversion of the profile in **EXP1**
- For the full 3D temperature field, the RMS error is
  - 0.72°C for the Gaussian estimate
  - 0.83°C after the adjustment operation
  - 0.74°C for the truncated Gaussian estimate.

**With the TG observational update T values remain closer to the observations than with the adjustment operator**

# Conclusion

- Study of two **error parameterization** aspects
  - two different methods for **an improved application of the SEEK filter / DA systems**
- Both ensure :
  - a **statistically more optimal extrapolation** of the observation information
  - a **more physically consistent** solution
- Impact of **Q** realistic parameterization → **fundamental role of the error covariances parameterizations in DA system**
- **TG description** of error is sensible and more adapted
  
- These studies have to be expanded :
  - application of ensemble methods to estimate the **error model linked to other sources** + linear combination with realistic  $P_0$  for **realistic experiments**
  - **full TG process** to be used on the real size problem of the Bay of Biscay
- Combination and generalization of these procedures for other applications / other OGCM

**Thank you !**

