Westward Motion of Mesoscale Eddies*

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ABSTRACT

Since the pioneering work of Nof, the determination of the westward drift of mesoscale eddies under the planetary (beta) effect has been a recurrent theme in mesoscale oceanography, and several different formulae have been proposed in the literature. Here, recapitulation is sought, and, within the confines of a single-layer model, a generalized formula is derived. Although it is similar to Nof's, the present formula is established from a modified definition and with fewer assumptions. It also recapitulates all other formulae for the one-layer model and applies to a wide variety of situations, including cases when the vortex develops a wake of Rossby waves or undergoes axisymmetrization.

Following the derivation of the formula, a physical interpretation clarifies the migration mechanism, which can be divided between a self-induced propulsion and a reaction from the displaced ambient fluid. Numerical simulations with primitive and geostrophic equations validate the formula for a variety of length scales and amplitudes. The work concludes with an attempt to extend the result to systems with two moving layers.

1. Introduction

Since the dawn of dynamical meteorology, investigators have been concerned with the variation of the Coriolis parameter with latitude (nowadays called the beta effect) on extratropical cyclones and anticyclones. Considering only the particles swirling within the vortex formation, thus ignoring the effect of the surrounding fluid, Bjerknes and Holmboe (1944) were the first to note that this planetary gradient produces a mass imbalance in a circular vortex causing it to move westward, irrespective of polarity. Yet, instead of evaluating a rate of westward translation, they discussed how a distortion from the circular state could counteract the westward tendency. Their strictly kinematic argument does not, however, offer proof that such an arrested state can persist for a finite period of time.

Returning to this analysis with consideration of the balance of forces, Rossby (1948) concluded that the planetary gradient of the Coriolis parameter exerts a net meridional force on cyclones and anticyclones, respectively directed poleward and equatorward. Rossby then invoked energy dissipation and discussed how the resulting acceleration can lead to a finite meridional displacement of the vortex.

Since the energy of synoptic motions can only be slowly dissipated, one is led to revise Rossby's conclusion by reasoning that the equatorward force on an anticyclone would accelerate the latter during a fraction of an inertial period until the Coriolis effect takes over and causes the anticyclone to veer and migrate westward, all this occurring before energy dissipation can take hold. This is precisely what Nof (1981) deduced with a one-layer, lens-like model for oceanic anticyclones. He then derived a formula for the westward translation speed of such vortices. Killworth (1983) extended that analysis somewhat.

Because the net force on cyclonic formations is directed in the opposite direction, one is tempted to conclude that cyclones ought to translate eastward. As Nof (1983) demonstrated, this conclusion is erroneous, and both cyclones and anticyclones migrate westward. The reason for this behavior is the non-negligible influence of the displaced surrounding fluid on the vortex. This issue will be made clear later in the present article.

The applicability of Nof's formula is restricted by the double assumption of steady circular eddies and of constant drift velocities. In other words, the eddies considered by Nof must remain unchanged with time, except for their westward migration, which itself must proceed at an unchanging rate. These restrictions exclude a number of realistic situations such as cases when the migrating eddies generate a wake of Rossby waves or undergo their own evolution (merging, axisymmetrization, pulsation, nutation, etc.). Yet Nof's formula has been verified in several instances (Bowman 1985; Cornillon et al. 1989; our numerical experiments de-

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scribed below) and one may wonder whether the range of applicability of the formula may not extend beyond the limited conditions under which it was derived. A first aim of the present study is to address this question and to delimit more general conditions for the validity of Nof's formula.

A second purpose is to show that Nof's and other formulae proposed over the years and under different assumptions can all be grouped into one generalized derivation. Indeed, in the framework of geostrophic formalisms, a number of investigators have derived various expressions predicting the westward drift of isolated vortices. Using a numerical quasi-geostrophic model (weak amplitudes, any length scale), McWilliams and Flierl (1979) showed that the center of mass of a vortex in an infinite domain moves westward at the greatest linear Rossby-wave speed, irrespectively of the vortex polarity. Matsuura and Yamagata (1982), Malanotte-Rizzoli (1982) and Williams and Yamagata (1984) showed that, under certain conditions (weak amplitudes and long length scales), mesoscale formations obey the classic Korteweg–deVries equation, of which the soliton solutions translate westward at a rate known as a function of amplitude and length scale. The frontal geostrophic formalism of Cushman-Roisin (1986) and Shapiro (1986) (finite amplitudes, long length scales) yields a constant rate of westward propagation, proportional to the ratio of potential energy to volume of the eddy. All these results will be shown to be asymptotic expressions of a single expression.

To derive a unified formalism for the westward migration of oceanic mesoscale eddies, it has been found convenient to follow the lead of McWilliams and Flierl (1979) and Killworth (1983) by defining, from the start, a center of mass. The problem is then reduced to finding the evolution in time of the coordinates of this center of mass. Because all results mentioned above were established in the context of a single-moving layer, the following study will also be restricted to the one-layer model. Extension to systems with multiple moving layers will be sought toward the end of the article.

2. The formula

a. Derivation

The primitive equations of the one-layer, reduced-gravity model on the beta plane are

$$\mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u} + f\mathbf{k}\times\mathbf{u} = -g'\nabla h$$

$$h_t + \nabla\cdot(h\mathbf{u}) = 0,$$

where $\mathbf{u}$ is the velocity vector, ∇ is the gradient operator, $f = f_0 + \beta_0 y$ is the varying Coriolis parameter, g' is the reduced gravity, and h is the local layer thickness. Writing $h = H + \eta$, i.e., the sum of a mean layer thickness and an interfacial displacement, one can define the radius of deformation, $R_d = (g'H)^{1/2}/f_0$. With a length scale L (not necessarily equal to R_d), a vertical displacement scale δH (not necessarily much less than H), and a time scale T (not necessarily the advective time scale), the equations can be scaled to become

$$\omega\mathbf{u}_t + \epsilon(\mathbf{u}\cdot\nabla)\mathbf{u} + \mathbf{k}\times\mathbf{u} + \beta y\mathbf{k}\times\mathbf{u} = -\nabla\eta \quad (1)$$

$$\omega\eta_t + s\nabla\cdot\mathbf{u} + \epsilon\nabla\cdot(\eta\mathbf{u}) = 0, \quad (2)$$

where the dimensionless numbers have the following interpretations: $\omega = 1/f_0T$ compares the time scale with the inertial period, $\epsilon = g'\delta H/f_0^2L^2$ is the Rossby number, $\beta = \beta_0 L/f_0$ measures the importance of the beta effect, and $s = g'H/f_0^2L^2 = (R_d/L)^2$ is a stratification or Burger number. [The Froude number is ϵ^2/s.]

Mesoscale eddies in the ocean tend to fall in one of two categories, namely the quasi-geostrophic mesoscale eddies of the open ocean ($L \sim R_d$, $\delta H \ll H$; Kamenkovich et al. 1986) and the frontal-geostrophic rings in the vicinity of western boundary currents ($L > 3R_d$, $\delta H \sim H$; Olson et al. 1985; Chassignet et al. 1989). Although much smaller, the intrathermoclinic lenses have frontal characters ($\delta H \sim H$; McWilliams 1985) and can be grouped in the category of rings. Generally, the Rossby number and the beta number are small, and the eddies are primarily in geostrophic balance. With ϵ and β small, it is expected that ω, too, is small. It is thus assumed at this stage that $\omega \ll 1$, which will be verified a posteriori. Exploiting the smallness of ω, ϵ, β, the leading approximation to Eq. (1) is the f-plane geostrophic balance,

$$\mathbf{u} = \mathbf{k}\times\nabla\eta + O(\delta), \quad (3)$$

where the order of magnitude of the error is

$$\delta = \max(\omega, \epsilon, \beta). \quad (4)$$

Substitution of this expression for $\mathbf{u}$ in the small terms of Eq. (1) provides, at the next level of approximation,

$$\mathbf{u} = \mathbf{k}\times\nabla\eta - \omega\nabla\eta_t - \epsilon J(\eta, \nabla\eta) - \beta y\mathbf{k}\times\nabla\eta + O(\delta^2). \quad (5)$$

Replacement of $\mathbf{u}$ by (5) in Eq. (2) yields a single equation for η (Cushman-Roisin and Tang 1989, 1990), which is not of interest here, except to note that its leading or potentially leading terms have the following orders of magnitude ω, $s\delta$, and $\epsilon\delta$. The rule to determine the time scale, T, is that the largest term containing ω must be on the same order as the largest term not containing ω. Since $\epsilon \leq s$ (a consequence of $\delta H \leq H$), the result is

$$\omega = \frac{\max(s\epsilon, s\beta)}{\max(1, s)}. \quad (6)$$

From this expression, it is easily verified that ω is bounded by $\max(\epsilon, \beta)$. Hence, ω is always much smaller than unity, as it was anticipated, and the expression for δ reduces to

$$\delta = \max(\epsilon, \beta). \tag{7}$$

Integration of the continuity equation (2) over an infinite domain yields

$$\frac{dV}{dt} = 0, \quad V = \iint \eta dxdy,$$

as long as the feature (η distribution) is sufficiently well isolated (η decays toward zero sufficiently rapidly at large distances). Although the initial distribution of η is left unspecified for generality, it is restricted to being non-compensating, i.e., $V \neq 0$. If V is positive, the vertical displacement is mostly downward, and one would speak of an anticyclonic formation; if V is negative, the feature is mostly an upwelled interface and is referred to as a cyclonic formation.

With the notation $\langle(\cdots)\rangle = V^{-1} \iint (\cdots)dxdy$, the coordinates of the center of mass of the feature are naturally defined by

$$X = \langle x\eta \rangle, \quad Y = \langle y\eta \rangle. \tag{8}$$

In addition to following its own evolution (pulsation, axisymmetrization, etc.), the feature may drift and this drift is provided by the velocity of its center of mass, which can be evaluated with the use of Eq. (2) (Killworth 1983):

$$\frac{dX}{dt} = \langle x\eta_t \rangle = \frac{s}{\omega}\langle u \rangle + \frac{\epsilon}{\omega}\langle \eta u \rangle \tag{9}$$

$$\frac{dY}{dt} = \langle y\eta_t \rangle = \frac{s}{\omega}\langle v \rangle + \frac{\epsilon}{\omega}\langle \eta v \rangle, \tag{10}$$

where (u, v) are the components of the velocity vector $\mathbf{u}$. A second differentiation yields (Killworth 1983)

$$\omega \frac{d^2X}{dt^2} - \frac{dY}{dt} = +\frac{s\beta}{\omega}\langle yv \rangle + \frac{\epsilon\beta}{\omega}\langle y\eta v \rangle, \tag{11}$$

$$\omega \frac{d^2Y}{dt^2} + \frac{dX}{dt} = -\frac{s\beta}{\omega}\langle yu \rangle - \frac{\epsilon\beta}{\omega}\langle y\eta u \rangle. \tag{12}$$

Note that in the absence of a beta effect ($\beta = 0$), these equations imply inertial oscillations of the center of mass (Ball 1963). With the addition of the beta effect, a net drift will likely be superimposed, and in the limit of geostrophic regimes (ω small), this drift will overcome the inertial oscillations.

The geostrophic approximation is now used as a simplification by replacing u and v in the previous expressions by their geostrophic values given in (3). The result is

$$\omega \frac{d^2X}{dt^2} - \frac{dY}{dt} = O\left(\frac{s\beta\delta}{\omega}\right), \tag{13}$$

$$\omega \frac{d^2Y}{dt^2} + \frac{dX}{dt} = -\frac{s\beta}{\omega}\langle \eta \rangle - \frac{\epsilon\beta}{2\omega}\langle \eta^2 \rangle + O\left(\frac{s\beta\delta}{\omega}\right). \tag{14}$$

The former equation implies $dY/dt = O(s\beta\delta/\omega, \omega)$, and the use of this result in the latter equation provides

$$\frac{dX}{dt} = -\frac{s\beta}{\omega}\langle \eta \rangle - \frac{\epsilon\beta}{2\omega}\langle \eta^2 \rangle + O\left(\frac{s\beta\delta}{\omega}, \omega^2\right). \tag{15}$$

A return to the dimensional variables then provides the final expression for the drift velocity

$$\frac{dX}{dt} = -\frac{\beta_0 g'}{f_0^2} \frac{\iint \left(H\eta + \frac{1}{2}\eta^2\right)dxdy}{\iint \eta dxdy}, \tag{16}$$

$$\frac{dY}{dt} = 0. \tag{17}$$

The error on the zonal drift speed is on the order of

$$\beta_0 R_d^2 \max\left(\delta, \frac{\omega^3}{s\beta}\right). \tag{18}$$

Typical orders of magnitude for open-ocean mesoscale eddies are $g' = 0.02$ m s^{-2}, $H = 400$ m, $\delta H = 100$ m and $L = 50$ km, which yield $R_d \approx 28$ km, $s \approx 0.3$, $\epsilon \approx 0.08$, $\beta \approx 0.01$. The westward drift of a Gaussian-shaped eddy is estimated to be about 1.6 cm s^{-1} with an error of about 8%. Gulf Stream rings are characterized by $g' = 0.02$ m s^{-2}, $H = \delta H = 500$ m and $L = 100$ km, or $R_d \approx 32$ km, $s \approx 0.1$, $\epsilon \approx 0.10$, $\beta \approx 0.02$, westward drift of about 2.0 cm s^{-1} within an error of 10%. Finally, intrathermoclinic eddies have $g' = 0.02$ m s^{-2}, $H = \delta H = 50$ m and $L = 30$ km, or $R_d \approx 10$ km, $s \approx 0.11$, $\epsilon \approx 0.11$, $\beta \approx 0.006$, westward drift of about 0.2 cm s^{-1} with an error of 11%.

b. Discussion

When the eddy leaves a trail of Rossby waves in its wake, it is evident that the center of mass defined in (8) does not coincide with the vortex center. But this situation is not particularly irksome because either the interfacial displacements in the Rossby-wave wake are small compared to that of the main eddy and the discrepancy is small, or they are large in which case the eddy loses its significance and finding the path of its center becomes a futile exercise.

Since $h = H + \eta \geqslant 0$, the integral in the numerator of dX/dt always has the same sign as that in the denominator, and expression (16) indicates that vortices in near-geostrophic balance all migrate westward. Since dY/dt vanishes at first order according to (17), there is no appreciable meridional drift. For weak, quasi-geostrophic ($\eta \ll H$) eddies, the term $\eta^2/2$ can be neglected compared to $H\eta$, leaving

$$\frac{dX}{dt} = -\frac{\beta_0 g' H}{f_0^2} = -\beta_0 R_d^2. \tag{19}$$

In other words, quasi-geostrophic vortices propagate at a constant speed, none other than that of the long, nondispersive Rossby waves. There is no dependency upon the amplitude, and, consequently, there is no distinction between cyclonic and anticyclonic formations. Note that the long Rossby wave speed is evaluated based on the interfacial depth outside of the eddy, not on the eddy central depth.

For finite-amplitude eddies such as Gulf Stream rings and intrathermoclinic lenses, η is a substantial fraction of H, and expression (16) can no longer be simplified. For anticyclonic vortices, η is positive, the terms in the numerator of Eq. (16) add to each other, and the westward drift speed is larger than the long-Rossby-wave speed. For cyclonic vortices, it is the opposite, and the drift speed cannot exceed the long-Rossby-wave speed. The rule is: the larger the amplitude, the faster the anticyclones, the slower the cyclones. There are, however, upper and lower bounds to the westward drift speeds. For anticyclones, $0 \leqslant \eta \leqslant h_{\max} - H$ where $h_{\max}$ is the maximum interfacial depth at the eddy's center, and it follows that $H\eta + \eta^2/2 \leqslant (H + h_{\max})\eta/2$ and that the zonal drift speed cannot exceed $\beta_0 g'(H + h_{\max})/2f_0^2$. This upper bound is the average between two values of $\beta_0 R_d^2$, evaluated with the interfacial depths taken first outside and then at the center of the eddy. For cyclones, a rise of the interface cannot reach beyond the surface, and $-H \leqslant \eta \leqslant 0$. It follows that $H\eta + \eta^2/2 \leqslant H\eta/2 \leqslant 0$, and the magnitude of the drift velocity cannot be less than half the long-Rossby-wave speed.

A physical interpretation of expression (16) is now proposed. Two processes are analyzed individually: the imbalance of the Coriolis force on the particles swirling within the eddy, which induces a first component of zonal motion, and the reaction onto the eddy by displaced particles of the surrounding fluid, which generates an additional zonal propagation. The net zonal speed is the sum of the two components.

Consider a circular eddy with azimuthal velocity U, pressure anomaly $g'\delta H$ and radius L (Fig. 1a). In the absence of zonal drift, the geostrophic balance ($fu = -\partial p/\partial y$) on the north and south sides yields: $f_N(-U_N = -(-g'\delta H)/L$ and $f_S U_S = -g'\delta H/L$. Since the same waters circulate around the eddy without loss of mass, $U_N = U_S = U$ and the two requirements are incompatible as long as f_N differs from f_S. This discrepancy is that at the base of the arguments of Bjerknes and Holmboe (1944) and of Rossby (1948). With the addition of a bulk zonal velocity c (positive eastward), the momentum balances become

$$f_N(c - U_N) = -\frac{-g'\delta H}{L}, \quad f_S(c + U_S) = -\frac{g'\delta H}{L}$$

on the north and south flanks, respectively. Again, with $U_N = U_S = U$, elimination of $g'\delta H/L$ yields $c = (f_N - f_S)U/(f_N + f_S)$, or, since $f_N = f_0 + \beta_0 L$, $f_S = f_0 - \beta_0 L$,

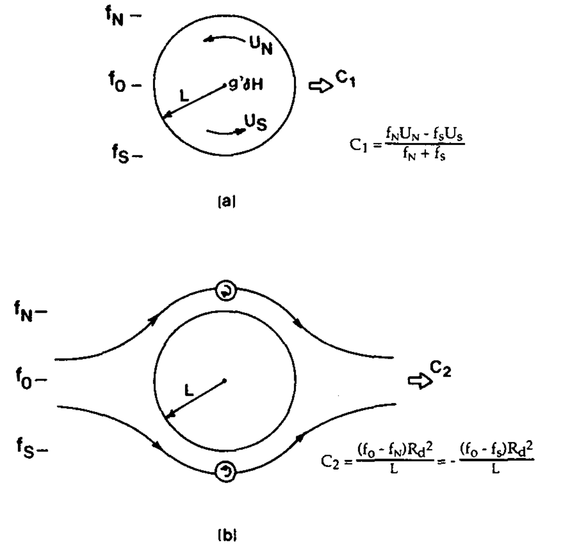


FIG. 1. Physical interpretation for the westward drift of eddies. (a) The imbalance of Coriolis force within the eddy causes zonal translation, to the east for cyclones and to the west for anticyclones. (b) As a result of this translation, surrounding particles are displaced meridionally, acquiring relative vorticity and inducing a westward translation on the eddy. The net zonal drift is the sum of both components, and since the latter is always greater than the former, all eddies propagate westward.

$$c = \frac{\beta_0 L U}{f_0}. \quad (20)$$

For cyclones, U as defined is positive and c is directed eastward, while for anticyclones U is negative and c is directed westward.

Now, this first zonal motion will cause displacements in the surrounding fluid, some parcels being moved around the northern edge of the eddy and some others around its southern edge, irrespective of the zonal direction along which the eddy propagates. The Coriolis parameter of the former particles changes approximately from f_0 to $f_N = f_0 + \beta_0 L$, while for the latter particles, it decreases from f_0 to $f_S = f_0 - \beta_0 L$ (Fig. 1b). If vertical stretching does not dominate the makeup of the potential vorticity, changes in relative vorticity will be equally important, if not dominant, and changes in relative vorticity on the order of $\beta_0 L$ must be expected, anticyclonic to the north and cyclonic to the south. Over a length scale on the order of the radius of deformation, R_d, the induced circulation (integrated vorticity) is $\Gamma = \beta_0 L R_d^2 = \beta_0 g' H L/f_0^2$, and the center of the eddy at a distance L from these vorticity patches is subjected to an entraining velocity $\Gamma/L = \beta_0 g' H/f_0^2$. With anticyclonic vorticity to the north and cyclonic vorticity to the south (Fig. 1b), this entrainment velocity is directed westward.

The sum of both contributions to the zonal drift of the eddy is

$$c = -\frac{\beta_0 g'H}{f_0^2} + \frac{\beta_0 LU}{f_0} = -\frac{\beta_0 g'H}{f_0^2} - \frac{\beta_0 g'\delta H}{f_0^2}, \quad (21)$$

where the second expression is obtained after approximation of the azimuthal velocity by its geostrophic value.

The parallel between expressions (16) and (21) is evident. Always dominant, the first term makes both cyclones and anticyclones drift westward. The previous physical interpretation indicates that the reason for this component is the reaction onto the eddy by the particles it displaces. This important effect was overlooked by Bjerknes and Holmboe (1944) and Rossby (1948). The second term, which can be of either sign, results from the self-induced drift of the particles within the eddy; it decelerates cyclones ($\delta H < 0$) and accelerates anticyclones ($\delta H > 0$). For weak, quasi-geostrophic eddies ($\delta H \ll H$), this self induction is negligible compared to the entrainment by the displaced fluid in the surroundings. Finally, it should be noted that the entrainment by the outside fluid is the contribution coined "planetary lift" by Nof (1983, 1985).

3. Comparison with previous formulae

Several expressions predicting the speed of westward migration of eddies have been proposed in the literature for various asymptotic dynamics. The purpose of this section is to show that all can be derived from the general formula (16).

McWilliams and Flierl (1979) established a formula providing the velocity of the center of mass for the one-layer version of a classic, quasi-geostrophic model. The result [Eq. (3.3) in their article] can be derived here again in just a few lines. The single-layer quasi-geostrophic equation in a dimensional form and with the present notation is:

$$\left(\nabla^2\eta - \frac{1}{R_d^2}\eta\right)_t + \frac{g'}{f_0} J(\eta, \nabla^2\eta) + \beta_0\eta_x = 0.$$

Multiplying this equation successively by x and y and integrating over the infinite domain, one obtains

$$-\frac{1}{R_d^2}\frac{d}{dt}\iint x\eta dxdy - \beta_0 \iint \eta dxdy = 0,$$

$$-\frac{1}{R_d^2}\frac{d}{dt}\iint y\eta dxdy = 0.$$

Invoking definition (8) of the center of mass then yields the expected result

$$\frac{dX}{dt} = -\beta_0 R_d^2, \quad \frac{dY}{dt} = 0,$$

which is the result of McWilliams and Flierl (1979) and the asymptotic expression of (16)–(17) in the quasi-geostrophic context [$\eta \ll H$, see (19)].

Nof (1981) investigated the westward drift of anticyclonic lenses and, later (Nof 1983), generalized the result to non-lens-like anticyclones as well as to cyclones. His most general formula, expressed in the present notation, is

$$c = \frac{\beta_0}{f_0} \frac{\int_0^L dr \int_r^L (H+\eta)vr'dr'}{\int_0^L \eta r dr} \quad (22)$$

where r is the radial coordinate, L the eddy radius (out to a certain streamline), and v the azimuthal velocity. To order β^2 in his analysis, Nof made the following assumptions: the eddy's shape and structure are unchanging (i.e., $\omega = 0$), the eddy is approximately circular, the beta effect is small ($\beta \ll 1$), the balance of forces in the eddy may include the centrifugal force (ϵ not necessarily much smaller than unity), and, finally, the drift speed is assumed to be constant in time.

When the geostrophic approximation ($\epsilon \ll 1$) is used to eliminate v in terms of η ($f_0 v = -g'\eta_r$), Nof's expression (22) becomes identical to the present result (16). One consequence of retaining the centrifugal force as a potentially important advective effect is to modify the upper and lower bounds for the drift speed, from $\beta_0 g'(H + h_{max})/2f_0^2$ to $2\beta_0 g'h_{max}/3f_0^2$ for lens-like anticyclones (Nof 1981) and from $\beta_0 g'H/2f_0^2$ to $2\beta_0 g'H/3f_0^2$ for cyclones (Nof 1983).

Unlike Nof's formula, the present result was derived without the assumptions of an unchanging eddy ($\omega \neq 0$) and of a constant drift speed. There is therefore a trade-off between both theories, but, when one notes (i) that oceanic eddies are not perfectly circular and are subject to time-dependent behavior such as pulsation, axisymmetrization and, even, merging, (ii) that the presence of important ageostrophic effects leads to circular inertial oscillations of the center of mass (Ball 1963), (iii) that cyclonic eddies with important ageostrophic effects are unstable (Cushman-Roisin and Tang 1990), and finally (iv) that most oceanic eddies are in nearly geostrophic balance, serious doubts can be entertained about the significance of the one ageostrophic effect (centrifugal force) that Nof chose to retain among all other possible candidates. Without that term, both theories provide identical results, but the present one has the advantage of showing that the assumptions of an unchanging eddy and of a constant drift rate are not necessary to derive the formula predicting the drift velocity. In other words, it has been demonstrated that Nof's formula happens to be valid under conditions far more general than those under which it was initially derived.

In the context of frontal geostrophic dynamics (one layer, $\delta H = H$, $L^2 \gg R_d^2$), Cushman-Roisin (1986) derived the following expression for the westward drift of any isolated feature

$$\frac{dX}{dt} = -\frac{\beta_0 g'}{2f_0^2} \frac{\iint h^2 dxdy}{\iint h dxdy}, \quad (23)$$

where the integrals cover the finite domain within which fluid is in motion ($h > 0$) and extending to a deformable front ($h = 0$). The same formula was derived by Shapiro (1986) in the more restrictive context of a single, circular vortex with presupposed constant drift speed.

With $H = 0$, i.e., no upper layer beyond the front, $h = \eta$ and expression (23) is found to be identical to (16). It is worth noting that, in frontal geostrophic dynamics, potential energy is far greater than kinetic energy and is thus conserved in first approximation; consequently, the numerator of (23) is an invariant, and the drift rate assumes, for this regime, a constant value, irrespective of eventual distortions of the eddy or eddies within the frontal line (Pavia and Cushman-Roisin 1988).

Intermediate geostrophic dynamics (Matsuura and Yamagata 1982; Malanotte-Rizzoli 1982; Williams and Yamagata 1984) are characterized by both small vertical displacements ($\delta H \ll H$) and long length scales ($L^2 \gg R_d^2$). The governing equation can be obtained by substituting (5) in (2) and setting ω to $s\beta$. For meridional bands (no y derivative), it reduces to

$$s\eta_{xxt} - \eta_t + \eta_x + \frac{\epsilon}{s}\eta\eta_x = O(\beta, \epsilon). \quad (24)$$

Since the vertical displacements are small ($\epsilon \ll s$) and the length scale is large ($s \ll 1$), the middle two terms dominate, the eddy formation translates westward without deformation, and, on a longer time scale, the evolution is governed by a KdV equation (Matsuura and Yamagata 1982; Malanotte-Rizzoli 1982). The expansion need not be reproduced here, for it suffices to note that if the eddy feature translates at a uniform speed c (η function of $x - ct$), a double integration of (24) yields:

$$(1 + c)\int_{-\infty}^{\infty} \eta dx + \frac{\epsilon}{2s}\int_{-\infty}^{\infty} \eta^2 dx = 0. \quad (25)$$

After a return to dimensional variables and with $c = dX/dt$, this latter result is found to be identical to (16). An axisymmetric eddy would be governed by the modified equation

$$s\left(\eta_{rr} + \frac{1}{r}\eta_r\right)_t - \eta_t + \eta_x + \frac{\epsilon}{s}\eta\eta_x = O(\beta, \epsilon) \quad (26)$$

where $r^2 = (x - ct)^2 + y^2$, and the expression for c is again identical to (16).

4. Numerical verification

The generality of the context within which expression (16) for the drift speed was established invites testing against a variety of numerical experiments. Such experiments permit variation of the radius and amplitude of the eddy and examination of the distinction between center of mass and center of eddy. Also, it is desirable to investigate numerically whether the error estimation (18) is reliable and under which conditions it is sufficiently small for formula (16) to give a drift-speed value of practical use. Finally, having two numerical models at our disposal, a primitive-equation model (Bleck and Boudra 1986; Chassignet et al. 1989) and a generalized geostrophic model (Cushman-Roisin and Tang 1989), we wish to evaluate the performance of the latter against the former insofar as the westward eddy migration speed is concerned.

In the first series of numerical experiments, the two-layer, primitive-equations, isopycnic-coordinate model of Bleck and Boudra (1986) was run with the second layer virtually infinitely deep ($H_2/H_1 = 1000$) in order to behave as a reduced-gravity model (Chassignet and Cushman-Roisin 1990). Simulations were initialized with a circular anticyclonic vortex of Gaussian profile. The fixed parameters were $f_0 = 10^{-4}\ \mathrm{s}^{-1}$, $\beta_0 = 2 \times 10^{-11}\ \mathrm{m}^{-1}\ \mathrm{s}^{-1}$ and $g' = 2 \times 10^{-2}\ \mathrm{m\ s}^{-2}$, while the undisturbed upper-layer depth H (and, consequently, the deformation radius L_R), the maximum interfacial displacement at the eddy center δH, and the distance from center to maximum azimuthal velocity L were varied from run to run, in order to span realistic mid-latitude oceanic eddy conditions.

Table 1 lists the parameters of each numerical experiment, while Table 2 presents the theoretical values of the drift speed, the allowed absolute errors, and the numerical results. The theoretical predictions using the initial Gaussian profile agree with the numerical findings within 10% to 20%, and, more often than not, this discrepancy does not lie within the expected error. Close examination of the eddy evolution reveals, however, that the eddies undergo a rapid adjustment following the initial conditions and, consequently, drift westward while maintaining a structure somewhat different from the Gaussian profile. Hence, we found it more appropriate to compare the observed drift speeds with the theoretical prediction (16) using not the initial Gaussian profile but the actual adjusted structure. Absolute errors were calculated from (18) using the parameter values listed in Table 1. Table 2 shows excellent agreement, and it can therefore be concluded that the theory performs according to its anticipated performance.

In run 2 ($\delta H = 0.25H$, $L = R_d$), the predicted error is very large (in excess of 100%) while the theoretical estimate is extremely close to the numerical realization (within 3.3%). Tracking the source of the predicted

TABLE 1. Parameters of the numerical experiments.

Run number	H (m)	δH (m)	R_d (km)	L (km)	$\frac{\delta H}{H}$	$\frac{L}{R_d}$	s	ϵ	β	ω
1	1000	100	45	45	0.1	1	1	0.1	0.009	0.1
2	1000	250	45	45	0.25	1	1	0.25	0.009	0.25
3	1000	250	45	90	0.25	2	0.25	0.06	0.018	0.016
4	1000	500	45	90	0.5	2	0.25	0.13	0.018	0.031
5	500	250	32	95	0.5	3	0.11	0.06	0.019	0.006
6	500	500	32	60	1.0	2	0.25	0.25	0.012	0.063
7	500	500	32	95	1.0	3	0.11	0.11	0.019	0.012
8	250	500	22	65	2.0	3	0.11	0.22	0.013	0.025

error shows that the higher-order time derivatives in Eqs. (13)–(14) (the ω terms on the left-hand sides) are not small ($\omega = 0.25$). Equation (15) is thus a poor approximation in this case. However, these higher-order time derivatives are responsible for circular inertial oscillations of the center of mass (Ball 1963), which can merely be superimposed on the zonal translation. The particular initialization used here, i.e. a single circular eddy, imparts no net momentum to the flow and, hence, does not trigger any inertial oscillation. This explains the good agreement between theory and numerical experiment despite a generous allowance for error.

Aside from run 2, the rule for the prediction of the error in all other cases is

$$\text{relative error} = \epsilon = \frac{g'\delta H}{f_0^2 L^2} = \frac{\delta H}{H}\frac{R_d^2}{L^2}, \tag{27}$$

since $\omega^3/s\beta \leqslant \delta$ and $\beta \leqslant \epsilon$ for those runs. Because the theory yields a better prediction when the error is smaller, it can be seen for (27) that the smaller the eddy amplitude ($\delta H/H$) and/or the larger the radius (L/R_d), the more accurate the theory.

A second series of numerical experiments was performed with a generalized geostrophic equation solved by a spectral method (Cushman-Roisin and Tang 1989). Identical parameters were selected for all eight runs, and the results are reported in the last columns of Table 2 for comparison. The drift rates agree with both the theoretical predictions using the initial Gaussian profile

$$-\frac{dX}{dt} = \frac{\beta_0 g' H}{f_0^2}\left(1 + \frac{1}{4}\frac{\delta H}{H}\right) \tag{28}$$

and the findings of the more complete primitive-equation model, within the allotted error interval. In fact, for the last six runs, the generalized-geostrophic model yields values extremely close to the theoretical predictions, but this is not surprising because the model and the theory are based on the same assumption of a small Rossby number, which is indeed small for those runs.

With both the primitive-equation and the generalized-geostrophic models, we compared the drift rates of both the center of mass and the eddy center (point of maximum interfacial depth). For most runs, the difference is small. There is one exception: in run 1 when the eddy is quasi-geostrophic (small amplitude and radius), ample Rossby waves are radiated, and the two centers are propagating at significantly different rates (difference of about 1 cm s^{-1}). Although the difference appears significant, such quasi-geostrophic eddies propagate slowly relative to their swirl velocities,

TABLE 2. Results of the numerical experiments using a primitive-equation (PE) model and a generalized geostrophic (GG) model, and comparison with the theoretical predictions. Values in parentheses denote the percentage of deviation from the corresponding theoretical predictions, based on the actual adjusted eddy structure (PE model) or the initial Gaussian profile (GG model).

	PE model		GG model	
Run number	Prediction (using adjusted profile)	Numerical finding	Prediction (using Gaussian profile)	Numerical finding
1	4.63 ± 0.44	4.65 (.05%)	4.10 ± 0.44	3.11 (24%)
2	4.76 ± 6.92	4.92 (3.3%)	4.25 ± 6.92	3.26 (23%)
3	4.80 ± 0.25	4.75 (1%)	4.25 ± 0.25	4.08 (4%)
4	5.07 ± 0.50	5.08 (.1%)	4.50 ± 0.50	4.37 (3%)
5	2.54 ± 0.11	2.65 (4.2%)	2.25 ± 0.11	2.23 (.9%)
6	2.76 ± 0.50	3.03 (9%)	2.50 ± 0.50	2.42 (3%)
7	2.81 ± 0.22	2.98 (6%)	2.50 ± 0.22	2.48 (.8%)
8	1.65 ± 0.22	1.82 (9%)	1.50 ± 0.22	1.48 (1.3%)

and the difference becomes a moot point. For the present run-1 eddy, for example, the turnaround time for the particle at the radius of maximum velocity (45 km) is 7.4 days, and during such time the difference between the displacement of the center of mass and that of the eddy center is only 3.9 km, i.e., less than a tenth of the eddy radius.

Finally, one additional run was performed with the generalized-geostophic model to test the theory in the case of significant eddy evolution. The initialization consisted of two slightly overlapped anticyclonic formations which underwent merging concurrently with westward translation. With the parameters $\delta H = 0.5$ and $L = 2R_d$, the drift speed of the center of mass was found to be 9% larger than the theoretical rate before merging and 5.3% larger after merging, while the predicted error on the theoretical estimate is 11%. The theory is thus verified to be correct in at least this instance of transient behavior.

5. Extension to two-layer systems

The formulae proposed over the years as well as the preceding generalization to predict the westward drift speed of oceanic eddies were restricted to systems with a single moving layer. The principal reason for this systematic limitation is the complication brought by baroclinic processes arising when several layers are in motion. An attempt is made here to remedy this shortcoming and to discuss the modifications resulting from the additional physics.

The theory starts with the equations for every layer, which, without lack of generality, can be written as

$$\mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u} + f\mathbf{k}\times\mathbf{u} = -\nabla p \qquad (29)$$

$$h_t + \nabla\cdot(h\mathbf{u}) = 0, \qquad (30)$$

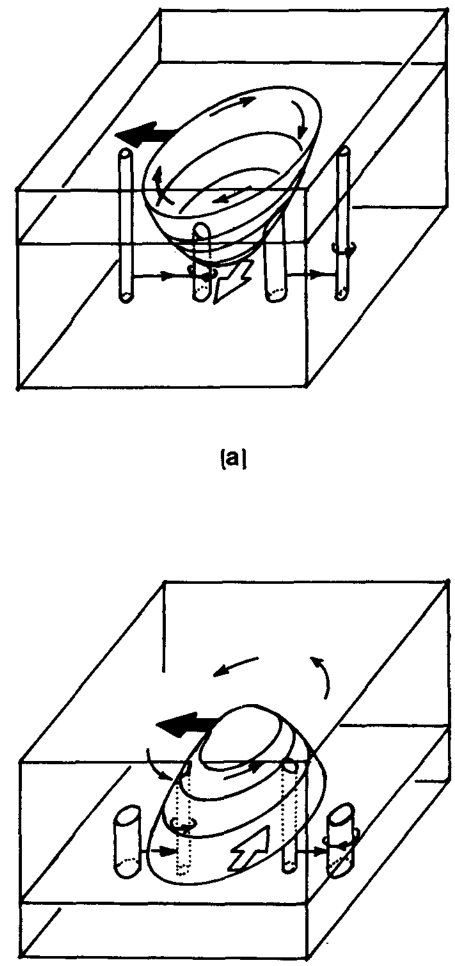
where a subscript indicating the layer number is understood. The requirement of hydrostaticity provides relations between the pressures, p, and thicknesses, h, of the layers, and these close the system of equations. In a two-layer system over a flat bottom and under a rigid lid, these relations can be expressed via the interfacial displacement η and the bottom-layer pressure π:

$$h_1 = H_1 + \eta, \quad p_1 = \pi + g'\eta$$

$$h_2 = H_2 - \eta, \quad p_2 = \pi.$$

In analogy with the developments in section 1, variables are scaled (adding $g'\delta H$ as the pressure scale), and the momentum equations are expanded about geostrophy,

$$\mathbf{u} = \mathbf{k}\times\nabla p - \omega\nabla p_t - \epsilon J(p, \nabla p) - \beta y\mathbf{k}\times\nabla p + O(\delta^2). \qquad (31)$$



(a)

(b)

FIG. 2. Impact of the westward migration of an upper-layer eddy on the lower layer and the reaction of the induced lower-layer relative vorticity on the eddy drift: squeezing and stretching under (a) an anticyclone and (b) a cyclone.

Then, multiplication of each continuity equation successively by x and y, integration over the infinite domain, and replacement of the velocity components by (31) provide the results. For two layers, one obtains

Upper layer:

$$\frac{dX}{dt} = -\frac{s_1\beta}{\omega}\langle\eta\rangle - \frac{\beta\epsilon}{2\omega}\langle\eta^2\rangle - \frac{s_1\beta}{\omega}\langle\pi\rangle - \frac{\epsilon}{\omega}\langle\eta\pi_y\rangle \qquad (32a)$$

$$\frac{dY}{dt} = \frac{\epsilon}{\omega}\langle\eta\pi_x\rangle \qquad (32b)$$

Lower layer:

$$\frac{dX}{dt} = \frac{s_2\beta}{\omega}\langle\pi\rangle - \frac{\epsilon}{\omega}\langle\eta\pi_y\rangle \quad (33a)$$

$$\frac{dY}{dt} = \frac{\epsilon}{\omega}\langle\eta\pi_x\rangle, \quad (33b)$$

where $s_i = g'H_i/f_0^2L^2$ and X and Y are still defined as in (8). Note that (33b) repeats (32b), while (32a) and (33a) can be solved for dX/dt and $\langle\pi\rangle$. In terms of dimensional variables, the relationships are

$$\frac{dX}{dt} = -\frac{\beta_0 g'}{f_0^2}\frac{H_2}{H_1+H_2}\frac{\iint (H_1\eta + \frac{1}{2}\eta^2)dxdy}{\iint \eta dxdy} - \frac{1}{f_0}\frac{\iint \eta\pi_y dxdy}{\iint \eta dxdy} \quad (34)$$

$$\frac{dY}{dt} = +\frac{1}{f_0}\frac{\iint \eta\pi_x dxdy}{\iint \eta dxdy} \quad (35)$$

$$\frac{1}{g'}(H_1+H_2)\iint \pi dxdy + H_1\iint \eta dxdy + \frac{1}{2}\iint \eta^2 dxdy = 0. \quad (36)$$

The diagnostic relationship (36) is akin to the requirement of compensated angular momentum discussed by Flierl et al. (1983) and will not be discussed further here. The other expressions, (34) and (35), are the two-layer extensions of (16) and (17). Two differences are noteworthy. First and foremost, a twisting term proportional to the barotropic signal π has emerged in each equation, modifying the westward drift velocity and introducing a meridional drift. A second modification is the multiplication of the former integral expression by the factor $H_2/(H_1 + H_2)$, thus reducing this component of the drift. The second modification is unimportant while the first is of primary importance.

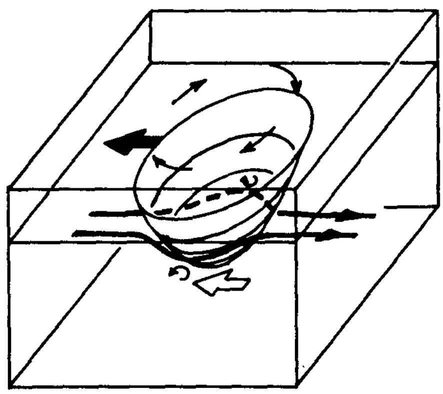
Although crucial, the new twisting terms correlating η with the derivatives of π cannot be discussed in all generality. The difficulty arises from the seeming impossibility to derive, from the original equations, other integral constraints involving the same twisting terms, so that eliminations could be performed. If it is true that no other useful integral constraint can be found, the only way to determine the signs and amplitudes of these twisting terms is by solving the equations themselves.

In light of this difficulty, one can now appreciate why the previous studies were mostly limited to one-layer systems. Three exceptions stand out, but these provide little assistance. Flierl (1984) considered the limit of a very deep but finite lower layer $[H_1/(H_1 + H_2) \ll \beta \ll 1]$, although he acknowledged that actual oceanic parameters $[\beta < H_1/(H_1 + H_2) < 1]$ hardly justify this approximation. Assuming in addition that the eddy is almost circular and steadily translating, that the potential vorticity in the lower layer has no closed contours, and that the Rossby wake has reached a steady state, Flierl could then show that the meridional drift of the eddy is directed southward and, surprisingly, increases with decreasing β.

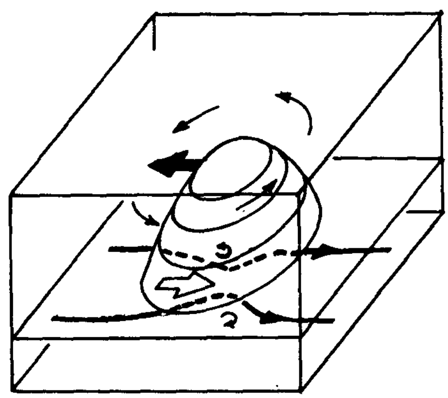
The second analysis involving multiple layers is due to Nof (1985). However, it was assumed there that the vortex formations in every layer are, at the leading order, vertically aligned so that the pressure contours are identical from layer to layer and the twisting terms identically vanish. The last analysis, due to Killworth (1986), ignores the presence of a barotropic component. In this case, the sum of equations similar to (32)–(33) over all moving layers yields complete cancellation of the twisting terms, therefore sidestepping an important process.

The view of the present authors is that little can be said about the effect of baroclinicity on the migration of eddies beyond the writing of the twisting terms [Eqs. (34) and (35)]. Indeed, as we shall attempt to show here, the signs and magnitudes of these terms are apt to depend strongly on the initial conditions. Typically, numerical simulations are initialized with a moving upper layer and a resting lower layer. Although this choice is well justified from the point of view of wanting to minimize the number of free parameters, it imposes a very specific potential-vorticity (PV) distribution in the lower layer, namely one that reflects the initial interfacial topography.

For example, an initial anticyclonic eddy in the surface layer implies a deepening of the interface and, hence, a bottom layer with shorter water columns (higher PV) below the eddy than elsewhere (lower PV). While the eddy begins its westward migration under the beta effect, the fluid below the eddy is being partially flushed, and, consequently, some surrounding, lower PV fluid is being squeezed below the eddy and some higher PV fluid is being stretched as it begins to trail behind the eddy (Fig. 2a). This generates a nonuniform distribution of relative vorticity in the lower layer, anticyclonic under the eddy and cyclonic behind the eddy (to the east), which induces a southward advection. Vice versa, an initial cyclonic eddy implies a raised interface, whose westward displacement generates stretching underneath and squeezing behind the eddy (Fig. 2b). The resulting relative-vorticity pattern now advects the eddy northward. Mathematically, the squeezing of the lower layer beneath a moving anticyclone ($\eta > 0$) generates a high pressure ($\pi > 0$), while the stretching behind it (larger x) sets a low pressure ($\pi < 0$); the product $\eta\partial\pi/\partial x$ is negative, and Eq.



(a)



(b)

FIG. 3. Impact of the westward migration of an upper-layer eddy on the lower layer and the reaction of the induced lower-layer relative vorticity on the eddy drift: meridional displacements caused by (a) divergence around an anticyclone and (b) convergence under a cyclone.

(35) yields a negative dY/dt, viz. a southward migration. In the case of the cyclonic eddy ($\eta < 0$), expansion below the eddy ($\pi < 0$) and squeezing behind it ($\pi > 0$) leads to a positive $\partial\pi/\partial x$; since η is negative, both numerator and denominator of (35) are negative, dY/dt is now positive, and the eddy moves northward.

Although the scenario described above seems to match numerical simulations (McWilliams and Flierl 1979), it should now be evident to the reader that the conclusions are intimately related to the initial potential-vorticity distribution in the lower layer. Other choices of initialization might well yield different, if not opposite, behaviors.

Perhaps less dependent upon the initial conditions is the effect of the baroclinic term on the zonal migration speed [last term of Eq. (34)]. This process is illustrated on Fig. 3a–b. Owing to its penetration in the lower layer, an anticyclone can be viewed as an obstacle, which, as it moves, forces surrounding fluid to go around it. Because the migration is primarily westward, the lower-layer parcels are displaced northward and southward (Fig. 3a), and the argument developed for the upper-layer surrounding fluid (Fig. 1b) applies here, leading to an acceleration of the westward drift. Mathematically, η and the pressure gradient $\partial\pi/\partial y$ are both positive, forming a negative contribution to (34) and, hence, a westward migration adding to the original beta effect. On the contrary, a cyclone behaves as a gulf of lower-layer fluid, which, as it moves, forces surrounding fluid to converge laterally to accommodate vertical stretching (Fig. 3b). The resulting meridional displacements generate cyclonic and anticyclonic relative vorticity on the northern and southern flanks, respectively. The net effect is an eastward induction. Mathematically, η and $\partial\pi/\partial y$ are both negative, forming a positive contribution to (34) and, hence, a retardation of the westward migration.

6. Conclusions

In the framework of a single-layer, reduced-gravity model, a simple formula has been proposed to estimate the westward drift of the center of mass of a mesoscale eddy formation. Although similar formulae have previously been proposed in the literature, the present one has two advantages. First, it is applicable to a wide variety of conditions; there is no restriction of axisymmetry or of steadiness, i.e., multiple eddies can be interacting, and wakes of Rossby wakes are not excluded (as long as they do not extend to infinity). Second, an estimate of the error on the formula is provided. The only restrictions are that the reduced-gravity model be applicable and that the Rossby number of the flow field be small.

If the Rossby number is not small, the formula is not applicable, and its failure will be indicated by a large relative error. If, however, the Rossby number is not small because of a large centrifugal force (as in some Gulf Stream rings) while the shape of the eddy remains close to a circle, the formula of Nof (1983) is still applicable. The latter formula involves the radial profile of the azimuthal velocity, which typically is not directly measured but inferred from the isopycnal slopes. In this estimation, the centrifugal correction to geostrophy is essential while observational uncertainties, stemming mostly from differentiating in the radial direction a profile that has been averaged azimuthally, may lead to an error level capable of jeopardizing the estimation of this ageostrophic correction. Hence, it is not clear whether Nof's (1983) formula can adequately supplement the present formula when the latter fails.

When the reduced-gravity model is not applicable, new terms must be added to the formula. Section 5 mostly dealt with the two-layer system, but the results

[Eqs. (34) and (35)] still hold if the additional pressure field [π] is merely that in the layer immediately below the surface layer, irrespective of the number of moving layers. Contrary to the terms obtained for the single-layer system, the additional terms cannot be discussed with a satisfying degree of generality. In particular, it does not seem possible to demonstrate that anticyclones will always migrate equatorward and cyclones poleward. However, it is not excluded that in certain realistic oceanic circumstances data will permit an estimation of these terms.

The restriction to small Rossby numbers is not too severe. Indeed, for the Rossby number to be on the order of unity, and thus for the flow to be fully ageostrophic, the eddy size must be comparable to the deformation radius, and the amplitude must simultaneously be finite. This only occurs for small eddies, which are quite insensitive to the beta effect. The center of mass of such eddies describes inertial circles, according to formulas first derived by Ball (1963) and corrected here for the beta effect [Eqs. (11) and (12)]. In his formulas, Nof (1981, 1983) has retained one ageostrophic effect, the centrifugal force. Although this component is important in fully ageostrophic vortices, it is not the sole ageostrophic effect that should be retained when the vortex departs from a steady, circular state. In the spirit of Nof, one could have retained here an additional term in the expansion of the velocity field, namely the term $-\epsilon J(\eta, \nabla\eta)$ of (5) before substituting the velocity components in (11) and (12). The result includes new terms of the form $-(\beta\epsilon/\omega)\langle(s + \epsilon\eta)\eta_x{}^2\rangle$ and $-(\beta\epsilon/\omega)\langle(s + \epsilon\eta)\eta_x\eta_y\rangle$ in the expressions for dX/dt and dY/dt, respectively. Like the presence of the azimuthal velocity in Nof's formula [Eq. (22) here], the presence of derivatives in these last terms requires a more detailed knowledge of the eddy formation before applying the formula than if only the leading terms are retained.

The formula constructed here includes, as asymptotic cases, previous formulae obtained for quasi-geostrophic, frontal-geostrophic and intermediate-geostrophic dynamics. Finally, new numerical simulations verify the applicability of the formula and indicate that the theoretical error estimate is generally appropriate.

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