A New Mapping Method for Sparse Observations of Propagating Features

Using Complex Empirical Orthogonal Function Analysis for spatial and temporal interpolation with applications to satellite data

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In oceanographic research, the relatively high cost of observations often prohibits scientists from collecting observations of a sufficient quantity to resolve the phenomena of interest. Efforts are usually made to space measurements as evenly as possible, but for a variety of reasons, station spacing is often problematic and almost invariably leads to unwanted gaps in the data set. The launch of many scientific satellites over recent years has provided scientists access to enormous volumes of data, presenting new opportunities for finding innovative ways to use them. One such instrument valuable to oceanographers is the satellite altimeter. Satellite altimeters use microwave radar to determine the instantaneous elevation of the sea surface height (SSH) of the ocean relative to an Earth-centered coordinate system, providing an absolute reference frame for studies of sea level rise. Satellite altimeters also provide frequent near-global data coverage of the world's oceans. This allows altimeter-derived estimates of SSH to be based on data collected over much of the ocean surface, in contrast to the geographically sparse coastal data provided by tide gauges. The TOPEX/Poseidon (T/P) satellite altimeter has been providing SSH measurements with a root mean square error of less than five centimeters since its launch in August of 1992. The satellite has a 10-day repeat orbit, with 6.2 kilometers between observations along the ground track and a 315-kilometer distance between neighboring ground tracks at the equator. However, there are still large gaps between the tracks compared to mesoscale oceanic features. For example, oceanic eddies have measurable sea level anomalies and diameters of roughly 100 kilometers. Loop Current eddies in the Gulf of Mexico are large (typically 150 to 200 kilometers) anticyclones with anomalously high sea levels in the center commonly over 50 centimeters. These, along with cyclonic eddies and some smaller features, generally propagate toward the west. While observing these eddies with T/P data, major difficulties arise when these eddies are located between satellite tracks. Traditional spatial interpolation of these satellite data will result in intermittent occurrences of these eddies in spatial maps of SSH constructed from the T/P data. While some useful computational techniques have been developed for unevenly spaced data, as reported in previous publications, most analysis methods require data values that are uniformly distributed in time or space. As a consequence, it is generally necessary to use an interpolation procedure to create the required regular set of data values as part of the data processing. This article will discuss an alternative to the conventional interpolation method, based on complex empirical orthogonal function (CEOF) analysis.

CEOF Mapping Method

CEOF analysis extracts information about propagating features from a two-dimensional data array, D, of N points in space and M points in time. With the use of CEOF analysis the component eigenmodes of D are obtained and consist of M pairs of complex vectors T_i and S_i, where T_i is a vector of length M and S_i is a vector of length N. Here, the S_i's will be referred to as the spatial functions (SFs) and T_i's as the temporal functions (TFs). T_i and S_i have the property that the sum of the products T_i x S_i for each mode recovers the original data array. Using a common vector identity, the complex vectors T_i and S_i can be written as functions of phase and amplitude.

The most significant modes (modes with bigger eigenvalues compared to others) are picked among all modes and contain most of the variability. For the TF and SF associated with each mode, the phase and amplitude information is mapped onto a regular finer grid in time and/or space. From these remapped phase and amplitude functions, the new TFs and SFs are rebuilt and finally the data set is reconstructed by summing the products of each TF and SF. This method could be further used for simultaneously interpolating data in both time and space.

Application of the Method

As a first demonstration, the method is tested with a simple analytical experiment requiring only time interpolation (Fig. 1a). In this case, a sinusoidal wave is propagating to the right and is sampled at each one fourth of its period (i.e., there are four observations, or samples during each cycle). A traditional interpolation technique would be to simply linearly interpolate in time between the samples. For the new technique, the CEOF analysis is conducted for the sampled data and the phase and amplitude of the time series for each of the top five modes is interpolated over time to a finer time grid. The new time series for each mode is reconstructed, and the product of it and the spatial pattern for each mode is summed. The new method does indeed identify the propagating information and retains the wave form and amplitude at the interpolated time better than the linear interpolation method, which results here in a reduced amplitude of the wave.

The next experiment involves a more realistic analytical approximation to ocean eddies, in order to demonstrate spatial interpolation with the new method (Fig. 1b). Here, a synthetic eddy with a positive SSH anomaly is propagating to the left. The SSH field is sampled at locations simulating the orbiting satellite altimeter observational pattern. The CEOF technique is applied to the sampled data and the top six modes are selected for interpolation. The phase and amplitude of the spatial pattern for each mode are interpolated from the coarsely distributed satellite track observational pattern onto a regular fine-mesh grid and the new spatial pattern for each mode is reconstructed using the new phase and amplitude information. Finally, the products of the newly mapped spatial pattern and the original time series for each mode are summed. This experiment clearly shows that the new method recovers the eddy and its propagation remarkably well.

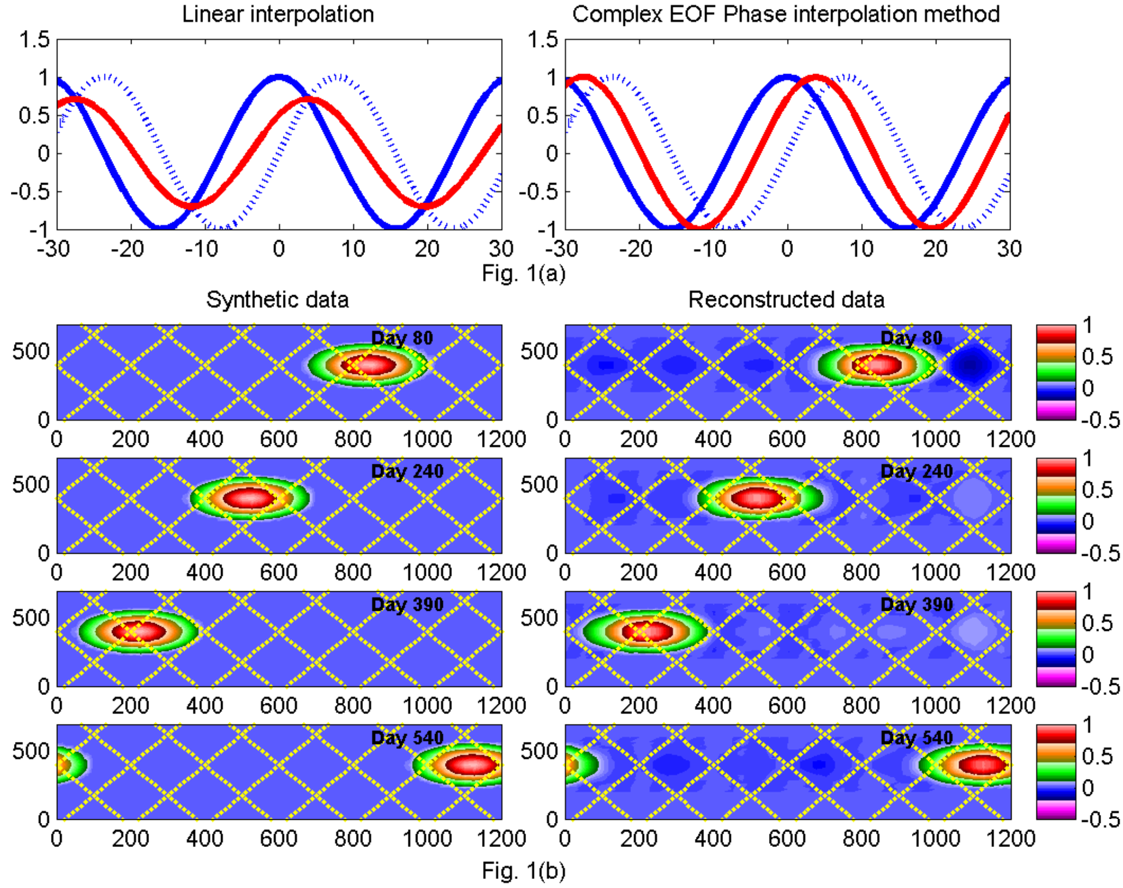

FIG. 1. A: Interpolation of a sinusoidal wave with an eight-second period propagating to the right, sampled every two seconds. The solid blue and hatched lines show the original wave during subsequent sampling at times t=0 s and t=2 s respectively, and the red lines show the positions of the wave interpolated to time t=1 s using linear interpolation (left), and the new method (right). B: A synthetic eddy propagating to the west. The yellow dots are the simulated satellite tracks. The right column shows the reconstructed data based on the simulated satellite observations.

The method is now applied to a realistic oceanic SSH field from a numerical circulation model of the Gulf of Mexico. The COAPS/Florida State University Gulf of Mexico simulation uses the Navy Coastal Ocean Model (NCOM), a 3D primitive equation ocean-circulation model. This 1/20º horizontal resolution simulation has been shown to reproduce the characteristics of the eddy field in the Gulf. Model SSH data are sampled at the actual observational locations and temporal frequency along the T/P orbital ground tracks. The new CEOF-based interpolation method is applied to these data in an attempt to recover the model's actual eddy field. The resulting interpolated field from the sparse observations locates the cyclones and anticyclones (with low and high SSH signatures, respectively) in generally the correct locations, even when these eddies are positioned between satellite tracks and have sizes comparable to the gaps in the spatial observational pattern (Fig. 2a).

Next, the method is applied to the real T/P satellite altimetry SSH anomaly data over the Gulf of Mexico. Each of the satellite tracks in the Gulf occurs at a different time during the10-day T/P repeat cycle, so all of the tracks must be synchronized prior to application of the spatial interpolation. This is accomplished by computing the CEOFs from the data along each track and interpolating the TFs to the specified times in the same way as the time interpolation of the moving sine wave example.

Synchronization of the tracks in this manner also allows interpolation of the physical fields to a shorter time interval than the observations. Again, the CEOF modes are computed for the synchronized time interpolated satellite track data and the phase and amplitude functions of the SFs are interpolated to a high-resolution uniform grid. The products of interpolated SFs and the original TFs are summed together to construct maps of SSH anomalies. A time versus longitude plot along the 26ºN zonal section from the reconstructed data in the Gulf of Mexico clearly shows the continuous propagation of eddies between satellite observations (Fig. 2b).

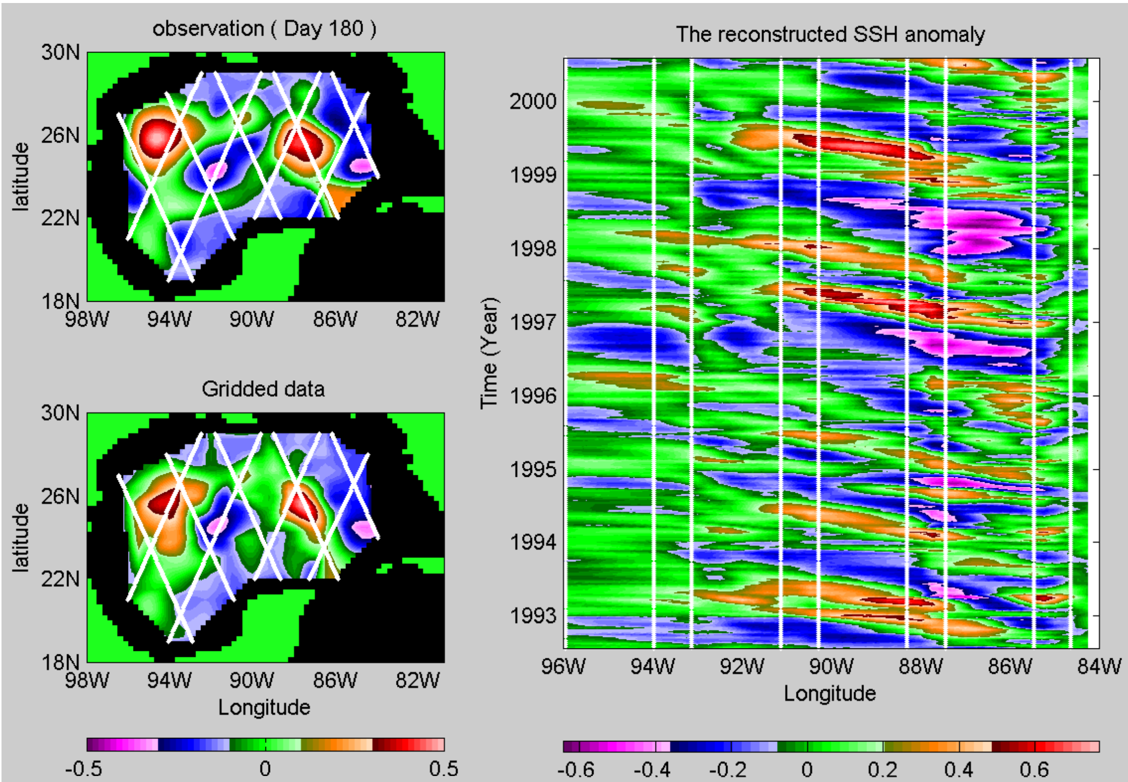

FIG. 2. A: SSH (meters) synoptic map from the NCOM GoM simulation (top) and the SSH map reconstructed using the new interpolation method from observations along the simulated satellite tracks (shown as white dotted lines). B: SSH anomaly (meters) of a zonal section (26ºN) in the Gulf of Mexico interpolated from the T/P satellite altimeter data. The westward propagation of cyclonic and anti-cyclonic eddies can be clearly seen between satellite observations (shown as white lines).

As another example, this same procedure is also applied to a region in the North Pacific near the Kuroshio Extension (Fig. 3). This current undulates producing strong, anomalous highs and lows that appear to propagate. SSH anomaly maps reconstructed from T/P satellite observations show that this interpolation method can capture information about eddies even at times when they are located between observational locations. A strength of this method is that it permits propagation of features in the interpolated grid without reduction in amplitude between satellite tracks.

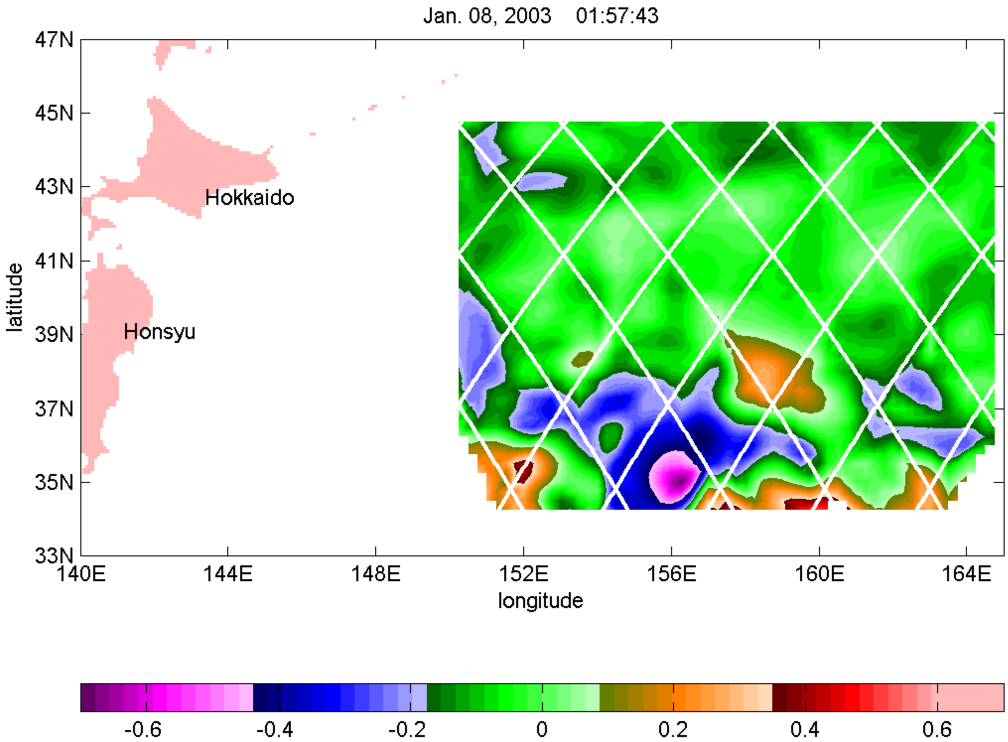

FIG. 3. SSH anomaly (meters) reconstructed from T/P satellite altimeter observations (shown as white lines) using the new interpolation method. The region lies to the east of Japan, along the northern edge of the Kuroshio Extension, where undulations in the current cause high and low SSH anomalies. Note the existence of anomalous highs and lows occurring between observations, which would be difficult to reproduce using traditional interpolation methods.

Conclusions

This work has focused on a new technique for interpolating irregularly spaced observations (both in space and time) to a finer-mesh grid retaining information of propagating features. The CEOF analysis extracts information about propagating modes from the observational data sets. The decomposed phases and amplitudes of the temporal and spatial functions can then be separately interpolated to the fine-mesh grid before reconstruction of the physical field through summation of the dominant modes.

This method has been applied to ocean eddies observed with satellite altimetry and has proven to be successful for identifying the life cycles of propagating eddies. The resulting uniformly mapped fields can then be quite useful for oceanographic studies, as well as ocean model initialization and data assimilation. Stationary features are problematic with this method, but this can be addressed with the higher spatial density of observations that can be achieved by combining data from multiple satellites. Further, this technique can be applied to other scalar fields and extended to vector fields, such as wind maps.

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