Assessment of a wetting and drying scheme in the HYbrid Coordinate Ocean Model (HYCOM)

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Acknowledgement

Thanks to this internship at COAPS, I learned the basics in physical oceanography, how to run a model and assess its performance. I improved my knowledge in computer languages (IDL, MATLAB, Shell, FORTRAN). I also improved my technical and relational English.
That is why I would like to thank Dr. Eric P. Chassignet for offering me the opportunity to live such an experience and discover the oceanography and a new country. I am grateful for his warm welcoming and for making me feel home during these 4 months. I would also like to thank all the people I met at COAPS, especially Flavien Gouillon and Alexandra Bozec for their constant help, Dmitry and Steve for their kindness, and James to have made me discover Florida. Then a special thank goes to Marie for being so nice with me. Finally, I would like to thank Magaly for coming even though I was working.

Contents

1 Motivation

To complete my second year in MATMECA Engineering School, I did an internship at the Center for Ocean-Atmospheric Prediction Studies (COAPS) from June 5^{th} to September 23^{rd} 2006. COAPS, which is a part of Florida State University, is located in Tallahassee, Florida state capital. COAPS performs research in air-sea interaction, ocean and coupled air-sea modeling, climate prediction, statistical studies, and predictions of social/economic consequences due to ocean-atmospheric variations. Students in COAPS come from a wide variety of departments including meteorology, mathematics, computer science, and physical oceanography. COAPS is funded by several federal agencies, producing original published papers that advance our understanding of the ocean and the atmosphere.

This internship aimed to validate and test the HYbrid Coordinate Ocean Model (HYCOM) with a wetting and drying (WAD) scheme. Numerically, wetting and drying means that a grid point can be considered either land or sea. Why do we need this scheme? It allows us to have a better understanding and resolution of coastal ocean model dynamics (tides, storm surges, tsunami, etc.) The need for such schemes has been tragically demonstrated in the December 2004 Indian ocean tsunami and in August 2005 floods and destruction caused by Hurricane Katrina in New Orleans and the northern Gulf of Mexico. It also adds value to large-scale ocean dynamics and in various regions would provide coastal and ocean models improvement. This capacity exists in HYCOM but has not been extensively tested. This report is divided as follows.

First I will briefly review the main forces that governed motion in the ocean. Then we will see what ocean modeling is. The third section focus on the analytical solution of a sloshing water in a parabolic channel. Finally, Section 4 presents the comparison with numerical experiment solution.

2 A brief overview of ocean dynamics

Earth's atmosphere and oceans exhibit complex patterns of fluid motion over a vast range of space and time scales. These patterns combine to establish the climate in response to solar radiation that is inhomogeneously absorbed by the materials comprising air, water, and land. Spontaneous, energetic variability arises from instabilities in the planetary-scale circulations, appearing in many different forms such as waves, jets, vortices, boundary layers, and turbulence. The ocean represent 70.8% of the world surface, and is one of the main actor in the earth climate system. So, to understand climate we need to understand the ocean motion.

The ocean is set in motion by winds in the atmosphere and differences in temperature and salinity between the poles and equator. The presence of an ambient rotation, due to the earth's spin around its axis, introduces two acceleration terms that we can interpret as forces: the Coriolis force and the centrifugial force. The effect of the Coriolis force on ocean currents is a deflection to the right in the Northern Hemisphere and on the left in the Southern Hemisphere. The centrifugial force is usually combined with gravity when geophysical flows are considered. There are lots of others forcing which modified the ocean motion like friction, tidal force and river which have also an important role in ocean dynamics.

In order to describe the dynamic of a fluid we usually use the Navier-Stokes equation. The full equation in a vectorial form is (*Cushman*,1994):

$$\underbrace{\frac{\partial \vec{U}}{\partial t} + \vec{U}.\nabla \vec{U}}_{1} + \underbrace{2\vec{\Omega} \times \vec{U}}_{2} = \tfrac{1}{\rho}(\underbrace{-\nabla p}_{3} + \underbrace{\vec{F_v}}_{4} + \underbrace{\mu \Delta \vec{U}}_{5})$$

where (1) is the acceleration off a water parcel; (2) is the coriolis force;(3) is the pressure gradient term; (4) are the voluminal forces and (5) are the viscous forces. But in the ocean it is impossible to fully resolve these equations. We need to simplify them by doing some approximations.

3 Ocean modeling

Ocean modeling deals with the discretization of the equations of motion written above. One important issue is to determine the grid which has to be used depending on the area of the ocean where we want to solve them. There are, at present, within the field of ocean general circulation modeling several classes of numerical models which have achieved a significant level of community management and involvement, including shared community development, regular user interaction, and ready availability of software and documentation via the World Wide Web.

Those different numerical models can be used for large scale studies (global models) as well as small scale studies (high resolution near coastal area), and from few days processing (tides) to centuries (ocean current). Each models use different ways to solve physical processes, which means different approximations but also different numerical schemes. The aims of those models are very wide. Indeed some of this model is used to forecast the climate and some are used to study coastal phenomenon like storm surges or tsunamis.
We can sort different models by their respective approaches of spatial discretizations and vertical coordinate treatments.

3.1 Different class of model

There are three different coordinate types used to discretize the ocean water column (Figure 1). The simplest choice is z-coordinate, which divide the water column in fixed level from the surface (z=0 of a resting ocean) and z=-H(x,y) corresponding to the bottom topography. Another choice for vertical coordinate is the potential density ρ referenced to a given pressure (isopycnal coordinate). This coordinate is a close analog to the atmosphere's entropy or potential temperature. The last one is the σ-coordinate. It is usually defined as:

$$\sigma = \frac{z - \zeta}{H + \zeta}$$

where $\zeta(x, y, t)$ is the displacement of the ocean surface from its resting position $z = 0$ and z=-H(x,y) the bottom topography. We can notice that $\sigma = 0$ at the ocean surface and $\sigma = -1$ at the bottom; this coordinate is called a terrain following coordinate.

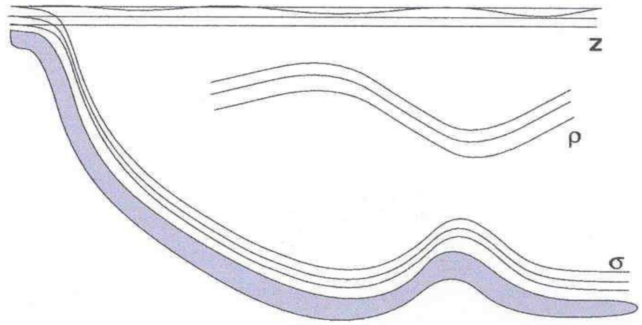

Figure 1: Schematic representation of the 3 vertical coordinates (after *Griffies et al.*, 2000)

The development of the first Oceanic General Circulation Model (OGCM) is credited to the Geophysical Fluid Dynamics Laboratory (GFDL) in the late 1960s. It was originally designed to use a z-based vertical coordinate, and to discretize the resulting equations of motion using low-order finite differences. During the 1970s, models started utilizing potential density and terrain following coordinates, but they still used low-order finite difference schemes. Today, several examples of isopycnal and σ coordinates models exist. For example, the Miami Isopycnic Coordinate Ocean Model (MICOM), created by the University of Miami, uses isopycnal coordinates. The disadvantage of all of these models is that they use a single coordinate type to represent the vertical but no single one can by itself be optimal everywhere in the ocean. This is why many developers have been motivated to pursue research into hybrid approaches, which is the subject of the following subsection.

3.2 The HYbrid Coordinate Ocean Model HYCOM

The HYbrid Coordinate Ocean Model (HYCOM) is the result of collaborative efforts among the University of Miami, the Naval Research Laboratory (NRL) and the Los Alamos National Laboratory (LANL) and combines all the three vertical discretization seen in the previous section (Figure 2). The optimal distribution is chosen at every time step: isopycnal (density tracking) layers are best in the deep stratified ocean, z-levels (constant fixed depths) are used to provide high vertical resolution near the surface within the mixed layer, and σ-levels (terrain following) is often the best choice in shallow coastal regions.

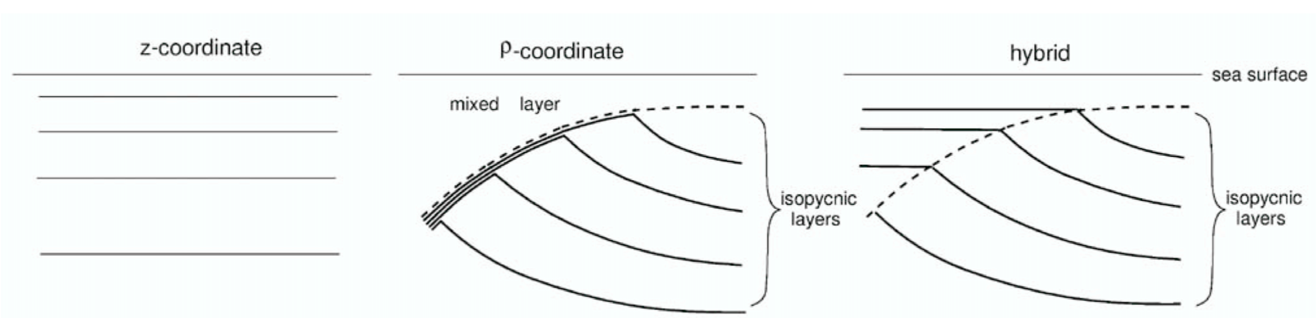

Figure 2: This schematic explain the hybrid coordinate between ρ and z.

The model makes a dynamically smooth transition between the coordinate types via the layered continuity equation. HYCOM is thus a highly sophisticated model, including a large suite of physical processes and incorporating numerical techniques that are optimal for dynamically different regions of the ocean. Several sophisticated vertical mixing turbulence closure schemes have also been implemented.

The purpose of my internship is to test a new version of HYCOM which include the wetting and drying scheme (WAD).

3.3 Standard HYCOM limitations

HYCOM assumes that prognostic variables, (horizontal velocities and pressure) can be divided in two components. The first component is the mean following z and called the barotropic component and the second one is the fluctuation around this mean called the baroclinic component.

$$u = u' + \overline{u}$$

where $\overline{u}$ is the barotropic component and, u' is the baroclinic component.
The time spliting that treats barotropic and baroclinic components separately in the implementation of the model in time is used in HYCOM in order to increase the code performance. The barotropic component which is independent on z are much easier to compute than the baroclinic. Assuming that the baroclinic component is smaller and slower than the barotropic one, the baroclinic component is then computed once every n barotropic time steps (Figure 3) (*Bleck and Smith*, 1990).

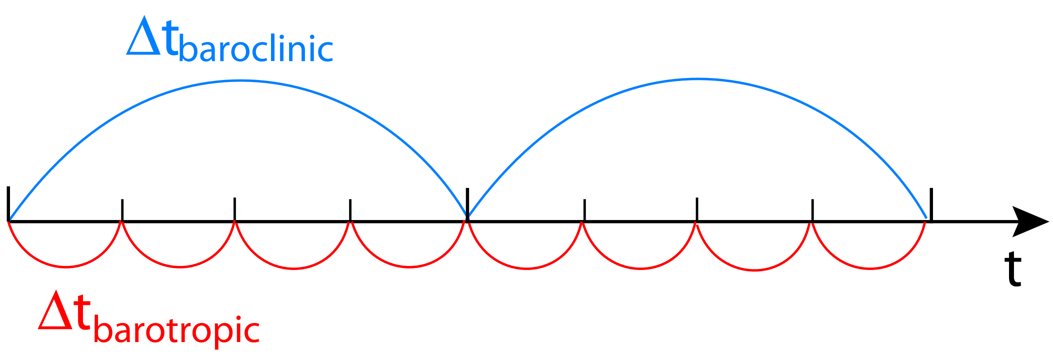

Figure 3: schematic of the time splitting in HYCOM. $\Delta t_{baroclinic}$ is the baroclinic time step and $\Delta t_{barotropic}$ is the barotropic time step. In general the baroclinic component is computed each 20 to 40 barotropic time steps.

Between the standard version of HYCOM and the wetting and drying version, the pressure term is treated differently at coastal areas.
The pressure is divided as follow:

$$p = (1 + \eta)p' \qquad \text{where} \qquad \eta = \frac{\zeta}{H}$$

where, p is the pressure, ζ is the sea surface elevation above the mean level and H is the ocean depth. In the deep ocean since $\zeta << H$ then $\eta << 1$.

The standard version of HYCOM has been adopted the convention of approximating $(1+\eta)$ by 1 wherever this expression appears as a factor. It means that, the standard HYCOM is making the assumption of $p = p'$ in some equation whereas the WAD HYCOM is not (Figure4).

Near coastal areas, the ocean depth can be of the same order as the sea surface perturbation and thus η can be not neglected as it is the case in standard HYCOM.

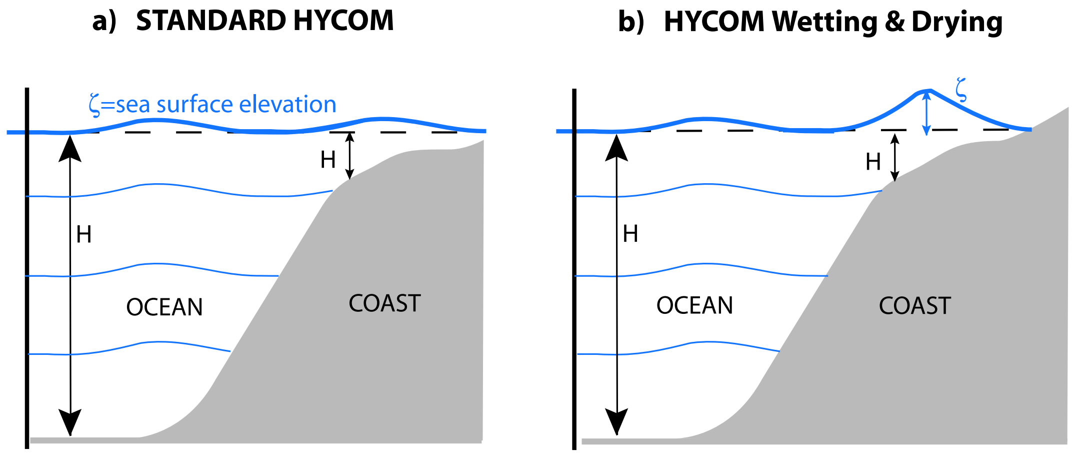

Figure 4: This schematic shows us the difference near the coastal area between the standard version of HYCOM a) and the WAD HYCOM b).

3.4 Wetting and drying

In this study we will particularly focus on coastal regions, since we have seen that the main differences appear in shallow water areas. The new version of WAD HYCOM profits of several developments previously tested in the MOUTON project (*Pichon and Baraille*, 2007) where a new time stepping has been implemented for the slow part of the barotropic fields.(see 3.3)

As we have seen before, the numerical definition of the wetting and drying is that a point can be considered land or sea. Thus the model topography is defined by an absolute land boundary over which the water can never spill over (brown line in figure 5). The dynamical boundary (light brown line) delimits wet points from dry points. People use to invoke blocking and deblocking conditions at cell's interface, for example we can set the flow across the interface to zero when the water depth becomes shallower than a pre-assigned depth, $H_{dry} \approx 0$.

Figure 5: Schematic of a coastal ocean region that can be flooded and drained (dried) by water-level variations from the open ocean (from *Oey*, 2004).

4 Thacker analytical solution for a sloshing water adjustment

In this section, we present the analytical solution of the shallow water equations in a parabolic basin which was first proposed by Thacker in 1981. Then in 4.2 it also presents the computation of the analytical solution for a parabolic channel basin. Finally the section 4.3 consists on providing initial condition and parameterization to get analytical solution.

The ocean's depth is very small compare to its length and width, so we can integrate over z this equation to have new one named "shallow water equation". So we have now, only two equations, one is following the x-direction (longitude) and the other one the y-direction (latitude). For this analytical problem, we neglect the viscous forces, but for the large scale phenomenon we must take into account the Coriolis force.

In his paper, Thacker considered the case where the motion in shallow water in a basin is governed by these equations:

$$\begin{cases} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = fv - g\frac{\partial \zeta}{\partial x} & \textbf{(1)} \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -fu - g\frac{\partial \zeta}{\partial y} & \textbf{(2)} \\ \frac{\partial \zeta}{\partial t} + \frac{\partial (h+\zeta)u}{\partial x} + \frac{\partial (h+\zeta)v}{\partial y} = 0 & \textbf{(3)} \end{cases}$$

where $\zeta(x, y, t)$ is the height of the water surface above mean water level; $z = -h(x, y)$ is the bathymetry; $u(x, y, t)$ is the depth averaged velocity component of the water current to the east; $v(x, y, t)$ is the depth averaged velocity component of the water current to the north; g is the acceleration due to gravity; f is the Coriolis parameter and t is time.

The Coriolis force depend only on the latitude an can be written as: $f = 2\Omega sin\phi$ where Ω is the angular velocity of the surface of the Earth ($\Omega = 7.29 \times 10^{-5} s^{-1}$) and ϕ the latitude of the water's parcel.

4.1 Topography used for the analytical solution

Thacker (1981) considered a parabolic bowl (Figure 6). We will adapt this bathymetry to get a parabolic channel configuration in order to have a simple analytical solution.

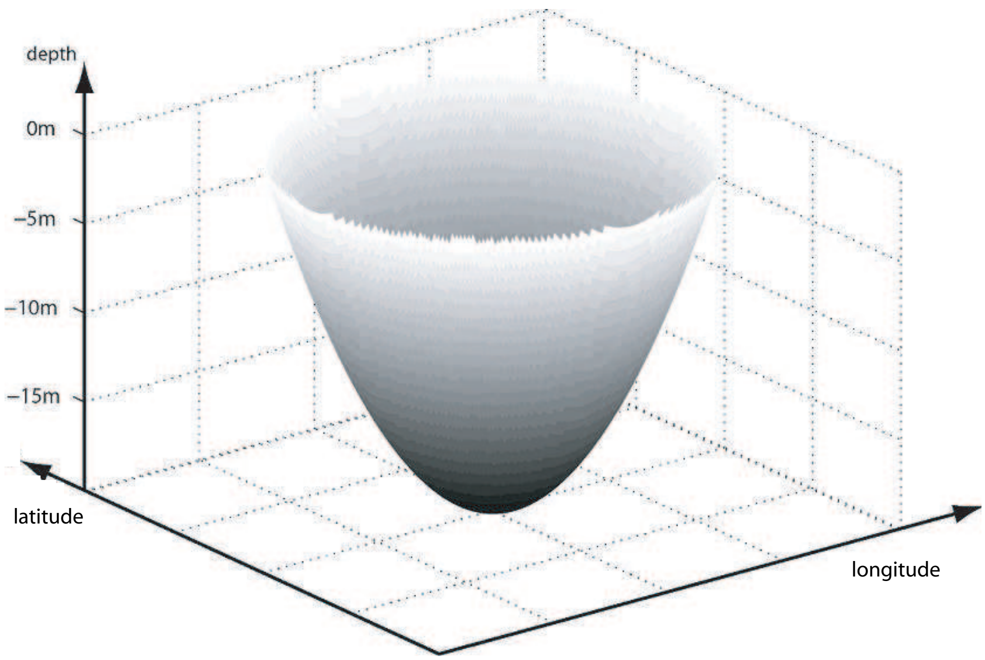

Figure 6: Schematic of the model domain. The horizontal dimensions (X, Y) are 201x201 and the bowl in itself has 160x160 grid points. The maximum depth of the bowl in this configuration is 20 meters but differs depending on the simulation run.

In a parabolic basin, the bathymetry is defined as:

$$h = h_0 \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)$$

Where h_0 is the deepest point of the channel, a is the radius of the bowl following the x direction, b is the radius of the bowl following the y direction.

Assuming that $b \longmapsto \infty$, the result is a parabolic channel (Figure 7). The equation for the bathymetry is now reduced to:

$$h = h_0 \left(1 - \frac{x^2}{a^2}\right) \qquad \textbf{(6)}$$

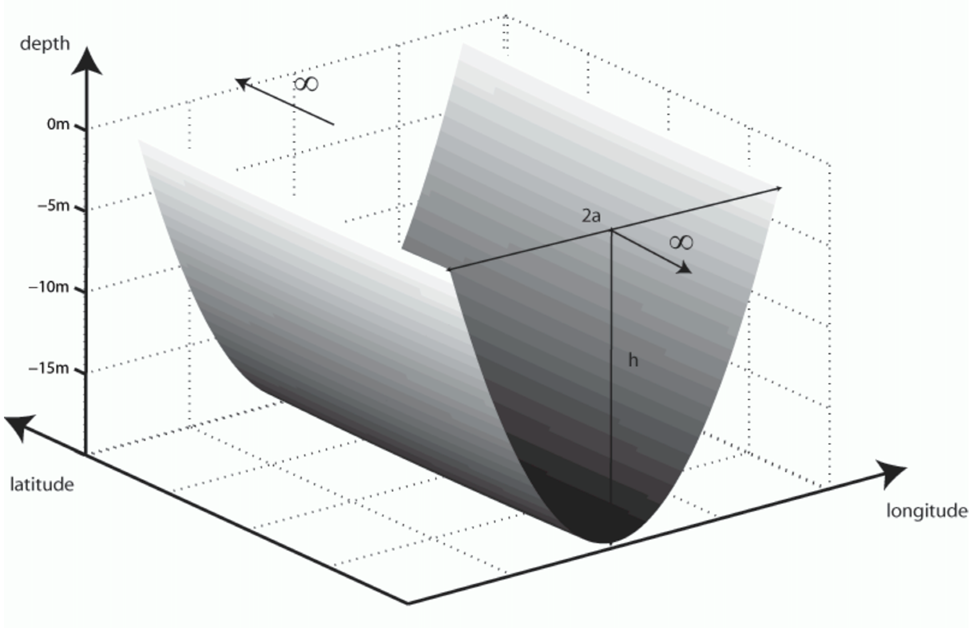

Figure 7: Schematic of the model domain. The horizontal dimensions (X, Y) are 201x201 and the canal in itself has 160x201 grid points. The maximum depth of the bowl in this configuration is 20 meters but differs depending on the simulation run.

Where h_0 is the deepest point of the channel, a is the width of the bowl following the x direction.

4.2 Analytical solutions of the sea surface elevation ζ in a parabolic channel

First, let assume that we are in a non-rotating case. The Coriolis parameter is then set to zero. The motion of the fluid is only following the x direction, so we set $v = 0$ (1D). In this case, the velocity is defined as:

$$u = u_0(t) \qquad \textbf{(7)}$$

$$v = 0 \qquad \textbf{(8)}$$

It's a zero order developpement follow x and y. This postulate will bring you to a 1^{st} order solution for the height of water surface above the mean water level ζ (i.e a straight line). As u depends only on the time, we have $\frac{\partial u}{\partial x} = 0$ and $\frac{\partial u}{\partial y} = 0$. The shallow water equations become:

$$\begin{cases} \frac{du_0}{dt} + g\frac{\partial \zeta}{\partial x} = 0 & \textbf{(9)} \\ \frac{\partial \zeta}{\partial y} = 0 & \textbf{(10)} \\ \frac{\partial \zeta}{\partial t} + u_0\frac{\partial (h+\zeta)}{\partial x} = 0 & \textbf{(11)} \end{cases}$$

Integrating over time (9) gives:

$$\zeta(x,t) = \zeta_0(t) + x\zeta_1(t) \qquad \textbf{(12)}$$

where

$$\zeta_1(t) = -\frac{1}{g}\left(\frac{du_0(t)}{dt}\right) \qquad \textbf{(13)}$$

Substituting (12) in (11), using (6) gives:

$$\frac{\partial\zeta_0(t)}{\partial t} + \frac{\partial\zeta_1(t)}{\partial t}x - \frac{2u_0(t)h_0}{a^2}x + u_0(t)\zeta_1(t) = 0 \qquad \textbf{(14)}$$

Given that the time-varying coefficients of the polynome are linearly independent, this equation can be solved by identifying terms of the same order (1 and x).

$$\begin{cases} \left(\frac{\partial\zeta_0(t)}{\partial t}\right) + u_0(t)\zeta_1(t) = 0 & \textbf{(15)} \\ \left(\frac{\partial\zeta_1(t)}{\partial t}\right) - \left(\frac{2h_0}{a^2}\right)u_0(t) = 0 & \textbf{(16)} \end{cases}$$

Substituting (13) in (16)

$$\left(\frac{d^2u_0(t)}{dt}\right) + \left(\frac{2gh_0}{a^2}\right)u_0(t) = 0 \qquad \textbf{(17)}$$

It is a second order differential equation where the solution is of the form:

$$u_0(t) = Acos(\Omega t) + Bsin(\Omega t) \qquad \text{with} \qquad \Omega = \sqrt{\frac{2gh_0}{a^2}}$$

Ω is the fequency of the water's oscillation.
if $u_0(t=0) = 0$ we have $A = 0$, so

$$u_0(t) = Bsin(\Omega t) \qquad \textbf{(18)}$$

Moreover, substituting (13) in (15) gives

$$\left(\frac{\partial\zeta_0(t)}{\partial t}\right) - \frac{1}{g}u_0(t)\frac{du_0}{dt} = 0 \qquad \textbf{(19)}$$

Substituting (18) in (19)

$$\zeta_0(t) = -\frac{B^2\Omega}{4g}cos(2\Omega t) + C \qquad \textbf{(20)}$$

Substituting (18) in (13)

$$\zeta_1(t) = -\frac{B\Omega}{g}cos(\Omega t) \qquad \textbf{(21)}$$

Substituting (20) and (21) in (12), we have finally the exact solution for the height of water surface above the mean water level in a parabolic channel:

$$\boxed{\zeta(t) = -\frac{B^2\Omega}{4g}cos(2\Omega t) + C - \frac{B\Omega}{g}cos(\Omega t)x \quad \textbf{(22)}}$$

where B and C are two constants, determined by the initial condition.

4.3 Parameters and initial conditions for numerical experiment

4.3.1 Parameters

The depth of the parabolic channel bathymetry is given by (6), and we will choose the different parameters for the numerical depth. The four parameters are

$$\begin{cases} h_0 = 2 \times ddo \\ x = (i - imil) \times dxgrid \\ a = 80 \times dxgrid \\ g = 9.806 \end{cases}$$

for the numerical equation of the bathymetry

$$depths = 2 \times ddo \left(1 - \left(\frac{dxgrid \times (i - imil)}{80 \times dxgrid}\right)^2\right)$$

where $ddo = 10m$ is the total depth of water; $dxgrid = 1000m$ is the space grid size; i is the index and $imil$ is the middle index.

4.3.2 Initial conditions

The B and C values are found since we know the profile for ζ at $t = 0$. There are two points where the solutions are easy to find.
First:

$$\zeta(0,0) = -\frac{B^2\Omega}{4g} + C \qquad \textbf{(23)}$$

Second:

$$\zeta(x_m, 0) = -\frac{B^2\Omega}{4g} - \frac{B\Omega}{g}x_m + C \qquad \textbf{(24)}$$

Where x_m are the X-coordinate of minimum of the sea surface water.
It follows from (23)-(24):

$$B = g\frac{\zeta(0,0) - \zeta(x_m,0)}{\Omega x_m} \qquad \textbf{(25)}$$

Then

$$C = \zeta(0,0) + g\frac{(\zeta(0,0) - \zeta(x_m,0))^2}{4\Omega x_m^2} \qquad \textbf{(26)}$$

5 Numerical solution for the sloshing water adjustment

This part consists on some comparison between the numerical and the analytical solutions find in the section above, and some error analysis.

5.1 Analytical solutions vs. numerical solutions

5.1.1 Visualization

The first idea is to plot the analytical and the numerical solution on the same graph to visualize if they are similar. An animation is available on the COAPS ftp website:
ftp://coaps.fsu.edu/pub/gouillon/WAD/Comapraison_analy_numer.gif (run for 90 hours).

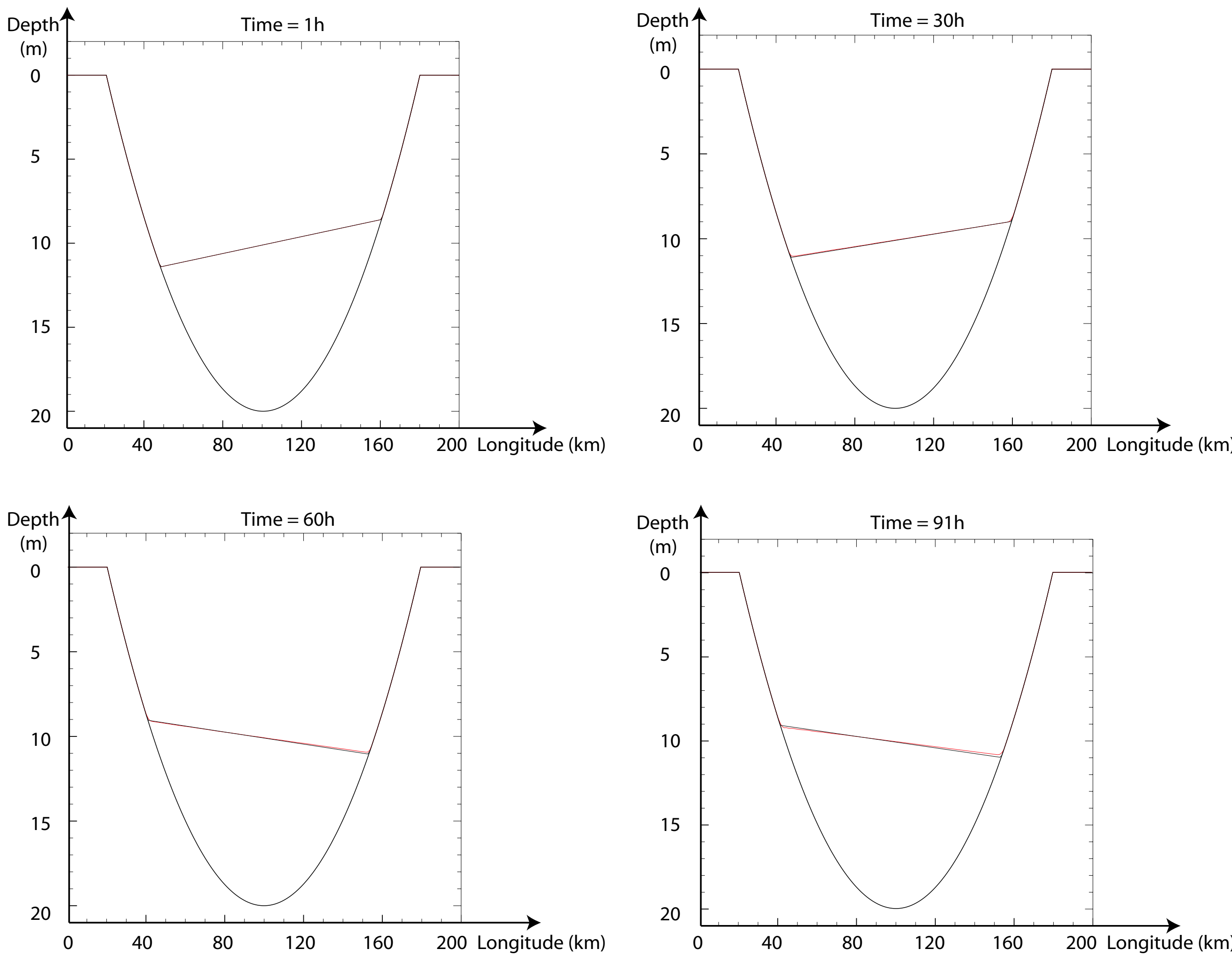

Figure 8: Comparison of analytic solutions vs. numerical solutions. The red line is the numerical solution and black solid line is the analytical solution

From this graph, we can see that one point can be wet or dry at different time and that the water can never spill over the absolute land boundary (0m). The solutions have the same behaviour, but after some time the numerical experiment diverge slowly from the analytical solution. Error estimate needs to be computed.

5.1.2 Error estimates

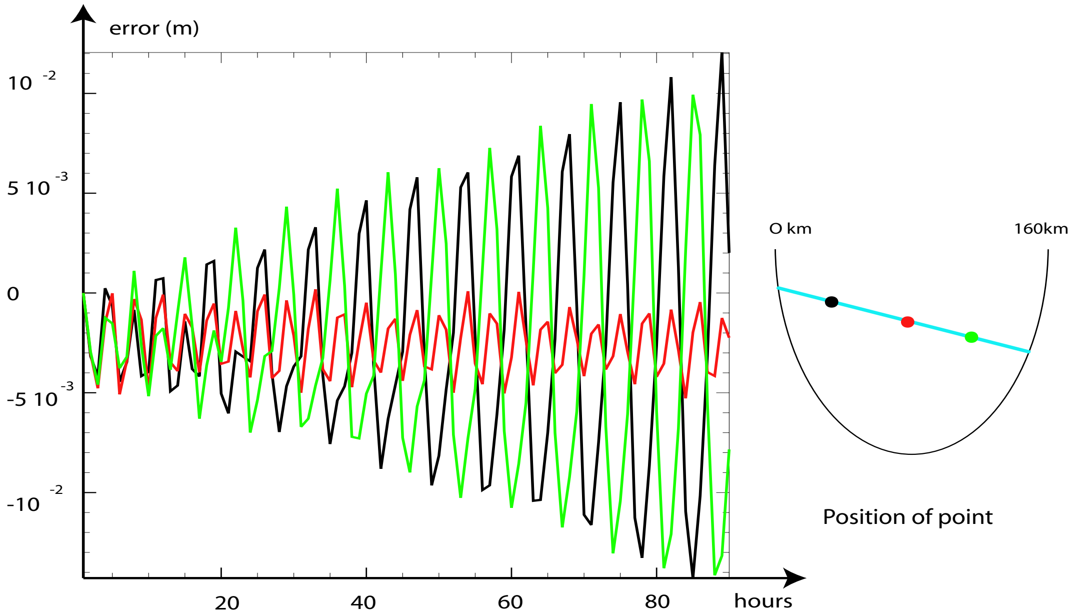

Figure 9: Error in meters between the analytical and numerical solutions for three points. The red line represents the middle point, the black line is at 40km form the left coast and the green line is at 40km from the right coast.

To quantitatively evaluate the error through time, we plot it for three points that differ by their locations (Figure 9). Errors consists in computing the distance (in meter) between two points of the same (x,y) coordinates, one with the standard HYCOM, the other one with the WAD HYCOM.

Figure 9 shows that the error is small, but it is empphasized by the presence of gravity waves along the interface generated by the topography discretization, the boundary wall effect and the stratification that perturbed the sea surface elevation.
To reduce this effect, we filter the surface gravity waves of the numerical solution using a linear regression and compare both slope coefficients of the sea surface height (Figure 10).

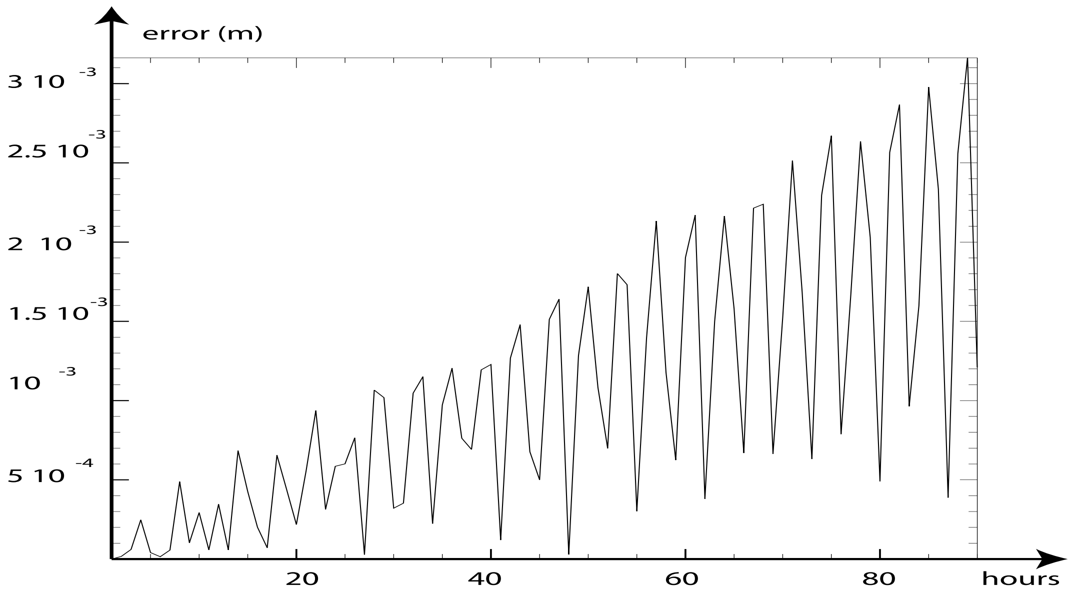

Figure 10: Smoothed errors estimate (absolute value) between analytical and numerical solutions for the slope coefficients.

The filtered error is smaller than unfiltered one and show as before that increase. Lahaye (2007) have made some similar experiment and the result is similar. Numerical dissipation exists explicitly in the model but weak enough to say that the model is well reproducing the sloshing water problem.

5.1.3 Velocity and period

The velocity time series of the middle point (Figure 11) shows that the velocity decreases over the time and the motion of the sloshing water is oscillating. We can evaluate that in 5 days the velocity have lost $0.15 m.s^{-1}$. If we assume that the velocity decreases linearly the total adjustment is reached after 31 days. This estimation is approximated because of two reasons: firstly the sampling is each hours (i.e we can miss the maximum sea surface elevation if it happen between two outputs, aliasing problem) and second in reality this phenomenon is non-leanear, i.e will reach asymptotically a state of rest.
As the motion is oscillating, we can also calculate the period of the oscillation. We find that the numerical period is $T_n = 7.06$ hr which is very close to the analytical period $T_a = 7.0500$ hr.

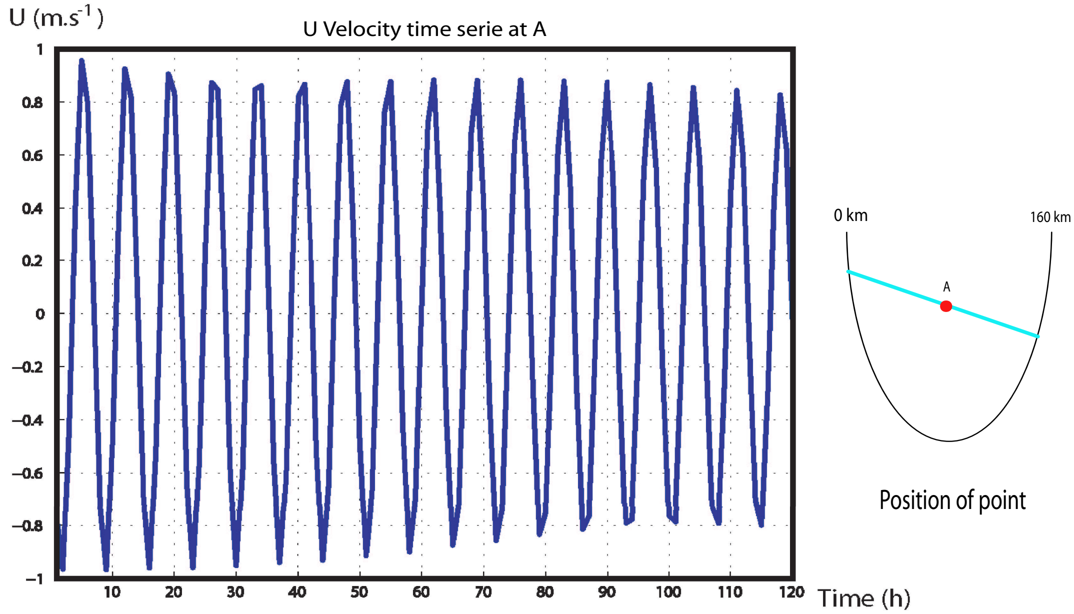

Figure 11: Velocity time series at A point which is in the middle of the bowl.

To conclude this part, it seems that a sloshing water problem with the WAD HYCOM is well representing the sea surface elevation and velocities. However, the system experienced some explicit dissipation in the numerical experiment while is not in the analytical experiment, but that are negligible on the time-scale of the simulation.

6 Conclusion

Through this report we have seen what the wetting and drying is in ocean modeling and why it improves the standard version of HYCOM. Then thanks to the Thacker analytical solution we have been able to evaluate the model. Finally, we have run the wetting and drying HYCOM in a parabolic channel and a bowl. The comparison with analytical solution (Thacker) shows that the model is working well.

Some numerical dissipation and wall effect attenuate the oscillation, but it is very weak. Now that the HYCOM model has been validated for a simple problem, it can be applied to more realistic cases. This work has already begun with some studies of the tides in the English Channel (i.e La Manche) by Remy Baraille and Yves Morel at SHOM (Service Hydrographique et Ocanographique de la Marine).

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