

# A Latent Heat Retrieval and Its Effects on the Intensity and Structure Change of Hurricane Guillermo (1997). Part I: The Algorithm and Observations

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## ABSTRACT

Despite the fact that latent heating in cloud systems drives many atmospheric circulations, including tropical cyclones, little is known of its magnitude and structure, largely because of inadequate observations. In this work, a reasonably high-resolution (2 km), four-dimensional airborne Doppler radar retrieval of the latent heat of condensation/evaporation is presented for rapidly intensifying Hurricane Guillermo (1997). Several advancements in the basic retrieval algorithm are shown, including 1) analyzing the scheme within the dynamically consistent framework of a numerical model, 2) identifying algorithm sensitivities through the use of ancillary data sources, and 3) developing a precipitation budget storage term parameterization. The determination of the saturation state is shown to be an important part of the algorithm for updrafts of  $\sim 5 \text{ m s}^{-1}$  or less.

The uncertainties in the magnitude of the retrieved heating are dominated by errors in the vertical velocity. Using a combination of error propagation and Monte Carlo uncertainty techniques, biases are found to be small, and randomly distributed errors in the heating magnitude are  $\sim 16\%$  for updrafts greater than  $5 \text{ m s}^{-1}$  and  $\sim 156\%$  for updrafts of  $1 \text{ m s}^{-1}$ . Even though errors in the vertical velocity can lead to large uncertainties in the latent heating field for small updrafts/downdrafts, in an integrated sense the errors are not as drastic.

In Part II, the impact of the retrievals is assessed by inserting the heating into realistic numerical simulations at 2-km resolution and comparing the generated wind structure to the Doppler radar observations of Guillermo.

## 1. Background and motivation

The main driver of tropical cyclone (TC) genesis and intensity change is the release of latent heat in clouds where the source of moist entropy flux comes from the thermodynamic disequilibrium at the ocean–atmosphere interface (Charney and Eliassen 1964; Kuo 1965; Emanuel 1986). In the eyewall region, convective clouds dominate the core structure with a mix of stratiform and convective features extending out to the bands of the system. Integrated cloud heating over the entire volume

of the storm is believed to be responsible for intensity and structure change (Cecil and Zipser 1999; Tory et al. 2006), although full-physics modeling studies (Braun 2002) and observational composites (Black et al. 1996) show that small-scale, intense convection contributes the largest percentage of the total upward mass flux ( $\sim 65\%$  from updrafts stronger than  $2 \text{ m s}^{-1}$ ).

Despite the fundamental importance of latent heat release, little is known of its structure in both space and time during all phases of storm evolution. To make matters worse, balanced nonlinear models of the vortex response to heating show large sensitivity to its structural characteristics (Hack and Schubert 1986). Most observational estimates of latent heat are from satellites, which have coarse resolution in both space (due to the height of the instrument as well as the limiting factors of antenna diameter and frequency choice) and time (due to orbit selection). Thus, the eyewall and rainband

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regions of a TC with embedded deep convective clouds are poorly resolved, leading to large errors in the latent heat field.

Early satellite estimates were made using passive microwave radiometers with horizontal resolutions of approximately 25 km at nadir (Adler and Rodgers 1977). The use of passive instruments for estimating latent heat release is difficult because of the broad, overlapping weighting functions and the complexity of the radiative transfer in clouds, especially those with mixed phase regions (Petty 2006). As a result, the specific details of hydrometeor distributions contributing to an observed brightness temperature can have large uncertainty. In addition, Adler and Rodgers (1977) and others (e.g., Sitkowski and Barnes 2009) use an estimate of the rainfall rate to compute latent heat; this approach represents a vertically integrated quantity and thus less information on cloud structure is obtained. More recent satellite estimates use the Tropical Rainfall Measuring Mission (TRMM) Microwave Imager (TMI), which has a much higher horizontal resolution of approximately 4–5 km at 85 GHz. Rodgers et al. (2000) were the first to use the TMI to compute vertical profiles of latent heat in a TC and found that as the storm intensified, heating rates increased in the inner core and extended upward into the mid- to upper troposphere. Recently, the TRMM Precipitation Radar (PR) has been used to estimate approximately 4.3-km horizontal and 0.25-km vertical resolution latent heating rates in TCs with three-dimensional (3D) capabilities (Tao et al. 2006).

Active instruments such as radars are not without errors either as many different drop size distributions and values of derived water content parameters, such as rainfall rate, can be associated with a measured value of reflectivity (Doviak and Zrnica 1984). As a result, latent heat estimates that rely solely on reflectivity-derived parameters can be expected to contain significant random error (a factor of nearly 4 for mean rainfall rate; Doviak and Zrnica 1984). As the TRMM PR is non-Doppler, critical information needed in the computation of latent heat (three components of the wind, especially vertical velocity) is unknown. In addition, the approximately 4.3-km surface footprint of the PR is still too coarse to resolve the important details of vigorous, deep convection in TCs (Black et al. 1996; Guimond et al. 2010).

Dual-polarization radar has been used to estimate warm rain and mixed phase microphysical processes in Florida convection (Tong et al. 1998). From an area-integrated perspective, Tong et al. (1998) found that warm rain processes (condensation and evaporation) dominated the total latent heat budget with a small component attributed to mixed phase processes (freezing/melting). Although very few dual-polarization observations of

TCs have been published, intuition suggests that the findings of Tong et al. (1998) extend to convection in TCs. For example, the modeling study of Zhang et al. (2002) showed the dominance of warm rain over mixed phase processes in the total thermal energy budget of simulated Hurricane Andrew (1992). Based on these results, the present study focuses on warm rain microphysics although the inclusion of mixed phase processes will alter the vertical profile of heating in the upper troposphere.

There are not many published Doppler radar estimates of latent heat in TCs. Gamache et al. (1993) used the National Oceanic and Atmospheric Administration (NOAA) WP-3D (P-3) tail radars to calculate the water budget of decaying Hurricane Norbert (1984). Although no latent heat estimates were calculated, Gamache et al. (1993) showed 3D distributions of condensed water that were retrieved using the steady-state continuity equation for water. An important result from Gamache et al. (1993) was that azimuthal asymmetries accounted for nearly half the net condensation of the storm. In addition, they noted significant departures from saturation in their full 3D retrievals whereas in the axisymmetric mean, the entire storm was saturated except in the eye. These results, for a decaying storm, indicate that computing the latent heat field within the inner core of TCs is not as simple as taking the product of the upward mass flux and the vertical derivative of the saturation mixing ratio. As part of the present work, the utility of determining saturation in the TC inner core is examined in detail.

In addition to the above observational studies, several investigators have documented considerable sensitivity to numerical model microphysical schemes when simulating TC intensity and structure. McFarquhar et al. (2006) found that choice of microphysics parameterization (including alterations to the basic condensation scheme) led to variations in simulated storm intensity by nearly 10 hPa. Uncertainty in graupel characteristics were found to also produce large changes in storm intensity and are likely one of the culprits behind the consistent and significant over prediction of radar reflectivities when compared to observations (McFarquhar et al. 2006; Rogers et al. 2007).

The goal of the first part of this work is to perform a comprehensive, high-resolution, 4D, airborne Doppler radar retrieval of the latent heat of condensation in a rapidly intensifying TC. New additions to existing retrieval methods will be highlighted including detailed error characteristics. Besides providing insight into the TC intensification problem, the latent heat fields presented in this study may prove useful for the validation of space-based algorithms and provide motivation for future satellite sensors (i.e., Doppler in space).

The paper is organized as follows. In the next section, the Doppler radar platforms and data used for computing the latent heat are described. In section 3, the latent heat retrieval algorithm is presented including enhancements to existing retrieval methods. In section 4, the algorithm is applied to observations of rapidly intensifying Hurricane Guillermo (1997) and uncertainty estimates are computed. Finally, in section 5, a summary of the algorithm, conclusions, and connections to Part II of the work are presented.

## 2. Doppler radar platform and data

The primary remote sensing instrument used in this work is airborne Doppler radar using the NOAA P-3 tail (TA) system. The P-3 TA radar operates at approximately 10 GHz and scans  $360^\circ$  in a plane perpendicular to the flight track. Frequently, the antenna is configured to scan in a cone offset from the track-normal plane alternately fore and aft of the aircraft, called the fore/aft scanning technique (FAST). The aircraft typically flies between 3- and 4-km altitude and does not penetrate convective cores, relying on side-looking views of high reflectivity regions. The along-track sampling of the P-3 TA radar in normal-plane scanning mode and FAST mode is  $\sim 0.75$  and  $\sim 1.5$  km, respectively, with 0.15-km gate spacing (Gamache et al. 1995; Black et al. 1996). Taking into account the  $1.9^\circ$  vertical and  $1.35^\circ$  horizontal beamwidths of the TA antennas and the sampling intervals using FAST, grid resolutions from the P-3s range from 1.5–2.0 km in the horizontal to 0.5–1.0 km in the vertical (Reasor et al. 2000, 2009). The main advantage of the P-3 radar sampling is the ability to provide essential information on the three wind components through the use of a retrieval technique (Gamache 1997; Gao et al. 1999; Reasor et al. 2009). However, the relatively coarse resolution of the analyses, the need to solve for the vertical velocity, and contamination of the boundary layer from ocean surface backscatter are the primary drawbacks of the P-3 TA radar. Additionally, the attenuation of the beam at 10 GHz through strong convective cores can be significant.

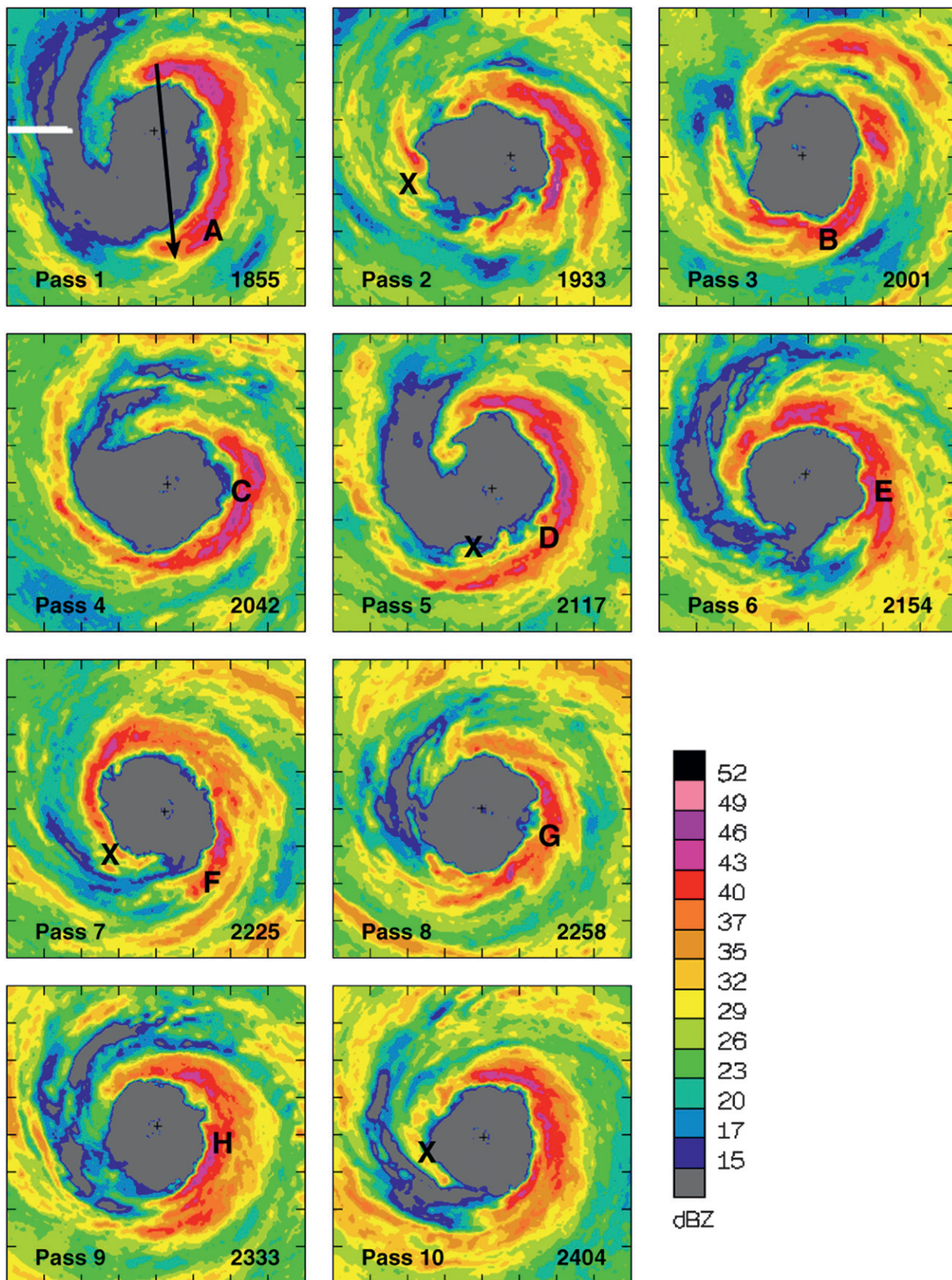
The P-3 data analyzed here were collected by two aircraft in the core of eastern Pacific Hurricane Guillermo on 2 August 1997 for approximately 5.5 h (10 composite periods with approximately 34-min sampling frequency), coincident with a rapid intensification episode of the storm (Reasor et al. 2009). Weak to moderate vertical wind shear ( $7\text{--}8\text{ m s}^{-1}$ ) resulted in a preference for convection in the downshear left quadrant of the storm during this period. Low wavenumber vorticity asymmetries propagating around the vortex were found to excite strong convective bursts that coincided with the

greatest intensification (Reasor et al. 2009). Figure 1 shows reflectivity scans from the NOAA P-3 lower fuselage radar (5.3 GHz) at 3-km altitude during 10 eyewall penetrations on 2 August 1997. Oscillations in the structure of the reflectivity from asymmetric to slightly more axisymmetric can be seen in Fig. 1 along with evidence of the presence of several convective bursts.

Given the consecutive sampling and spatial coverage, the Guillermo dataset provides the opportunity for studying fundamental problems associated with the impacts of deep convection and the role of the asymmetric mode in TC intensification. However, relatively coarse resolution of the Doppler analyses in space and time still limits the interpretation of the basic physics. The storm-centered radar domain is a box extending 120 km on a side with 2-km grid spacing and 20 km in the vertical with 1-km grid spacing. The first level of useful data is at 1-km height because of ocean surface contamination. The TA radar reflectivity field used in this study has not been corrected for attenuation. We focus our attention mostly on the inner portion of the domain (the eyewall, which is about 30 km from the radar on average) to minimize these effects. Note that, for many passes, two aircraft were used to construct the radar analysis, which will help reduce attenuation effects (see Table 1 in Reasor et al. 2009). Guillermo's 3D wind field was retrieved using a variational approach on a system of equations that includes the radar projection equations, the anelastic mass continuity equation, and a Laplacian filter, among others, including boundary conditions for the surface and just above the echo top (Gamache 1997; Gao et al. 1999; Reasor et al. 2009). Regions of the domain that do not have Doppler velocity information, such as portions of the eye, are effectively interpolated/extrapolated from regions where Doppler velocity was observed through a Laplacian filter (Reasor et al. 2009). The radar scanning strategies employed in the Guillermo sampling require a finite time separation between radial wind measurements in order to construct an accurate wind vector. Reasor et al. (2009) found a maximum time separation of about 6 min on the edges of the Guillermo Doppler domain and about 3 min in the eyewall region, indicating relatively small impact on the present analysis, which focuses on the eyewall. This radar dataset is used to perform a latent heat retrieval, described in detail in the next section.

## 3. Latent heat retrieval algorithm

The technique for retrieving latent heat from airborne Doppler radar is based partly on the method of Roux (1985) and Roux and Ju (1990). These studies used simplified forms of the momentum and thermal energy



equations, with individual terms or forcings estimated from radar observations, to deduce the pressure and temperature fields of squall lines. We focus our attention on the computation of saturation [see appendix B of Roux and Ju (1990)] and the use of the thermal energy equation. Several advancements in the basic algorithm are developed and presented below, including (a) analyzing the scheme within the dynamically consistent framework of a numerical model, (b) identifying sensitivities through the use of ancillary data sources, and (c) developing a water budget storage term parameterization.

### a. Theory

To prove the efficacy of the retrieval method, output from a nonhydrostatic, full-physics, quasi-cloud-resolving model simulation of Hurricane Bonnie (1998) at 2-km horizontal grid spacing (Braun et al. 2006; Braun 2006) is examined. The focus here will be on a 1-h period of the simulation (domain size of approximately  $450 \text{ km}^2$  in the horizontal extending to 17.2 km in the vertical, with the first model level at 40 m above the ocean), where model variables and precipitation budget terms were output every 3 min. At this time, the simulated storm was intensifying despite the influence of northwesterly vertical wind shear that resulted in an asymmetric distribution of convection [see Braun et al. (2006) and Braun (2006) for a detailed description of Hurricane Bonnie and the numerical simulation].

Although the simulated TC does not replicate the observed storm, the dynamically consistent nature of the model budgets allows the assessment of the qualitative and, to some degree, quantitative accuracy of the method. Gao et al. (1999) used numerical model output to test the accuracy of a Doppler radar wind retrieval algorithm and found errors (see their Table 1) that are consistent with those computed from in situ data using a similar retrieval algorithm (e.g., see Table 2 of Reasor et al. 2009). More real cases are needed to determine if the quantitative aspects of the Gao et al. (1999) results are valid, but the qualitative accuracy appears robust. As a result, we believe that testing the latent heat retrieval algorithm in the context of a numerical model provides a useful first step toward a reliable product.

The release of the latent heat of condensation occurs when water vapor changes phase to liquid water, which requires the air to be saturated. Therefore, for strong

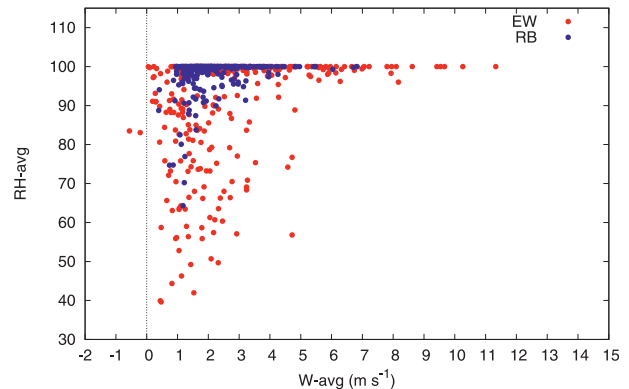


FIG. 2. Aircraft (P-3) flight-level (1.5–5.5-km altitude) measurements of updraft core magnitude as a function of relative humidity from the eyewall and rainband regions of intense TCs. Note that there are 620 data points in the figure. The convective-scale updraft events shown in the figure were defined after a scale separation method was applied (Eastin et al. 2005). As a result, the total vertical velocity can be negative when a convective updraft is superimposed upon a stronger mesoscale downdraft. The figure is courtesy of Dr. Matt Eastin; see Eastin et al. (2005) for more details.

updrafts, analysis of the vertical momentum equation reveals that local buoyancy from the release of latent heat must be present to generate significant vertical wind speeds and accelerations (Braun 2002; Eastin et al. 2005). Therefore, an important question is the following: does a threshold of vertical velocity exist where saturation and the release of latent heat can be assumed? Figure 2 shows 620 updraft cores (defined as convective-scale vertical velocities that exceed  $1.0 \text{ m s}^{-1}$  for at least 0.5 km along the flight track) as a function of relative humidity from the P-3 flight-level (1.5–5.5-km altitude) measurements in the eyewall and rainband regions of 14 intense TCs (Eastin et al. 2005). At  $5.0 \text{ m s}^{-1}$  and below, large variability in relative humidity is observed, while above  $5.0 \text{ m s}^{-1}$  nearly all updraft cores are saturated. Levels above approximately 5.5 km are not sampled by the aircraft. These data suggest that using a vertical velocity saturation threshold of approximately  $5.0 \text{ m s}^{-1}$  is reasonable although the sample size is small.

The numerical simulation of Hurricane Bonnie is used to calculate basic statistics on saturated vertical velocities on a grid point by grid point basis to support the observational data. Over the 1-h portion of the simulation analyzed here, approximately 52% of grid points

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FIG. 1. NOAA P-3 lower fuselage radar (5.3 GHz) reflectivity at 3-km height during the center of each aircraft pass through Hurricane Guillermo (1997). The domain is 120 km on each side with tick marks every 15 km. The solid arrow in pass 1 represents the time-averaged local shear vector and the capital letters denote details of the convective bursts. Figure is taken from Reasor et al. (2009). Used with permission of the American Meteorological Society.

with an updraft are unsaturated, with 94% of these coming from values less than  $1 \text{ m s}^{-1}$ . More importantly, approximately 92% of grid points with updrafts greater than  $5 \text{ m s}^{-1}$  are saturated (95 635 out of 104 074 points), which corroborates the observational data shown in Fig. 2. A similar result is found for downdrafts. Based on these data, we conclude that a threshold of  $|w| > 5 \text{ m s}^{-1}$  is reasonable for assuming saturation. Above  $5 \text{ m s}^{-1}$ , vertical accelerations are dominated by local buoyancy forcing while below  $5 \text{ m s}^{-1}$  various physical processes may play a role in the evolution such as perturbation pressure gradient forces (which are not generated by heating) and turbulence (Braun 2002; Eastin et al. 2005). This threshold should only be used as a guide as updrafts likely do not obey strict rules, but rather evolve through a continuum. Furthermore, the saturation threshold has uncertainty: the observational data shown in Fig. 2 have a small sample size and the model statistics are likely dependent on grid spacing and parameterized physics.

Statistics computed from the Bonnie simulation revealed that approximately 99% of updrafts are found to be less than or equal to  $5 \text{ m s}^{-1}$ , which carries the vast majority of the upward mass flux ( $\sim 70\%$ ; Black et al. 1996; Braun 2002). As a result, saturation cannot be assumed for the vast majority of updrafts and a large percentage of the total mass flux, which motivates the need for the determination of saturation through the algorithm described below.

The simplified form of the full model equation for the continuity of total precipitation mass (rain, snow, and graupel) can be written in a manner similar to Braun (2006):

$$\frac{\partial \rho q_p}{\partial t} = -\nabla \cdot (\rho q_p \mathbf{u}) - \frac{\partial(\rho q_p w)}{\partial z} + \frac{\partial(\rho q_p V_t)}{\partial z} + \rho Q_+ - \rho Q_- + \rho D + \rho Z, \quad (1)$$

where  $\rho$  is the dry air density,  $q_p$  is the total precipitation mixing ratio ( $\text{kg kg}^{-1}$ ),  $V_t$  is the hydrometeor fall speed ( $\text{m s}^{-1}$ ),  $Q_+$  and  $Q_-$  are respectively the total precipitation sources and sinks ( $\text{kg kg}^{-1} \text{ s}^{-1}$ ),  $D$  is the diffusive tendency of  $q_p$ , and  $Z$  is an artificial model offset for negative mixing ratios. The horizontal winds  $\mathbf{u}$  are storm relative and  $w$  is the vertical velocity ( $\text{m s}^{-1}$ ). Examination of each budget term [see Braun (2006) for a description] on the convective and grid point scales revealed that the turbulent diffusion of precipitation and model-offset terms are small and can be neglected. These results are consistent with Braun (2006). The reduced form of the continuity equation for total precipitation mass used in this study becomes

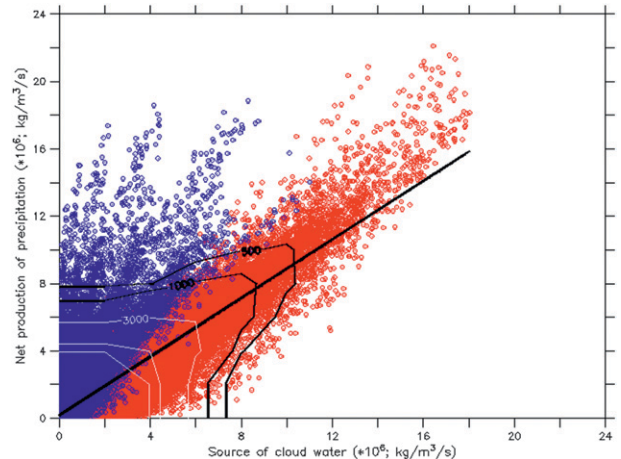


FIG. 3. Relationship between the source of cloud water and the net production of precipitation for grid points where precipitation is produced in the model domain between 0- and 10-km height and over a 9-min period (with 3-min output). Data are from the numerical simulation of Hurricane Bonnie (1998; Braun 2006). The black line shows the linear fit to the data with an  $R^2$  of  $\sim 0.70$  found using all 60 min of model output. The red and blue circles denote warm- ( $T > 0^\circ\text{C}$ ) and mixed/ice- ( $T \leq 0^\circ\text{C}$ ) phase precipitation processes, respectively. An  $R^2$  of  $\sim 0.87$  is found applying the linear fit to the red points only over a 60-min period. The contour lines show the number of points in dense regions of the scatterplot. Black contours start at 500 with a 500-point interval while white contours start at 3000 with a 6000-point interval. There are a total of 828 611 points in the figure with the largest percentage located in the lower left corner ( $>15\ 000$  points).

$$\frac{\partial \rho q_p}{\partial t} = -\nabla \cdot (\rho q_p \mathbf{u}) - \frac{\partial(\rho q_p w)}{\partial z} + \frac{\partial(\rho q_p V_t)}{\partial z} + \rho Q_{\text{net}}, \quad (2)$$

where the sources and sinks of precipitation mass are combined into a net precipitation source term  $Q_{\text{net}}$ . We note that the second and third terms on the right-hand side of (2) can be combined to yield a vertical Doppler velocity flux divergence of precipitation, which reduces errors in the budget (avoids estimation of hydrometeor fall speeds). However, combining these terms can only be done when the radar antenna is positioned in vertical incidence, which was not the case for the P-3 sampling of Guillermo (FAST was employed).

Figure 3 shows a scatterplot of the relationship between  $Q_{\text{net}}$  (output from model) and the source of cloud water (saturation) at model grid points where precipitation is produced between 0- and 10-km heights in the first 9 min of the 1-h simulation period. This subset of data is representative of the entire simulation and includes 828 611 points. A height of 10 km is used as a cap for points in Fig. 3 because the simulation revealed that the source of cloud water ceased at this level in deep

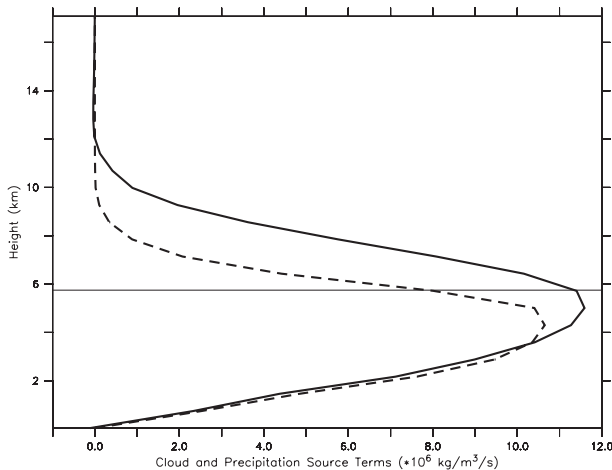


FIG. 4. A typical profile of the net source of precipitation (thick solid line) and the source of cloud water (dashed line) through deep eyewall convection from the numerical simulation of Hurricane Bonnie (1998; Braun 2006). The profiles are averaged over a  $10 \text{ km} \times 10 \text{ km}$  horizontal region centered on a convective cell for one snapshot in time. The thin solid line indicates the freezing level.

convection (see Fig. 4 for an example). The points in Fig. 3 are colored by temperature with red points greater than  $0^\circ\text{C}$  (rain microphysics) and blue points less than or equal to  $0^\circ\text{C}$  (ice microphysics). There is a linear relationship between the two variables in Fig. 3 with approximately 70% of the variability (statistic computed for the entire 1-h period) in  $Q_{\text{net}}$  explained by the source of cloud water for rain and ice points. The dominant mode of precipitation growth shown in Fig. 3 is rain microphysics and the associated collision-coalescence process (Rogers and Yau 1989), with the source of cloud water explaining 87% of the variance in  $Q_{\text{net}}$  for rain microphysics (red points) only.

Braun (2006) showed that in the azimuthal mean, the source of cloud water in the eyewall is immediately removed by precipitating hydrometeors, shown here on the grid point scale. The off-linear scatter in Fig. 3 is explained by ice microphysics (blue points) taking over the net production of precipitation. Note that there is some overlap between the red and blue points near the freezing level because no discrete threshold for rain/ice microphysics exists. Indeed, observations suggest that supercooled cloud liquid water can exist at altitudes of 12 km in deep convection located in the TC eyewall (Black et al. 2003).

Figure 4 shows an example of the vertical structure of the relationship between  $Q_{\text{net}}$  and the source of cloud water averaged over a representative eyewall convective cell in the Bonnie simulation. The source of cloud water matches very well with the net production of

precipitation up to 5–6-km height (melting zone). Above 6-km height, ice phase microphysics begins contributing to the formation of precipitation. A similar vertical structure is found for other regions of the simulation domain.

Figs. 3 and 4 demonstrate that by acquiring information on  $Q_{\text{net}}$  and determining where  $Q_{\text{net}} > 0$ , we are able to distinguish where the air is saturated, which is required before the release of latent heat can take place. Knowledge of the possible microphysical sources of precipitation (Rogers and Yau 1989) suggests that this is also true of TCs in nature. Note that (2) and the association of  $Q_{\text{net}} > 0$  with saturation are valid *instantaneously*. That is, assuming that information on the water content and winds are available quasi-instantaneously, the saturation state of the air and the associated magnitude of the latent heat release (described below) can be determined at the same time. Therefore, by using the signal to which the radar responds (precipitating hydrometeors for 10 GHz), information on the saturation state at each grid point in the 3D Doppler domain can be retrieved.

There are errors in associating the net production of precipitation with saturation in mixed phase regions of convection and for small values of  $Q_{\text{net}}$  that could occur near cloud boundaries, for example. When applying the theory to radar observations, instrument errors are also possible because of resolution, nonhomogeneous beam filling, attenuation, and calibration. Another source of error is the time separation between radar beam intersections discussed in section 2, which violates the instantaneous assumption. However, the algorithm presented here is somewhat insensitive to these errors because information is only required on the *condition* of saturation, not the *magnitude* of that saturation. Put another way, the algorithm is only dependent on knowing if precipitation is being produced, not on the precise value of precipitation production. Relying on  $Q_{\text{net}}$  for quantitative purposes (such as computing the latent heat magnitude) can lead to significant errors because of the large uncertainty in single-frequency radar-derived water parameters (see introduction; Gamache et al. 1993). We focus on the qualitative nature of  $Q_{\text{net}}$  to reduce the consequences of these errors, although dual-frequency radars show promise for quantitative retrievals of  $Q_{\text{net}}$  in future studies. With the P-3 radar used in this study, substantially reduced errors in the latent heat magnitude can be achieved by using the radar estimates of vertical velocity directly (described below) rather than relying on  $Q_{\text{net}}$  quantitatively.

Once the saturation state is determined, the magnitude of the latent heat can be calculated according to the entropy form of the first law of thermodynamics,

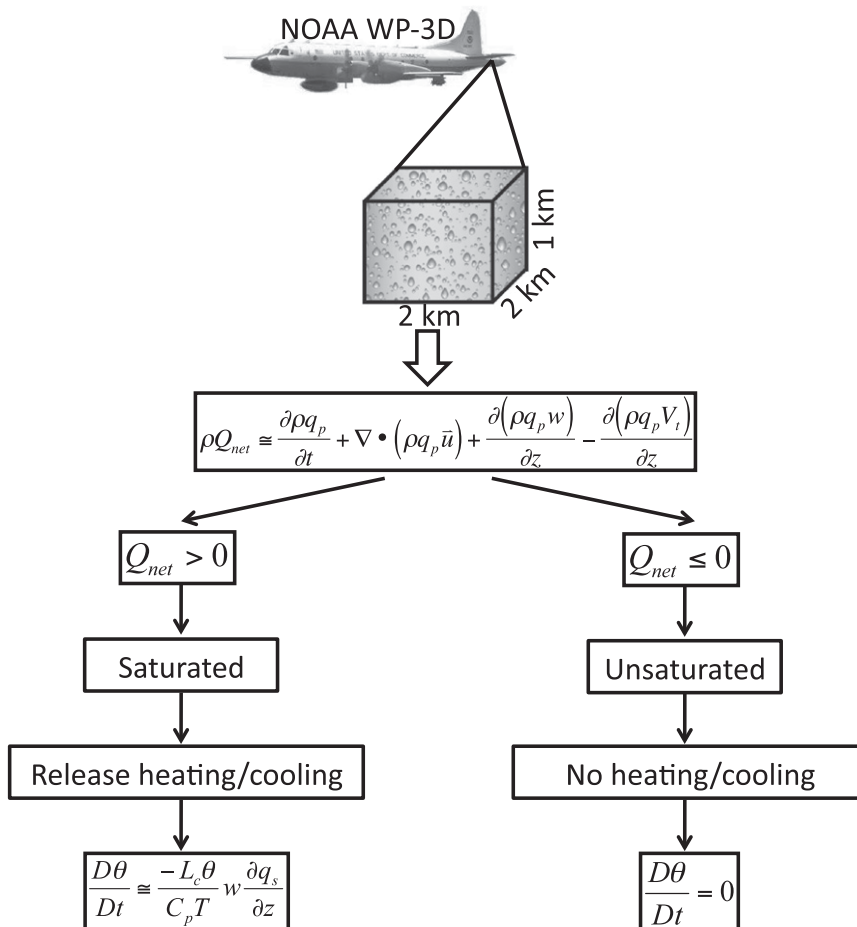


FIG. 5. Flowchart summarizing the basic steps in the latent heat retrieval algorithm. These steps are performed at each grid point in the Doppler analysis domain. Note that  $q_p$  and  $V_i$  are determined from the radar reflectivity using the empirical relationships described in section 4a. All variables and equations, including approximations, are defined in the text.

$$C_p \frac{D \ln \theta}{Dt} = \frac{J}{T}, \quad (3)$$

$$\frac{D\theta}{Dt} \cong -\frac{L_c \theta}{C_p T} w \frac{\partial q_s}{\partial z}. \quad (4)$$

where  $C_p$  is the specific heat of dry air at constant pressure ( $1004 \text{ J K}^{-1} \text{ kg}^{-1}$ ),  $\theta$  is potential temperature (K),  $T$  is temperature (K), and  $D/Dt$  is the material derivative. The heating rate term in (3) takes the form  $J = -L_c(Dq_s/Dt)$ , where  $L_c$  is the latent heat of condensation at  $0^\circ\text{C}$  ( $2.50 \times 10^6 \text{ J kg}^{-1}$ ) and  $q_s$  is the saturation mixing ratio ( $\text{g kg}^{-1}$ ). The material rate of change of the saturation mixing ratio, which is a function of temperature and pressure, is dominated by the vertical advection, yielding an approximate expression for the heating rate term:  $J \cong -L_c w (\partial q_s / \partial z)$ . Note that other diabatic contributions to the thermal energy budget such as radiative effects have been neglected. Substituting the approximate heating rate term into (3) and rearranging yields the expression used to calculate the magnitude of the latent heat release:

This method provides information on the latent heat of condensation/evaporation only and does not include mixed phase processes. However, as mentioned in the introduction, the overwhelming contribution to the total latent heat and thermal energy budget in convection comes from warm rain processes (Tong et al. 1998; Zhang et al. 2002).

Figure 5 presents a flowchart summarizing the main steps in the latent heat retrieval algorithm described above. The two main steps are (a) to solve (2) for  $Q_{net}$  and identify regions of saturation ( $Q_{net} > 0$ ) and, (b) in regions of saturation, to compute the magnitude of latent heating or evaporative cooling using (4). Note that when applying the theory to radar observations,  $q_p$  and  $V_i$  shown in (2) are determined from the reflectivity using the empirical relationships described in section 4a.

*b. Examining the assumptions in computing saturation*

Previous studies employing a form of the retrieval method outlined above have been unable to calculate the storage term in (2) because of inadequate Doppler radar sampling and thus have assumed the system or the clouds were in a steady state (Roux 1985; Roux and Ju 1990; Gamache et al. 1993). In a storm-relative reference frame, both the cloud and system scales of motion are not steady state and significant error can be expected if assuming stationarity (Gamache et al. 1993), especially on a local scale. The Guillermo dataset is unique in that composite Doppler radar sampling was completed on average every 34 min, allowing estimation of the storage term. However, it is found that using a 34-min time increment for computing the storage term added no more information (order of magnitude smaller than other terms) to the precipitation budget than using the steady-state assumption. This result is not surprising considering that the life cycle of a cloud is on the order of 30 min (Houze 1993).

How important is the storage term in the current latent heat retrieval algorithm for more accurate or shorter time update values? To address this question, a parameterization of the storage term is derived using output from the Bonnie numerical simulation. For model grid points where precipitation is produced, a linear relationship between the total horizontal advective flux of precipitation (largest contribution from tangential component) and the storage of precipitation (output directly from model with a time step of a few seconds) is found (see Fig. 6). Note that Fig. 6 only includes data at one snapshot while the linear fit used to derive the regression relation ( $R^2 = 0.78$ ),

$$\frac{\partial \rho q_p}{\partial t} = 0.802 \times [-\nabla \cdot (\rho q_p \mathbf{u})], \quad (5)$$

utilized an average of the fits at 3-min intervals over a 1-h period. The strong relationship in (5) does not imply that the saturation signal  $Q_{net}$  is a small residual and therefore prone to large error. The magnitude of  $Q_{net}$  is analyzed on several scales (grid point, spatial, and temporal averages in convective/stratiform regions) and is found to have a significant signal relative to the other terms in (2). Physically, the relationship in (5) confirms the fact that the tangential advective transport of precipitation in a mature TC controls the storage of precipitation to a large degree (a consequence of the divergence theorem). This relationship indicates that morphing (advecting precipitation features forward in time; Wimmers and Velden 2007) the radar reflectivity and derived precipitation fields using the Doppler wind

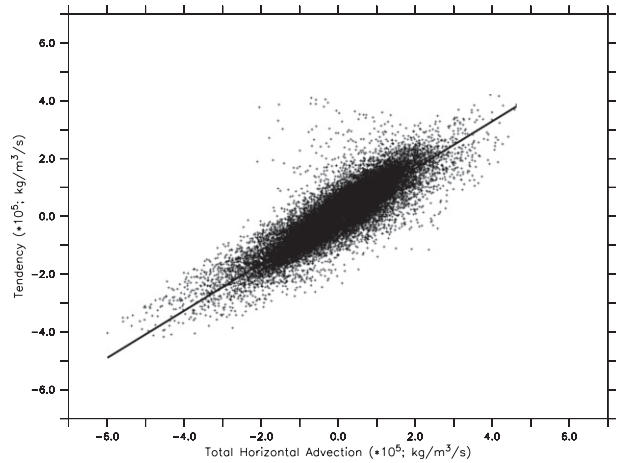


FIG. 6. The relationship between the horizontal advective flux of precipitation [in brackets on the rhs of (5)] and the storage of precipitation for model grid points where precipitation is produced at one snapshot in time. Model data are from Hurricane Bonnie (1998; Braun 2006) using 2-km horizontal resolution. The fit (see text) explains 78% of the variance in the data.

analyses to generate a storage term tendency shows promise. The storage term values produced through the model-based parameterization are very similar to those calculated by the authors using ground-based radar (refresh time of ~5 min) and P-3 LF radar (refresh time of 30 s) observations of mature TCs (not shown).

For the Bonnie simulation, Fig. 7 shows that using the storage term parameterization in (5) reduces the root-mean-square error (RMSE) in  $Q_{net}$  by more than a factor of 2 relative to the steady-state case. This result can also be expressed in terms of a cylindrical volume integrated error,

$$\text{Error} = \frac{\left| \int_0^z \int_0^r \overline{X^P} r dr dz - \int_0^z \int_0^r \overline{X^O} r dr dz \right|}{\int_0^z \int_0^r \overline{X^O} r dr dz}, \quad (6)$$

where  $\overline{X^P}$  is the azimuthal mean of the predicted variable (in this case, the estimate of  $Q_{net}$  under various approximations),  $\overline{X^O}$  is the azimuthal mean of the observed variable (in this case,  $Q_{net}$  output directly from the model), and  $r$  and  $z$  are the chosen outer (200 km) and upper (17 km) boundaries of the domain, respectively. Figure 8 depicts a time series of (6) for  $Q_{net}$  revealing that, in the temporal mean, the storage term parameterization reduces the error in  $Q_{net}$  by approximately 16% with improvements of nearly 30% at various times using the numerical model output. Also shown in Figs. 7 and 8 is the error using the approximate form of the precipitation continuity equation in (2) with  $Q_{net}$

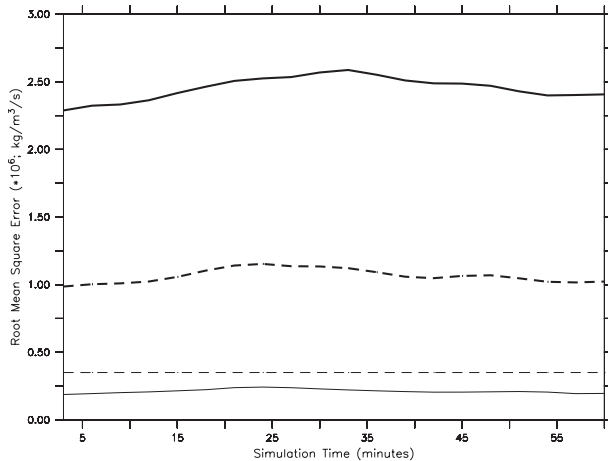


FIG. 7. The impact of the model-derived storage term parameterization on computations of  $Q_{\text{net}}$  [using (2)] in terms of the RMSE (averaged over the model domain). The control is  $Q_{\text{net}}$  output directly from the model. The thick solid line shows the results for computing  $Q_{\text{net}}$  using the steady-state assumption and the thick dashed line using the parameterization [(5)]. In addition, the thin solid line shows the impact of using the complete reduced form of the precipitation continuity equation [(2)] to calculate  $Q_{\text{net}}$ . The thin dashed line shows the mean value of  $Q_{\text{net}}$  for reference.

output directly from the model serving as the control. The errors in using (2) are low, which is consistent with the scale analysis already discussed.

We have demonstrated that  $Q_{\text{net}}$  is a very good proxy for saturation in a numerical setting. In addition, a reduced form of the precipitation continuity equation with a parameterization of the storage term has been shown to provide a good diagnosis of the actual  $Q_{\text{net}}$  output from the model. An obvious question presents: what is the impact of these approximations on the derived latent heating?

Figure 9 shows the impact of the storage term parameterization in terms of the azimuthal mean latent heating at the radius of maximum wind (RMW) for the Guillermo Doppler radar observations (shown in the next section). This sensitivity analysis is shown here because our ultimate goal is to apply the algorithm to observations. Large changes to the azimuthal mean heating relative to the steady-state case are found when using the parameterization (see Fig. 9) with differences of approximately 20% at midlevels to over 100% at lower (3 km) and upper (10 km) levels. Recent research has shown that for simplified TC-like vortices, the azimuthal mean heating dominates the dynamics of TC intensification (Nolan and Grasso 2003; Nolan et al. 2007). In light of these results, the storage term sensitivity shown in Fig. 9 is not only quantitatively significant for the accurate retrieval of latent heating in TCs such as Guillermo; it is physically significant as well.

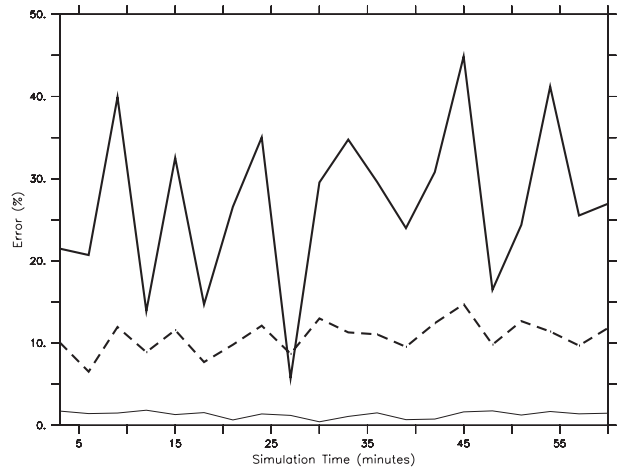


FIG. 8. As in Fig. 7, but the chosen measure of error is the azimuthal mean integration for  $Q_{\text{net}}$ . The mean values over time for each case are: steady state (thick solid line;  $\sim 27\%$ ), parameterization (dashed line;  $\sim 11\%$ ), and reduced form (thin solid line;  $\sim 1\%$ ).

Figure 10 shows the errors [according to (6)] in computing latent heat by determining saturation using (2) and (5) and identifying where the values of  $Q_{\text{net}}$  are greater than zero. The control is computing latent heat at grid points that are producing cloud water, which is required for air to be saturated. Note that latent heating rates computed from the model's microphysical scheme were not available, so the diagnostic latent heating rate (considering updrafts only) in (4) is used instead. Differences between the latent heating rates should be small and the expression in (4) is currently the only practical way to compute them from radar observations. The temporal mean error in Fig. 10 is approximately 8%,

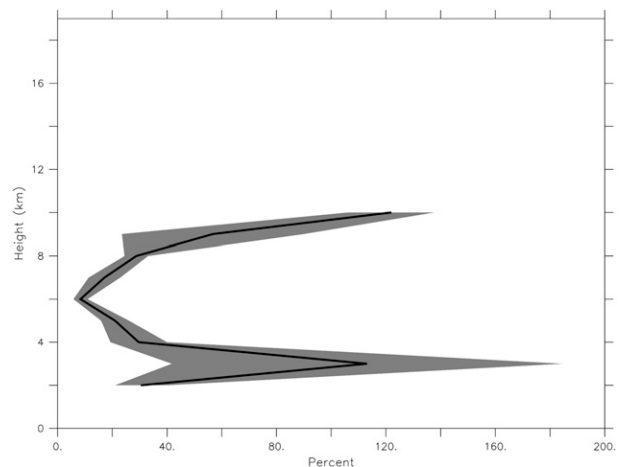


FIG. 9. The impact of the model-derived storage term parameterization on the azimuthal mean heating at the RMW for the Guillermo Doppler analyses. The thick black line shows the time mean and the shading depicts the standard error of the mean.

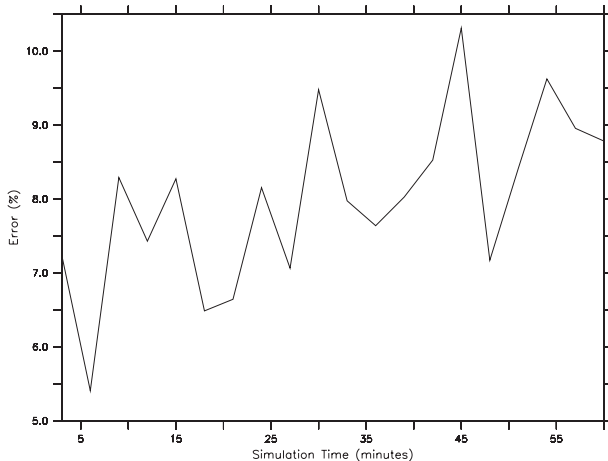


FIG. 10. The error [according to (6)] in computing latent heat by determining saturation using the algorithm described in the text [using (2) and (5) to determine where the values of  $Q_{\text{net}}$  are greater than zero]. The control is computing latent heat where model grid points are producing cloud water. The heating rates are computed according to (4) with the figure showing results for updrafts only. The temporal mean error is  $\sim 8\%$ .

with approximately 93% of the variance in the azimuthal mean heating explained by the retrieval method. Errors computed using both updrafts and downdrafts showed similar results, albeit with a weaker explained variance ( $\sim 87\%$ ).

The results described above demonstrate that the method for determining saturation in the latent heat retrieval is quite reasonable. Validating this result using observations is difficult because of the lack of in situ data over the large swaths sampled by the radar. Using a combination of flight-level data and dropsondes offers the best avenue for validation and is left for future work. Sensitivity tests and observational error analyses of the diagnostic heating expression in (4) are detailed in the next section.

#### 4. Observations and errors

##### a. Doppler radar-derived latent heating

To compute  $Q_{\text{net}}$  from Doppler radar, the total precipitation mixing ratio must be known, which is a summation of liquid water content (LWC) and ice water content (IWC). To derive this quantity, in situ cloud particle data collected by NOAA P-3 aircraft near 4-km altitude in the intense stages of Hurricane Katrina (2005) are analyzed. The cloud particle data are averaged over a period of 6 s in an attempt to match the sampling volumes of the particle probe and Doppler radar pulses (R. Black 2008, personal communication). Using the cloud particle data, the radar reflectivity factor

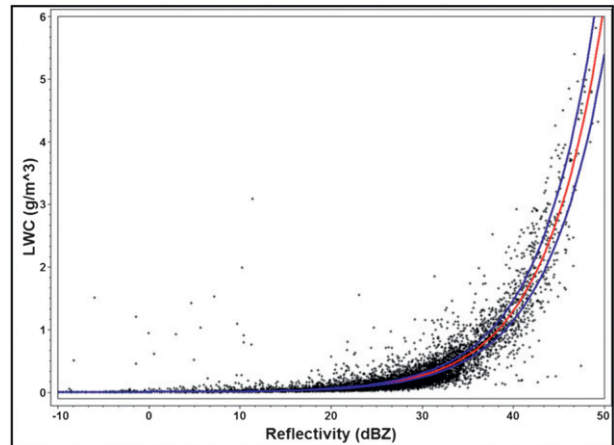


FIG. 11. The relationship between radar reflectivity factor (dBZ) and liquid water content using cloud particle data ( $\sim 7000$  data points) from NOAA P-3 aircraft flying near a 4-km altitude in Hurricane Katrina (2005) during a mature stage of the storm. The red line shows the best-fit nonlinear model ( $Z = 402 \times \text{LWC}^{1.47}$ ) and the blue lines represent the 95% confidence interval. The correlation coefficient is 0.88.

$Z$  and LWC are computed and the coefficients  $A$  and  $B$  of the power law ( $Z = A \times \text{LWC}^B$ ) are determined. Figure 11 shows a scatterplot of the relationship between reflectivity factor and LWC for the Katrina data. The red line shows the best fit ( $Z = 402 \times \text{LWC}^{1.47}$ ) with a correlation coefficient of 0.88 while the blue lines depict the 95% confidence interval, which gets larger with higher reflectivities (partly due to sampling). The relationship  $Z = 402 \times \text{LWC}^{1.47}$  is used below the melting layer while the IWC parameterization  $Z = 670 \times \text{IWC}^{1.79}$  (Black 1990) is used above the melting layer with linear interpolation of the two expressions within the melting layer.

Note that relationships between radar reflectivity factor and water content parameters are not unique and therefore uncertainty in  $Q_{\text{net}}$  will exist. This uncertainty is similar to rainfall rate (discussed in section 1) with random errors as large as a factor of 4 (Doviak and Zrnic 1984). As mentioned in the previous section, however, the algorithm presented here is *somewhat* insensitive to these errors because information is only required on the condition of saturation (sign of precipitation production term), not the magnitude of saturation (precise value of precipitation production term). Equation (2) is solved for  $Q_{\text{net}}$  using the Guillermo Doppler analyses, the storage term parameterization in (5), the computed precipitation mixing ratios described above, hydrometeor fall speed relations for a gamma distribution (Ulbrich and Chilson 1994; Heymsfield et al. 1999), and a composite eyewall density profile discussed below (perturbations to the density profile in all directions had a small effect on calculations). Based on Fig. 2 and the discussion

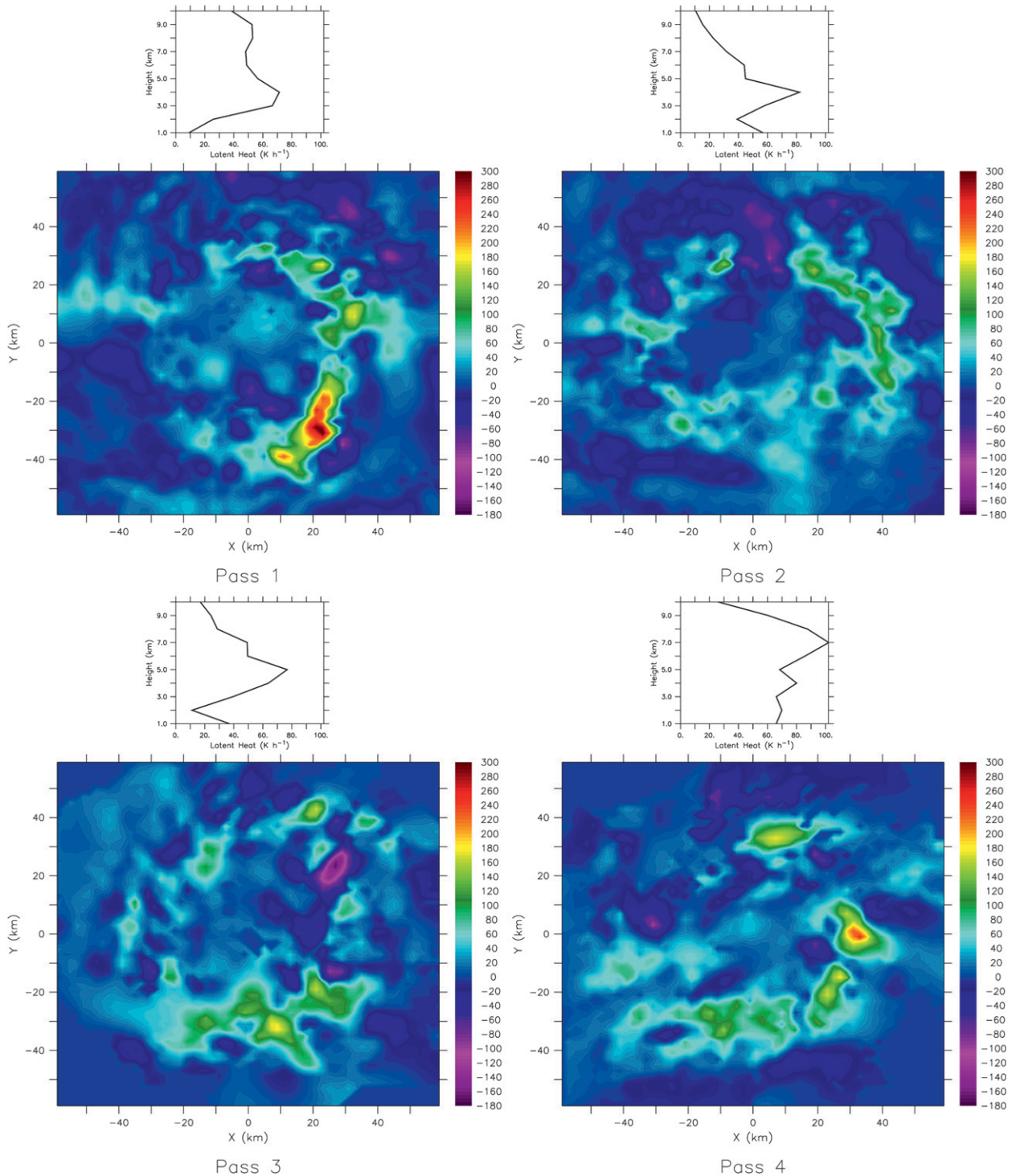


FIG. 12. Horizontal views (averaged over all heights) of the latent heating rate ( $\text{K h}^{-1}$ ) of condensation/evaporation retrieved from airborne Doppler radar observations in Hurricane Guillermo (1997) at the times shown in Fig. 1. Note that grid points without latent heating were assigned zero values after the vertical averaging. The vertical profile of the azimuthal mean latent heating rate at the RMW (30 km) is shown above each contour plot. The first level of data is at 1 km because of ocean surface contamination. See text for details on the radar domain.

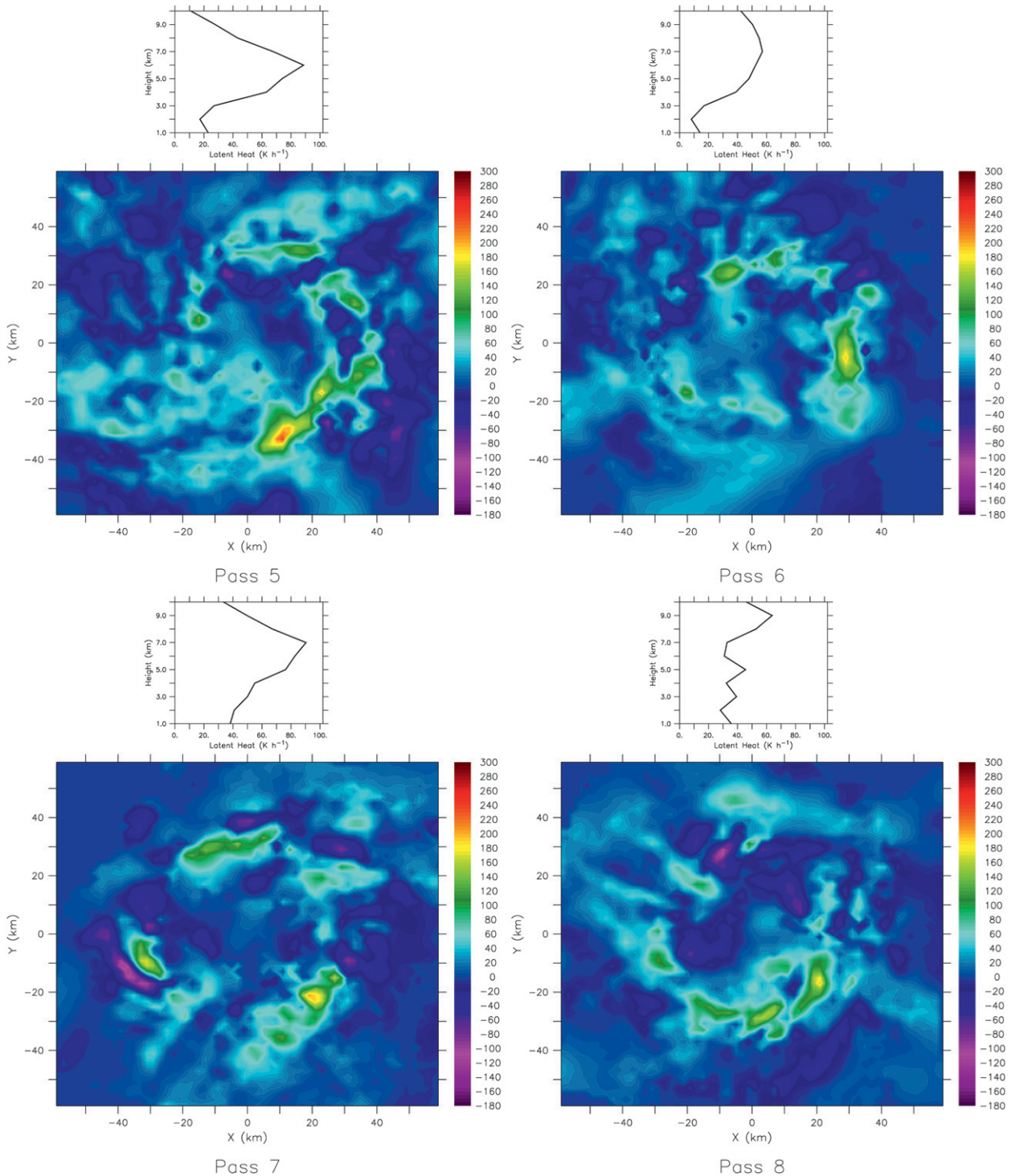


FIG. 12. (Continued)

in the previous section, grid points with  $|w| > 5 \text{ m s}^{-1}$  are assumed to be saturated.

To compute the magnitude of latent heat released at saturated grid points in the radar domain, knowledge

of the thermodynamic structure of convective cells is required, which is very difficult to obtain. To approximate the thermodynamic structure, a composite sounding derived from 10 high-altitude [using NASA

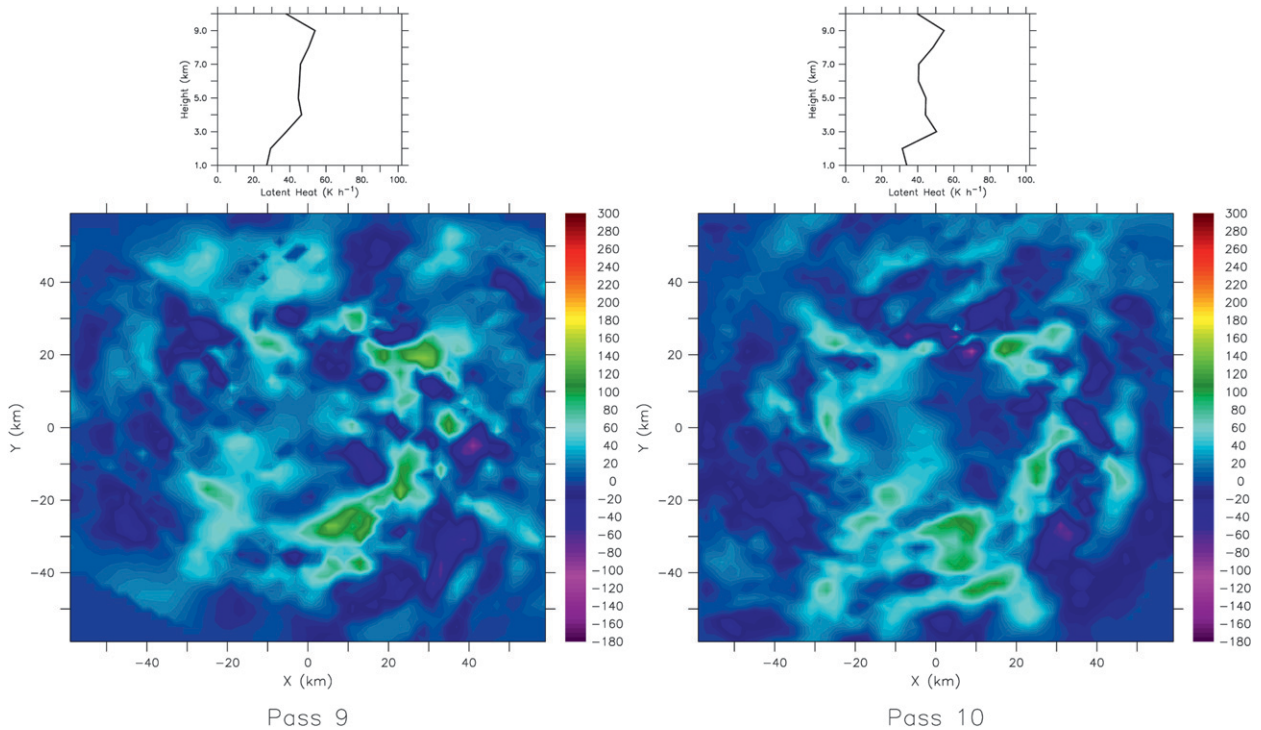


FIG. 12. (Continued)

aircraft that fly at altitudes of 10 and 20 km] dropsondes representative of eyewall convection in TCs is utilized. The storms sampled were Hurricane Bonnie (1998), Tropical Storm Chantal (2001), Hurricane Gabrielle (2001), Hurricane Erin (2001), and Hurricane Humberto (2001) yielding 10 independent thermodynamic profiles of eyewall convection. The sampling of eyewall convection is verified using winds and relative humidity from the dropsondes as well as satellite (infrared and passive microwave) observations. Discussion on the uncertainty associated with using a composite dropsonde is discussed below. To complete the latent heat calculation, the vertical velocities derived from the Doppler radar synthesis procedure are input to (4). The latent heat of condensation is capped at 10-km altitude based on independent numerical simulation experiments and the structure of the cloud water source shown in Fig. 4.

Figure 12 shows horizontal views of the derived latent heat field in Hurricane Guillermo (1997) averaged over all heights for each aircraft pass in Fig. 1. The structure in Fig. 12 is consistent with the 3-km altitude reflectivity scans shown in Fig. 1. Initially the latent heat field is quite asymmetric with convection displaced to the downshear quadrants of the storm due to the persistent vertical shear forcing of the vortex (Reasor et al. 2009). This low-wavenumber latent heat asymmetry

is coupled to the vorticity field (Reasor et al. 2009) as the features propagate around the eyewall of Guillermo, occasionally revealing a slightly more axisymmetric distribution of convection. Several passes in Fig. 12 show the emergence of large individual pulses of heating and cooling with peak magnitudes in the grid volume between 200 and 300  $\text{K h}^{-1}$ . Reasor et al. (2009) found that these strong convective bursts coincided with the largest intensification of Guillermo and were triggered by convergence associated with low-wavenumber vorticity asymmetries in the eyewall.

Figure 12 also displays the vertical profile of the azimuthal mean latent heating at the RMW (30 km) above the contour plot for each pass. The altitude of peak heating varies between approximately 4 and 9 km including a distinct double maximum (upper and lower levels) present in several passes. During the first four aircraft passes ( $\sim 2$  h), the azimuthal mean heating becomes larger in magnitude and the altitude of peak heating increases from approximately 4 to 7 km. During the next six passes ( $\sim 3$  h), the azimuthal mean heating decreases significantly and the maximum heating altitude generally remains above 5 km. The full observational period ( $\sim 5.5$  h) of the 3D latent heat retrievals presented in Fig. 12 is used as time-dependent forcing in a nonlinear numerical model in order to examine the impacts on the simulated intensity

and structure change of Guillermo. This is the topic of Part II.

*b. Uncertainty estimates*

There are two main calculations in the retrieval that require error analysis: the computation of the saturation state and the magnitude of the latent heat. The approximate errors associated with determining saturation are analyzed in section 3b, and thus the focus here is on the magnitude of the latent heat fields. The magnitude of the latent heat is essentially a function of thermodynamic information (temperature and pressure) and vertical velocity. The uncertainty in the thermodynamic information is assessed by first gathering soundings from various regions (eyewall and environment) of the numerical simulation of Hurricane Bonnie and eyewall dropsonde observations in several storms (see section 4a for the list of TCs). These thermodynamic profiles are then input to (4), revealing differences in the peak latent heat of only approximately 10%–15%. These results indicate that the magnitude of the latent heat is not very sensitive to the details of the thermodynamic information in the eyewall of TCs.

Sensitivity to the vertical velocity is much greater and is the most important parameter in the estimation of latent heat. Reasor et al. (2009) compared the Guillermo Doppler radar analyzed vertical velocities to flight-level in situ measurements and found an RMSE of  $1.56 \text{ m s}^{-1}$  in the eyewall region with a correlation coefficient of 0.61. Morrow (2008) compared a large set of P-3 derived wind fields with flight-level wind measurements, including those from Guillermo, and found that overall the intense and wide updrafts were captured well by the Doppler analysis while those that were narrow and weaker were not well represented. This result extends to downdrafts as well and is consistent with previous studies (Marks et al. 1992; Reasor et al. 2009).

Figure 13 shows a representative comparison of flight-level vertical velocities (near the 3-km altitude) to those computed from the Doppler analysis valid at approximately 2002 UTC 2 August 1997 in Hurricane Guillermo. The strong, wide updraft pulse at 30-km radius is represented well by the Doppler analysis, as are the general patterns of the vertical velocity field, but the narrow updrafts/downdrafts are clearly not captured. These errors are likely a result of 1) inadequate matching of the radar and flight-level sampling volumes and resolutions and 2) the need to use the anelastic mass continuity equation (more specifically, divergence) to solve for the vertical velocity. For the 2-km horizontal resolution of the Guillermo dataset (which relies heavily on FAST), the vertical velocity is estimated by computing divergence from

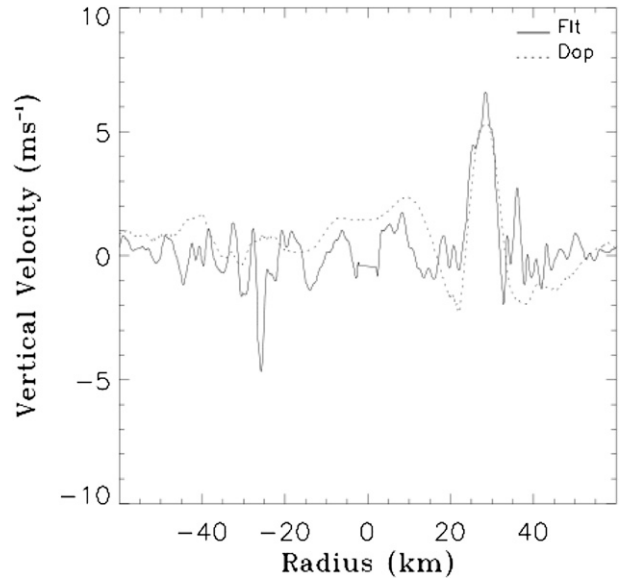


FIG. 13. Comparison of P-3 flight level (~3-km altitude) and Doppler radar-retrieved vertical velocity for a radial penetration into Hurricane Guillermo valid at ~2002 UTC 2 Aug 1997. Figure is from Morrow (2008). See Reasor et al. (2009) for details of the comparisons.

data over an area of  $16 \text{ km}^2$ , which effectively filters out smaller-scale perturbations (Marks et al. 1992). In addition, surface contamination does not allow adequate computation of divergence in the boundary layer, which will lead to errors in the vertical velocity aloft. Application of the latent heat algorithm to other airborne radars with higher resolution is being conducted.

The random error in the latent heat magnitudes can be estimated through an error propagation analysis. The general formula for error propagation is

$$\delta q^2 = \sum_i \left( \frac{\partial q}{\partial x_i} \right)^2, \tag{7}$$

where  $\delta q$  represents the Gaussian uncertainty in  $q$  (a function of  $x_i$ ), and each  $x_i$  denotes a variable with associated uncertainty  $\delta x_i$  that contributes to the calculation of  $\delta q$ . Applying (7) to (4) yields

$$\delta_{D\theta/Dt}^2 = \left( \frac{-L_c}{C_p} \right)^2 \left[ \left( \frac{w \partial q_s}{T \partial z} \delta_\theta \right)^2 + \left( \frac{\theta \partial q_s}{T \partial z} \delta_w \right)^2 + \left( \frac{-\theta}{T^2} w \frac{\partial q_s}{\partial z} \delta_T \right)^2 + \left( \frac{\theta}{T} w \delta_{\partial q_s / \partial z} \right)^2 \right]. \tag{8}$$

The uncertainties in each variable in (8) are determined from standard deviations in the dropsonde data described in section 4a and the RMSEs in vertical velocity from

the Reasor et al. (2009) study:  $\delta_T = 2.5$  K,  $\delta_\theta = 3.1$  K,  $\delta_{\partial q_s/\partial z} = 3.4 \times 10^{-7} \text{ m}^{-1}$ , and  $\delta_w = 1.56 \text{ m s}^{-1}$ . For all other variables in (8), characteristic values for the TC eyewall are chosen:  $T = 300$  K,  $\theta = 302$  K,  $\partial q_s/\partial z = -4 \times 10^{-6} \text{ m}^{-1}$ , and  $w = 5 \text{ m s}^{-1}$ . The second term on the right-hand side of (8) is larger than the other terms by at least an order of magnitude. Using this information and expressing the uncertainty in the latent heat magnitude as a percentage  $U_{D\theta/Dt}$  yields the following simplified equation:

$$U_{D\theta/Dt} \cong \left| \frac{\delta w}{w} \right| \times 100. \quad (9)$$

Plugging in the characteristic values chosen above, and assuming that the RMSEs computed in Reasor et al. (2009) are representative of a spectrum of vertical velocities, the uncertainty in the latent heat magnitude for updrafts of  $5 \text{ m s}^{-1}$  is approximately 32%. For smaller vertical velocities the errors can be large: a  $1 \text{ m s}^{-1}$  updraft has an uncertainty in latent heat magnitude of approximately 156%. The errors in the latent heat magnitude are dominated by random errors, although a slight positive bias of  $0.16 \text{ m s}^{-1}$  in the eyewall vertical velocities was found by Reasor et al. (2009), indicating that the latent heat retrievals may produce too much heat on average. However, when combining the biases in the latent heat magnitude and structure (through the calculation of saturation), the sign of the total bias in the retrievals is not clear although it is small compared to the random errors. Part II of this study will attempt to address the combined error issue by analyzing the ability of the latent heat fields to reproduce the observed wind speeds of Guillermo using a numerical model.

The discussion above estimates the uncertainties with computing the latent heat magnitude. Another source of uncertainty is discovered by asking this question: how well does the Guillermo dataset represent a larger distribution of convection and latent heat in TCs? This type of error is referred to as a sampling uncertainty. The updrafts and latent heat in Guillermo were log-normally distributed as are most TCs (Black et al. 1996) and require more advanced statistics than those of Gaussian distributions to describe their sampling uncertainties. We are interested in the sampling errors associated with deep convection; therefore, a subset of the latent heat field is selected for statistical analysis (vertical velocities greater than  $5 \text{ m s}^{-1}$ ) shown in the histogram in Fig. 14.

To estimate the sampling uncertainty in the mean value of this subset ( $117 \text{ K h}^{-1}$ ), a Monte Carlo-based method called the ‘‘bootstrap’’ is utilized. Estimation of the uncertainty in the mean (including the bootstrap method) is sensitive to the degrees of freedom in the

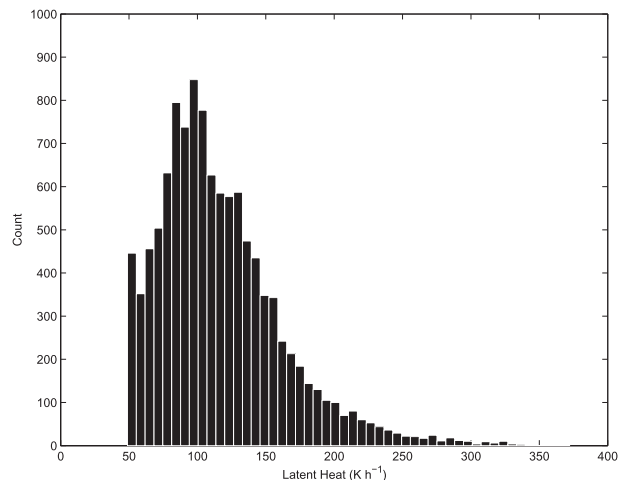


FIG. 14. Histogram of Doppler radar-retrieved latent heating rates for vertical velocities  $> 5 \text{ m s}^{-1}$  in Hurricane Guillermo on 2 Aug 1997.

dataset. To estimate the degrees of freedom (DOF) in the latent heat field over the full 3D domain and for all 10 composite periods, a combination of statistical (auto-lag correlation) and physical reasoning is employed. An auto-lag analysis in time reveals that each grid point in the Doppler domain has a time scale for independence of about 30 min (convective lifetime), while 1 degree of freedom in the vertical is assumed to represent a column of the atmosphere. An auto-lag analysis in the horizontal directions through deep convective cells reveals an independent spatial scale of approximately 12 km in each direction (approximate deep convective cell size in P-3 data). The number of DOF is then calculated as

$$\text{DOF} = \frac{T_x T_y T_z T_t}{I_x I_y I_z I_t} \alpha, \quad (10)$$

where  $T_i$  are the number of grid points in a dimension,  $I_i$  are the length and time scales for independence in each dimension, and  $\alpha$  is the percentage of the total sample being considered (3% for vertical velocities greater than  $5 \text{ m s}^{-1}$ ). Using the scales discussed above along with (10), 30 degrees of freedom or independent deep convective cells are found in the Guillermo radar dataset.

The bootstrap is performed through the following two steps. First, a random number generator with a discrete uniform distribution is used to create 1000 perturbed latent heat datasets each with a sample size of 30 (degrees of freedom) from the observed distribution shown in Fig. 14. Second, averages are computed for each dataset and they are sorted in ascending

order. Using the sorted data, the 25th and 975th values are selected, yielding the 95% confidence interval for the mean latent heat rate in the observed distribution ( $117 \text{ K h}^{-1}$ ; Fig. 14) of  $101\text{--}133 \text{ K h}^{-1}$  (or 14%). This sampling uncertainty is lower than the standard uncertainty found for updrafts of  $5 \text{ m s}^{-1}$  ( $\sim 32\%$ ) because of the distribution of data considered in the sampling case (updrafts  $> 5 \text{ m s}^{-1}$ ). Plugging the maximum updraft analyzed in the Guillermo P-3 dataset ( $\sim 30 \text{ m s}^{-1}$ ) into (9) along with the approximate uncertainty of  $1.56 \text{ m s}^{-1}$  yields an error of approximately 5%. Averaging this 5% error value with that for a  $5 \text{ m s}^{-1}$  updraft (32%) yields a mean error of 18.5%; this is close to the sampling uncertainty.

## 5. Summary and conclusions

In this paper, a revised algorithm for computing the latent heat associated with warm rain microphysics in TCs from airborne Doppler radar observations is presented. Several advancements in the basic algorithm (Roux 1985; Roux and Ju 1990) are developed, including (a) analyzing the scheme within the dynamically consistent framework of a numerical model, (b) developing a precipitation budget storage term parameterization, and (c) identifying sensitivities and errors in the retrievals through the use of ancillary data sources and uncertainty analysis.

The determination of the saturation state is shown to be an important part of the algorithm. While strong vertical velocities will virtually always be saturated in order to provide the necessary buoyancy forcing (Braun 2002; Eastin et al. 2005), weak to moderate vertical velocities require calculation or observation of the saturation state. Analysis of flight-level data in the inner core of intense hurricanes (courtesy of Eastin et al. 2005) as well as a high-resolution numerical model simulation of Hurricane Bonnie (1998; Braun 2006) advocates that for  $|w| > 5 \text{ m s}^{-1}$ , saturation can be assumed. Vertical velocities at or below  $5 \text{ m s}^{-1}$ , which contain the vast majority of the upward mass flux in TCs ( $\sim 70\%$ ; Black et al. 1996; Braun 2002) are shown to have larger variability in their saturation state, and thus more information is needed.

In the present algorithm, saturation is determined by solving for the net production of precipitation in a reduced form of the precipitation continuity equation. Cloud water production, which occurs when the air is saturated, is shown to explain approximately 71% of the variability in the net production of precipitation at all temperatures and approximately 87% at temperatures greater than  $0^\circ\text{C}$ . There are errors in the saturation computation due to more complicated physics (mixed-phase

regions and cloud boundaries) and when applying the algorithm to Doppler radars (i.e., resolution in time/space, attenuation, and calibration). A positive aspect of the saturation algorithm is the ability to accept some error because only the *condition* of saturation is necessary for the retrievals.

Latent heating rate sensitivity tests showed that random errors are small (mean of less than 10%) from the association of saturation with the net production of precipitation. The heating errors are larger from assuming steady state in the precipitation continuity equation (mean of approximately 20%). A parameterization for the storage term based largely on the tangential advective flux of precipitation (a consequence of the divergence theorem) was developed using output from the Bonnie numerical simulation, which shows promise for reducing the steady-state uncertainties in TCs.

Applying the new algorithm to NOAA P-3 airborne Doppler radar observations, the three- and four-dimensional structure of the latent heat of condensation/evaporation in rapidly intensifying Hurricane Guillermo (1997) is presented. Given the fact that latent heat is the primary energy source for TCs and that considerable uncertainty exists in previous observational studies and numerical model microphysics schemes, the new retrievals could prove quite useful for the community. The dominant source of error in the latent heating magnitude is the vertical velocity with minor contributions from the thermodynamic information. For characteristic errors in vertical velocity from the Guillermo P-3 analyses (Reasor et al. 2009), an uncertainty of approximately 32% in the heating magnitude is found for updrafts of  $5 \text{ m s}^{-1}$  and approximately 156% for updrafts of  $1 \text{ m s}^{-1}$ . Uncertainty in the retrievals due to sampling issues (for updrafts greater than  $5 \text{ m s}^{-1}$ ) is small (14%).

Overall, the algorithm does a reasonably good job of retrieving the latent heat field in TCs. Even though errors in the vertical velocity can lead to large uncertainties in the latent heating field for small updrafts/downdrafts ( $|w| \leq 1 \text{ m s}^{-1}$ ), in an integrated sense the errors are not as drastic. Furthermore, the majority (65%–85%) of the upward mass flux in TCs come from updrafts greater than  $1\text{--}2 \text{ m s}^{-1}$  (Braun 2002; Black et al. 1996), which have smaller errors. The real test of the usefulness of the latent heat retrievals comes from an analysis of their impacts on the predicted intensity and structure of TCs. To this end, the Guillermo latent heat retrievals are used as forcing in a nonlinear numerical model to examine their impacts on the simulated intensity and structure change of the storm relative to a case that relies on the model's microphysical scheme for forcing. This work is presented in Part II.

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