Generation of baroclinic topographic waves by a tropical cyclone impacting a low-latitude continental shelf

Dmitry S. Dukhovskoy*, Steven L. Morey, James J. O’Brien

Center for Ocean-Atmospheric Prediction Studies, Florida State University, Tallahassee, Leon County, FL 32306-2840, USA

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Abstract

Numerical model experiments have been performed to analyze the low-latitude baroclinic continental shelf response to a tropical cyclone. The theory of coastally trapped waves suggests that, provided appropriate slope, latitude, stratification and wind stress, bottom-intensified topographic Rossby waves can be generated by the storm. Based on a scale analysis, the Nicaragua Shelf is chosen to study propagating topographic waves excited by a storm, and a model domain is configured with simplified but similar geometry. The model is forced with wind stress representative of a hurricane translating slowly over the region at 6 km h\(^{-1}\). Scale analysis leads to the assumption that baroclinic Kelvin wave modes have minimal effect on the low-frequency wave motions along the slope, and coastal-trapped waves are restricted to topographic Rossby waves. Analysis of the simulated motions suggests that the shallow part of the continental slope is under the influence of barotropic topographic wave motions and at the deeper part of the slope baroclinic topographic Rossby waves dominate the low-frequency motions. Numerical solutions are in a good agreement with theoretical scale analysis. Characteristics of the simulated baroclinic waves are calculated based on linear theory of bottom-intensified topographic Rossby waves. Simulated waves have periods ranging from 153 to 203 h. The length scale of the waves is from 59 to 87 km. Analysis of energy fluxes for a fixed volume on the slope reveals predominantly along-isobath energy propagation in the direction of the group velocity of a topographic Rossby wave. Another model experiment forced with a faster translating hurricane demonstrates that fast moving tropical cyclones do not excite energetic baroclinic topographic Rossby waves. Instead, robust inertial oscillations are identified over the slope.

Keywords: Baroclinic motion; Topographic waves; Low-frequency internal waves; Hurricanes; Caribbean Sea

1. Introduction

This paper discusses a baroclinic shelf response to a tropical cyclone (in this paper, “tropical cyclone” is used as a generic term to include hurricanes of the Atlantic and Northeast Pacific, typhoons of the Northwest Pacific, and cyclones of the Indian and Southwest Pacific Oceans (Landsea and Gray, 1992)). Analytical and numerical models suggest that given appropriate slope and latitude, the ocean responds to a tropical storm with motions of sub-inertial frequencies trapped over a continental slope, the coastally trapped waves. It is speculated that in a low-latitude region a storm can excite bottom-intensified topographic Rossby waves, a special case of coastal-trapped waves, whose theory has been outlined by Rhines (1970). There is evidence that this class of waves can strongly modify dynamics in the deep ocean and can propagate long distances in the ocean (Hamilton and Lugo-Fernandez, 2001).

The ocean response to a tropical cyclone over regions located away from the continental margins has been studied intensively. Most attention has been paid to the baroclinic response such as upper ocean dynamics, upwelling processes and generation of inertial internal waves (O’Brien and Reid, 1967; Price, 1981; Greatbatch, 1984), interaction between the hurricane induced motions and ocean currents (Oey et al., 2006, 2007), upper ocean temperature changes and air–sea interaction (Morey et al., 2006b).

The theory of sub-inertial frequency wave motion trapped near the coastal boundary has been examined...
Extensively (see review in LeBlond and Mysak, 1978; Mysak, 1980; Allen, 1980; Brink, 1991). The first study of sub-inertial coastal-trapped waves in a continuously stratified fluid was conducted by Wang and Mooers (1976) and further continued in Clarke (1977) and Huthnance (1978). Several analytical studies have looked into the excitation of low-frequency wave motions on the shelf by large-scale weather systems (Gill and Clarke, 1974; Gill and Schumann, 1974). The low-frequency shelf response to a tropical cyclone is predominantly studied by means of numerical models (Gjevik, 1991; Gjevik and Merrifield, 1993; Morey et al., 2006a). Numerical models have the advantage of being able to give solutions to complicated nonlinear problems, such as storm events, that are intractable with analytical methods. In this study, excitation of bottom-intensified topographic Rossby waves along a continental slope by a tropical cyclone is investigated. There are speculations that topographic Rossby waves in the deep ocean (less than \(-2000\) m) can be generated by mesoscale oceanic features such as Loop Current Eddies in the Gulf of Mexico (Hamilton and Lugo-Fernandez, 2001; Oey and Lee, 2002) or Gulf Stream meanders on the continental slope (Pickart, 1995). Here, the generation of bottom-intensified wave motions at shallower depths by a tropical cyclone is discussed.

In order for a continental shelf to support baroclinic topographic waves, it should (1) respond as a baroclinic ocean and (2) have a slope steep enough to dominate the planetary \(\beta\)-effect, but small enough to prevent internal Kelvin-type modes. It will be shown in Section 2 that the low-latitude Nicaragua Shelf region in the Caribbean Sea matches those criteria. This region is often impacted by tropical cyclones (Elsner and Kara, 1999). In 2005, a record of eight tropical cyclones impacted the Western Caribbean Region and four of these cyclones were category 3 or stronger (based on the data from the Caribbean Hurricane Network, http://stormcarib.com). Notable category 5 storms impacting the region have included Hurricane Mitch in 1998 and Hurricane Felix in 2007.

A model domain representative of the Nicaragua Shelf region with a simplified topography (Fig. 1) is set up to analyze the low-frequency oceanic motion as a response to a hurricane. Model experiments are forced by wind stress representative of a hurricane translating over the region at different speeds. The model experiment forced by a slower moving storm generates baroclinic topographic Rossby waves along the deeper part of the slope. The faster moving hurricane excites weak topographic Rossby waves and strong (compared to lower frequency motions associated with the topographic waves) inertial oscillations. This study analyzes the characteristics of the resulting baroclinic

Fig. 1. Model domain: (a) bathymetry of the Nicaragua Shelf region. The dashed box marks the region approximated by the model domain. Contoured isobaths are \(-50\) and \(-700\) m. Axes inside the dashed box show the orientation of the model domain. (b) Bathymetry of the model domain: the domain is rotated clockwise in the \(xy\)-plane relative to part (a). The gray area is land. Solid lines are isobaths (every 200 m from \(-1000\) to \(-200\) m). The abscissa and ordinate are distances in km. The black box around the shelf indicates the region with 3 km horizontal grid spacing in the model. Outside the box, the resolution linearly increases up to 25 km near the boundaries. In the inset, the arrow \(\mathbf{h}\) indicates the slope gradient. Vector \(\mathbf{K}\) is the wavenumber vector and \(\mathbf{C}_g\) is the group velocity vector.
waves, energetics of the waves, and dependence of the wave properties on the forcing.

2. Coastally trapped waves and scale analysis for the study region

2.1. Review of coastally trapped waves

2.1.1. Classification of coastally trapped waves

In the case of a barotropic ocean and arbitrary monotonic depth profile, low-frequency free coastally trapped waves can be viewed as a superposition of a Kelvin wave mode plus an infinite set of (continental) shelf wave modes (Gill and Schumann, 1974; Huthnance, 1975). These waves are right-bounded in the northern hemisphere and, except for the barotropic Kelvin wave, their frequencies are always subinertial. In a simple limiting case of a stratified ocean with a flat bottom and wall at the coast, the coastally trapped waves propagate as internal Kelvin waves (Gill and Clarke, 1974). For arbitrary monotonic depth profile and continuous stratification, the waves are a hybrid of internal Kelvin waves and an infinite set of higher mode waves (Clarke, 1977; Mysak, 1978). For certain limiting cases, the wave solutions simplify as follows (Huthnance, 1978; Mysak, 1980). For weak stratification, the motions become depth independent and wave motions reduce to barotropic. For strong stratification, the wave characteristics are similar to the internal Kelvin waves. For short wavelengths (large longshore wavenumber), the motion is confined to the bottom and the waves become bottom-trapped topographic Rossby waves that are discussed in Rhines (1970). In the limit of a vanishing coastal wall, coastally trapped waves are restricted to topographic Rossby waves (Wang and Mooers, 1976). The modal structure of the topographic Rossby waves is determined by the effect of the density stratification on the motions. Wang and Mooers (1976) have demonstrated that the stratification effect can be estimated from the local (effective) baroclinic radius of deformation and the shelf width. If the stratification effect is felt, the motions are bottom-trapped topographic Rossby waves.

2.1.2. Overview of bottom-trapped topographic Rossby wave theory

Assume a rotating stratified fluid column in an ocean with a rigid lid over a sloping bottom \( h \) with slope \( z = |\nabla h|: \)

\[
h = -H_0 + zy. \tag{1}
\]

Assuming a wave solution proportional to \( \exp[i(kx + ly - \omega t)] \) and a constant buoyancy frequency \( (N^2 = \text{constant}) \), the following dispersion relationship is obtained for quasi-geostrophic motion over a sloping bottom (Pickart, 1995):

\[
\mu = f^{-1}N \left( K^2 + \frac{\beta k}{\omega} \right)^{1/2}, \tag{2}
\]

\[
\frac{\omega}{f} = -\left( \hat{K} \times \nabla h \right)_y \frac{N^2}{\mu f^2} \coth(\mu h),
\]

where \( \hat{K} = (k, l) \) is the horizontal wavenumber vector with magnitude \( K = |\hat{K}| \), and \( \beta = \partial f / \partial y \) represents the planetary \( \beta \)-effect or change of Coriolis parameter with latitude \( y \). The ratio \( 1/\mu \) is the vertical trapping scale of the wave. Stratification enhances the trapping and if \( N \to 0 \), meaning a homogenous ocean, the trapping scale \( \to \infty \) in agreement with Taylor–Proudman theorem (Pedlosky, 1987).

Rhines (1970) has showed that in order for the wave motion to be exponentially concentrated near the wall, the along-isobath component should have the sign such that \( \omega > 0 \) for the assumed wave solution. From the dispersion relation (2), it follows that for \( f > 0 \), the angle \( \theta \) between the wavenumber vector and the upslope direction should be within the interval \( (0, \pi) \) (Fig. 1b). If the bottom slope has a steep slope large enough so that the change in potential vorticity due to water column stretching across the slope (the topographic \( \beta \)-effect) dominates planetary \( \beta \) (Pedlosky, 2003), i.e.,

\[
\frac{f |\nabla h|}{H} \gg \frac{\partial f}{\partial y}, \tag{3}
\]

the planetary \( \beta \)-effect can be omitted resulting in an \( f \)-plane approximation so that the domain can be rotated arbitrarily with respect to north. With this and assuming \( f > 0 \), the dispersion relation (2) simplifies to the more familiar dispersion relation (Pedlosky, 1987):

\[
\omega = -\frac{k z N}{K} \coth\left( \frac{NHK}{f} \right). \tag{4}
\]

From Eq. (4), it follows that the maximum (cutoff) frequency for the baroclinic topographic Rossby waves is \( zN \). Wave motions which have higher frequencies are not supported by this theory.

2.2. Barotropic versus baroclinic response

An intense synoptic-scale atmospheric vortex may generate coastally trapped waves in shelf regions. In particular, at mid-latitudes the surface wind stress associated with large-scale weather systems (including cyclones and storms) that affect the shelf region can cause shelf waves (Mysak, 1980). The response of the ocean over a shelf to a cyclone depends on the characteristics of the cyclone (intensity, track, translation speed), but also on the latitude, shelf geometry and water stratification. The interplay between these parameters is characterized by a scaling called the Burger number (LeBlond and Mysak, 1978):

\[
Bu = \left( \frac{R_i}{L} \right)^2 = \left( \frac{NH}{fL} \right)^2,
\]

where \( R_i \) is the internal radius of deformation, \( L \) is the length scale, \( H \) is the average depth of the region, and \( f \) is the Coriolis parameter. \( N \) is the Brunt–Väisälä (buoyancy) frequency defined as

\[
N^2 = -\frac{g}{\rho_0} \frac{d\rho}{dz},
\]
where \( g \) is gravitational acceleration, \( \rho \) is water density, and \( z \) is the vertical coordinate. When \( Bu \ll 1 \) (this also includes the case of weak stratification mentioned earlier), the shelf responds to large-scale weather systems as a barotropic system (Clarke and Brink, 1985). When \( Bu \gg 1 \), the shelf response should predominantly be baroclinic. The Burger number dictates that shelves with the same stratification and slope at different latitudes will respond differently to the same atmospheric forcing.

As an illustration, taking scales for typical values of depth, buoyancy frequency, Coriolis parameter and cross-shelf horizontal lengths for the West Florida Shelf (eastern Gulf of Mexico) gives

\[
Bu = \frac{\left( \frac{O(10^{-2})}{s^{-1}} \times \frac{O(10^{3})}{m} \right)^2}{\left( \frac{O(10^{-4})}{s^{-1}} \times \frac{O(10^{5})}{m} \right)^2} \approx O(10^{-2}).
\]

From this scaling, one should expect that large-scale motions on the West Florida Shelf can mostly be described by barotropic dynamics in accordance with Mitchum and Clarke (1986). A recent example of the generation of a barotropic shelf wave by a hurricane occurred during Hurricane Dennis (2005) which translated along the West Florida Shelf and resulted in a shelf wave propagating northward (Morey et al., 2006a). For low latitudes (\( |\varphi| < 20^\circ \)), the same calculation of the Burger number results in a value near 1 which indicates the transition of barotropically dominated dynamics to baroclinic. Hence the internal modes should be considered for a similar study in a low-latitude sea such as Caribbean Sea.

### 2.3. Scale analysis for the Nicaragua Shelf slope

The Caribbean Sea shelf region has a sloping bottom of the continental margin which is a necessary property for supporting sub-inertial coastal-trapped waves (Rhines, 1970). The coastal regions of the Caribbean Sea are typically characterized by a narrow shelf which rapidly descends to several hundred meters and then gradually descends to the deep abyss. The continental slope offshore of the narrow-shelf coastal areas is steep. At low latitudes, steep continental margin may appear like a vertical wall. Thus, it is probable that this type of coastal region does not support topographic Rossby waves and coastally trapped perturbations in the density field propagate as internal Kelvin waves (Rhines, 1970; Allen, 1980). The area near Nicaragua and Honduras, the Nicaragua Shelf (Fig. 1a), has different geometric features that fit the goals of this study. The shelf is wide and protrudes far into the deep sea forming slopes stretching several hundred kilometers on both sides of the shelf, long enough to observe and analyze the propagation of baroclinic topographically trapped waves. The slopes of the Nicaragua Shelf are less steep and might allow the propagation of baroclinic topographic Rossby waves. The scale analysis based on Allen (1975) proves this speculation. If the internal Rossby radius of deformation (which is the scale of the Kelvin wave decay in the offshore direction) is larger than the width of shelf, the continental margin will act like a vertical wall to a wave allowing the existence of internal Kelvin wave type modes (Allen, 1980). The scale is \( \gamma = R_s/\delta_b \), where \( R_s \) is the internal radius of deformation and \( \delta_b = H/|\nabla H| \) is the scale of the bottom topography. For \( H \approx 500 \text{ m} \) and \( R_s \approx 50 \text{ km} \), the Nicaragua Shelf region has the scale \( \gamma \approx 1 \) and the other steeper coastal regions have the scale \( \gamma \approx 3 \). This confirms the idea that the internal Kelvin type waves may exist over the continental margins of the Caribbean Sea but the Nicaragua Shelf slope is not steep enough and should support different wave modes including bottom-intensified topographic Rossby waves.

### 2.4. Anticipated modal structure of coastally trapped motions over the Nicaragua Shelf

The geometry of the chosen Nicaragua Shelf allows one to consider coastally trapped motions over the slope in the limit of a vanishing coastal wall leaving topographic Rossby wave modes. The modal structure of the wave is determined based on Wang and Mooers (1976). The Nicaragua Shelf slope has an abrupt depth increase leading to very different local baroclinic radii of deformation over the upper and lower parts of the slope. For \( H = -200 \text{ m} \) and \( N = 8 \times 10^{-3} \text{ s}^{-1} \), the internal radius of deformation is \( R_s \approx 49 \text{ km} \). For the shelf width \( L = 55 \text{ km} \), \( R_s < L \) meaning that motions are barotropic in the upper part of the slope. For \( H = -700 \text{ m} \) and \( N = 3.5 \times 10^{-3} \text{ s}^{-1} \), \( R_s \approx 75 \text{ km} \) giving \( R_s > L \) meaning that motions are baroclinic in the lower part of the slope. Hence, from the scale analysis it follows that anticipated modal structure of the topographic Rossby wave motions over the Nicaragua Shelf will have barotropic mode over the upper part of the slope and baroclinic (bottom-intensified) mode over the deeper part of the slope.

### 3. Description of the model experiment

The simulated region includes a broad and shallow (~50 m) shelf which rapidly descends to \(-1000 \text{ m}\) on an \( f \)-plane (\( \varphi = 13^\circ \text{N} \)). The model bathymetry replicates major topographic features of the Nicaragua Shelf region (Fig. 1) without small-scale details which would complicate analysis of the model results. The slope around the shelf is roughly 0.015 which is close to the slope of the Nicaragua Shelf continental slope. The orientation of the \( y \)- and \( x \)-axes are shown in Fig. 1b. The domain is rotated relative to the geographic North direction. Since the model domain is on the \( f \)-plane, there is no dependence on compass direction in the wave dynamics. However, for ease of discussion the \( y \)-axis is assumed to point in the north direction (and \( x \) toward the east). Further in the text when the wave characteristics will be discussed the rotated coordinate system is used, noted as \( y_r \), \( x_r \). In the rotated coordinates, \( y_r \) is directed across isobaths and upslope and \( x_r \) is along isobaths.
The region is modeled using the Navy Coastal Ocean Model, which numerically solves the three-dimensional primitive equations with Boussinesq, incompressible, hydrostatic approximations (Martin, 2000). The vertical grid in the model experiments is a terrain-following sigma coordinate system. Twenty-one vertical sigma levels are non-uniformly (Gaussian) stretched with small vertical grid spacing near the bottom and the surface. The bottom stress for the momentum equations is parameterized by a quadratic drag law as

\[ K_M \frac{\partial \bar{u}}{\partial z} \bigg|_{z = -H} = c_b |\bar{u}| \bar{u}, \]  

(5)

where \( K_M \) is the vertical eddy coefficient calculated based on Mellor–Yamada Level 2 (Mellor and Yamada, 1974) turbulence scheme, \( \bar{u} \) is the horizontal velocity, and bottom drag coefficient is given as

\[ c_b = \max \left[ \frac{\kappa^2}{\log^2 (\Delta z_b/2z_0)}, c_{b\text{min}} \right], \]  

(6)

where \( \kappa \) is von Karman’s constant, \( \Delta z_b \) is the thickness of the bottommost grid cell, \( z_0 = 1 \times 10^{-5} \) m is the bottom roughness length scale, and \( c_{b\text{min}} = 1 \times 10^{-6} \) is a prescribed minimum value for \( c_b \). Open boundary conditions for the simulations use Orlanski (1976) radiation. No incoming signals are prescribed in the model.

The model equations are solved on the Arakawa C stencil. A quasi-third-order upwind advection scheme is used for momentum and scalar fields (Holland et al., 1998). Temporal differencing is performed with the semi-implicit leapfrog scheme with an Asselin (1972) filter to suppress time splitting. The free surface and vertical mixing are treated implicitly; the other terms are treated explicitly.

The model horizontal grid spacing is 3 km over the shelf (within the box in Fig. 1b). The region of interest is separated from the open boundaries by a region of stretched grid spacing to mitigate possible effects from the lateral boundaries. The horizontal grid spacing in this “buffer” zone around the shelf linearly increases from 3 to 25 km towards the boundaries.

The model is initialized with horizontally averaged temperature and salinity profiles (Fig. 2) derived from the World Ocean Atlas (Locarnini et al., 2002). The model is integrated from rest. The wind field forcing the model is computed from the gradient wind balance applied to the analytical pressure field

\[ P(r) = P_0 + (P_n - P_0) e^{-R/r}, \]

where \( P_0 = 955 \) hPa is the minimum central pressure, \( P_n = 1015 \) hPa is the ambient pressure at the periphery of the wind field, \( r \) is the radial distance from the storm center, and \( R = 30 \) km is the radius of maximum wind speed (O’Brien and Reid, 1967). For the given pressure distribution, the radius of the region with wind speed greater than 20 m s\(^{-1}\) (15 m s\(^{-1}\)) is \( \sim 200 \) km (270 km). Wind stress is calculated based on a bulk aerodynamic formula with a

\[ \rho N^2 \]

\[ T \]

\[ b \]

\[ \rho \]

\[ c \]

\[ N^2 \]

\[ S \]

\[ 35 \]

\[ 36 \]

\[ 1024 \]

\[ 1028 \]

\[ 2 \times 10^{-4} \]

\[ 0 \]

\[ 1 \]

Fig. 2. Initial profiles of (a) temperature (°C, black) and salinity (psu, gray), (b) potential density (kg m\(^{-3}\)), and (c) Brunt–Väisälä frequency, \( N^2 \) (s\(^{-2}\)).
Large et al. (1994) drag coefficient as in Morey et al. (2005), with wind velocity determined from the analytical pressure field.

For the primary experiment analyzed in this study, the storm translates over the region with speed $6 \text{ km h}^{-1}$ ($1.7 \text{ m s}^{-1}$). This parameterizes a slow-moving storm of category 3 according to the typical characteristics of tropical cyclones in the Caribbean Sea region based on the Caribbean Disaster Mitigation Project (www.oas.org/CDMP/hazmap/taos/atlastxt.htm). The storm translates from the lower (southern) part of the domain across the shelf to the upper (northern) boundary (the trajectory is shown in Fig. 3). Hurricane Mitch had a similar trajectory that impacted the Nicaragua Shelf in 1998. The model is

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**Fig. 3.** Evolution in time (in columns) of the simulated fields (in rows). All fields are shown after 168 h (a, d, g), 264 h (b, e, h), and 1176 h (c, f, i) of the model integration. Upper row (a, b, c): potential density field at 300 m depth. The trajectory of the hurricane center (black bullet in part (a)) is indicated with the dashed arrow. The gray area is shallower than $-300\text{ m}$. The contours represent the 0, $-50$, and $-100\text{ m}$ isobaths. Being trapped along the slope, density anomalies start propagating along the isobaths. In part (b), the negative density anomaly (purple spot) at the tip of the shelf is an indication of the coastally trapped baroclinic wave propagating from the northern edge of the shelf. Middle row (d, e, f): the 22, 11, and $6 \text{° C}$ isotherms. Lower row (g, h, i): potential density surface $1027.2 \text{ kg m}^{-3}$. In the middle and lower rows, the vertical axis is the depth in meters, and the $x$-$y$-axes are distances in km.
run for 51 days and model variables are saved every 6 h for analysis.

Before running the experiments with forcing, the model has been run for 21 days without any forcing to test for potential numerical truncation errors arising as consequences of using the sigma coordinate system. After 20 days of integration, the near-bottom velocity remains close to zero. A maximum speed of 0.04 m s\(^{-1}\) is simulated at several grid points along the “southern” open boundary between the \(-50\) and \(-1000\) m isobaths. Over the Nicaragua Shelf, the near-bottom speeds are less than 0.01 m s\(^{-1}\) and at the edge of the shelf (50 m isobath) the speeds do not exceed 0.02 m s\(^{-1}\). Over the shelf break slope (between 50 and 1000 m) velocities are less than 5 \times 10^{-3} m s\(^{-1}\). The model has demonstrated acceptable stable behavior for the purpose of studying the vastly more energetic ocean response to the storm.

It is worth noting that similar numerical model configurations in which \(z\)-level coordinates were applied below 500 m (as opposed to a fully terrain-following sigma vertical grid) result in a step-like representation of the topography. The wave solutions from these preliminary experiments (not shown) have dramatically different characteristics from the final results presented in this paper using the sigma grid, which permits a more realistic sloping seafloor. This discrepancy in the model solutions demonstrates an extension of the findings of Dukhovskoy et al. (2006), that the shelf wave solution in a barotropic ocean is sensitive to step-like representation of topography, to the baroclinic case.

4. Simulated ocean response to the hurricane

4.1. Temperature and density fields

The positive wind stress curl imposed on the sea surface causes upwelling of the deep dense water in the wake of the hurricane (Fig. 3). Dense water is intensively upwelled at the northern and southern edges. Formation of internal waves trapped along the slope is well observed in the plot of the potential density field at \(-300\) m depth (Fig. 3a–c) and three-dimensional diagrams of the temperature and potential density surfaces (in Fig. 3 middle (d–f) and lower (g–i) rows). Forced by the storm, water parcels move up- and downslope causing stretching of vortex tubes. By conserving their potential vorticity, water columns displaced from their equilibrium position form coastally trapped waves propagating along the slope with the shallow water to the right. In the plane view of the density field after 12 days of the model run, a negative density anomaly is evident at the tip of the shelf (shown by the purple region in Fig. 3b). This is a fingerprint of a topographically trapped wave propagating from the northern side of the shelf.

Three-dimensional plots of temperature and potential density surfaces (1027.2 kg m\(^{-3}\)) show the internal response of the deep ocean (middle and lower rows in Fig. 3). Early in the simulation (Fig. 3d and g), the deep water is forced up the slope (from \(-600\) m up to the \(-200\)-m isobath) by strong Ekman pumping induced by the moving storm. Following this, several topographically trapped waves begin propagating along the slope (Fig. 3e and h). After 50 days of the model run waves along the slope have significantly weakened amplitude but still can be viewed. Following the storm (Fig. 3f and i), deep-penetrating cyclonic eddies remain along the storm track and can be identified by elevated density surfaces.

4.2. Baroclinic topographic waves along the shelf edge

The numerical model experiment clearly shows low-frequency topographically trapped waves are forced by the slowly translating tropical cyclone. In the following sections, the wave motions are examined to determine if their characteristics match those of topographic Rossby waves. Some important features of this class of waves are as follows (Rhines, 1970; Hamilton, 1984):

1. The wave motion is bottom-intensified.
2. At any particular frequency, the motion is columnar, i.e., velocity at different depths co-oscillate
3. The maximum (cutoff) frequency is \(\frac{\Omega}{\omega}\).
4. Particle motions in the waves are transverse to the wavenumber vector, i.e., the principal axes of the velocity fluctuations for a given frequency band are perpendicular to the horizontal wavenumber vector.

4.2.1. Time series of velocity vectors at different depths

Time series of the near-bottom velocity vectors for several points (Fig. 4) show different responses to the storm at different depths across the shelf slope. The points are located across the slope at the \(-200\) (point 1), \(-400\) (point 2), \(-600\) (point 3), and \(-800\) (point 4) m isobaths. Analysis of simulated velocities at different depths and locations reveals that the ocean above the model pycnocline (\(-100\) to \(-200\) m) is strongly decoupled from the oscillations below the pycnocline. The ocean response could be approximated as a two-layer model which suggests the dominance of the first baroclinic mode in the dynamics. As an illustration, time series of the velocity vectors above \(-200\) m (upper parts in Fig. 5) demonstrate different behavior than the time series in the deep ocean (lower parts in Fig. 5). At point 1 (left column in Fig. 5), where the bottom depth is \(-200\) m, the vertical decoupling is not well pronounced. Even near the bottom the currents are generally coherent with motions at the upper levels.

After the storm has passed the region, motions in the deeper ocean form weak oscillations superimposed upon a low-frequency mode. The orientation of the velocity vectors is predominantly along the slope. The important feature of the deep ocean dynamics is columnar behavior of the currents below the pycnocline: fluctuations of the velocity vectors at different depth levels are coherent. This
is one of the features of the baroclinic topographic Rossby waves mentioned earlier. At points 2 and 3, amplification of the magnitude of the velocity vectors is observed towards the bottom. Hence, the model predicts bottom intensification of the currents at these locations which is also a characteristic feature of the topographic Rossby waves. At point 4, the intensification is not as obvious from visual inspection of the diagrams.

4.2.2. Wavelet analysis of the vector velocity time series

Time series of simulated velocities are non-stationary with large variations of the amplitude of the oscillations. Also, the periodicity of the internal waves may change in time and in space. Thus, spectral analysis directly applied to these time series might give wrong estimates of the frequencies of the waves of interest. An alternate approach is to examine the wavelet transform of the data.

The Morlet wavelet (Combes et al., 1989)

$$\psi(t) = e^{ijt}e^{-t^2/2}$$

(7)

is used for this analysis. To reduce ringing, each end of the time series has been extended by adding a trigonometric taper $$(1−\sin \phi)$$ (Emery and Thomson, 1998) such that the fields of the wavelet modulus shown are within the cone of influence (Meyers et al., 1993). The wavelet transforms (left column in Fig. 6; the first 180 h are not shown)
demonstrate that the motions are dominated by slow-oscillating modes (>100-h period). Inertial oscillations (~53 h) can be detected with a visual inspection of the diagrams although their amplitude is close to zero (green shades in the colorbar) which shows that the amplitude of inertial oscillations is negligibly small compared to the lower frequency oscillations. A distinct wave pattern is observed in the range of 150–250 h which may be an indication of the topographic Rossby waves. At point 2, the low-frequency oscillations are weakened after 600 h of the model run. At other locations, the oscillations remain evident through the end of the integration.
4.2.3. Spectral analysis

Before spectral analysis, the time series have been de-noised or decomposed into low-frequency (approximations) and high-frequency (details) components using wavelet decomposition and employing Daubechies family wavelets which allow signal reconstruction. The de-noised time series have been band-pass filtered leaving oscillations within the frequency band $7.7 \times 10^{-3} < \omega < 0.15 \text{rad h}^{-1}$ (oscillations with periods longer than ~2 days and shorter than 33 days). The kinetic energy spectra of time series of the along-isobath component of the velocity vector at points 1–4 shown in Fig. 4 are obtained (Fig. 7). The spectra show the distribution of kinetic energy in the frequency domain for different depth levels: near the bottom (black) and 120 m above the bottom (gray). To increase the statistical significance of the spectral estimates, the spectra have been smoothed in the frequency domain by band averaging.

The kinetic energy spectra show a concentration of energy within the frequencies from 0.01 to 0.1 rad h$^{-1}$ with maxima at 0.041 and 0.031 rad h$^{-1}$ (153 and 203 h). The spectral peaks are likely related to topographic waves simulated in the model. It is noteworthy that all these peaks are in the lower frequency band relative to the cutoff frequency. For the studied case, the cutoff frequency for the points below the pycnocline varies from 0.14 to 0.25 rad h$^{-1}$ which is high due to the relatively steep slope. For points 2–4, the spectra show higher energy for the near-bottom motions which indicates bottom-trapping of energy by the wave motions. At point 1, no energy trapping near the bottom is revealed by the spectral analysis. This suggests that at the shallower site the coastally trapped waves propagate without bottom intensification, i.e., not as topographic Rossby waves. From the spectra plots, the simulated topographic waves have periods 153 h (points 3 and 4), and 203 h (point 2) which is within the range of possible periods for topographic Rossby waves (Thompson and Luyten, 1976; Hamilton, 1984). The spectra indicate that bottom trapping of energy is simulated on the slope where the bottom depth is below the pycnocline (~200 m). No bottom trapping is identified in the spectra for point 1 which is located on the upper part of the slope. This suggests that the modal structure of the topographic waves over the slope in the model is such that over the upper part of the slope the wave motions are barotropic and over the deeper part the motions are affected by the stratification.

Fig. 7. Kinetic energy spectra for different depths at points 1–4. The black curve is the spectrum of the near-bottom kinetic energy. The gray curve is the spectrum at the shallower level (exact depths are in the legend boxes). Averaging over three frequency bands has been used to smooth the spectral estimates. The frequency resolution of the spectra is $5 \times 10^{-3}$ rad h$^{-1}$.
Results from the spectral analysis of the model output agree with the scale analysis in Section 2.4. Thus, the periods of the topographic waves and the bottom intensification imply that the baroclinic topographic Rossby wave theory holds for the oscillations over the deeper part of the slope and can be employed to derive wave characteristics.

To learn more about the simulated wave field in the frequency domain, rotary analysis has been conducted for the velocity vectors at the same points. Rotary analysis of currents is accomplished by separating the velocity vector for given frequencies into positive (counterclockwise) and negative (clockwise) rotating circular components with amplitudes $A^+$, $A^-$ and phases $\theta^+$, $\theta^-$, respectively. At every given frequency, motions are characterized by an ellipse obtained by vector addition of oppositely rotating circular components. The amplitudes and phases are used to calculate semi-major and semi-minor axes of the ellipse and orientation of the major axis. The one-sided spectra for positive and negative components are proportional to the squared amplitudes (for more details see Gonella, 1972; Mooers, 1973; Emery and Thomson, 1998).

The rotary spectra of the currents (not shown) replicate the kinetic energy spectra. For both positive and negative components, the peaks are at the same frequencies as in Fig. 7. For all points, the current ellipses derived from the rotary constituents are plotted for identified frequency peaks (Fig. 8). Not unexpectedly, the orientation of the semi-major axes of the ellipses is nearly along isobath. All velocity vectors rotate counterclockwise at these frequencies. The rotary coefficient characterizes the rotational nature of the flow in the given frequency band. This coefficient is $-1$ for clockwise rotation and $1$ for counterclockwise rotation. A coefficient of zero indicates unidirectional flow. The flows are almost unidirectional at points 1, 2, and 4. At point 3, the motions at the given frequency are more circular but still are predominantly along the isobath. Larger semi-axes of the ellipses for the shallower points (note the scale in the diagrams) reveal that the currents are very energetic along the shallower part of the slope which agrees with the origin of energy from the sea surface. Nevertheless, scales of the ellipses for the deeper points 3 and 4 are still roughly one-half that of the shallower locations meaning that the motions remain quite energetic even along the deeper part of the slope. The orientations of the major axes do not exactly match the orientation of the along-isobath axis; the ellipses are slightly tilted along the slope.

5. Analysis of model results: obtaining estimates of the wavenumber vectors

5.1. Methodology

Inspection of the evolution of the density fields in the model, kinetic energy spectra and rotary spectra suggest the existence of topographically trapped internal waves along the isobaths represented by points 2, 3, and 4. Since no characteristics of the topographic Rossby waves have been identified at point 1, it is excluded from further analysis. The characteristics or parameters of these long waves simulated in the model are estimated using the following methods. For frequencies identified from spectra, the wavenumber vectors are derived. The phase speeds along the isobaths are obtained from cross-correlations calculated between speed time series at the locations shown

![Fig. 8. Near-bottom current ellipses of the rotary constituent at the frequency ($\omega$) of the maximum spectral peak for points 1–4 shown in Fig. 4. Note the different scale of the diagrams. The horizontal bar indicates the length of the axis for the specified value (cm s$^{-2}$ cph$^{-1}$). The angle is the tilt of the major axis with respect to the $x$-axis. Values are given for depth ("Z"), frequency ("$\omega$"), rotary coefficient, and ratio of the minor to major semi-axes.](image-url)
in Fig. 4. This approach has been used in a number of studies of topographic waves to estimate the along-coast phase speed of the wave (e.g., Kundu and Allen, 1976; Thompson, 1977). Along-isobath \( k \) components of the wavenumber vector are readily obtained from the estimated phase speeds through the relation \( C_v = \omega / k \). In order to use this approach, the along-isobath direction of the wave propagation should be known \textit{a priori} to decide which point leads in the pairs along the same isobaths. According to the theory, the along-isobath component of the baroclinic topographic wave is always negative in the northern hemisphere (with the \( y \)-axis directed upslope). Hence, the first maximum of the sample cross-correlation function in the direction where points 2, 3, and 4 lead the points 6, 7, and 8, respectively (Fig. 4) will indicate the expected time the wave propagates along the isobath.

The cross-isobath component is derived from the dispersion relationship (4). From this technique, only absolute values of the components of the wavenumber vector can be determined. The direction of the cross-isobath component is assumed from considering the energy propagation. The direction of the energy flux coincides with the horizontal group velocity (Luyten, 1974; Pedlosky, 2003):

\[
\hat{\text{c}}_y = \left( \frac{\hat{\text{c}}_v \hat{\text{c}}_o}{\hat{\text{c}}_k \hat{\text{c}}_l} \right).
\]

The wavenumber vector and the group velocity are nearly perpendicular to each other. The cross-isobath component is positive when the group velocity vector points downslope (negative) and vice versa. Thus, when the energy flux has a positive cross-isobath component the cross-isobath wave-vector component \( l \) is negative (Hamilton, 1984; Oey and Lee, 2002). The direction of the cross-isobath component of the energy flux vector at a given frequency is estimated from the cross-spectra of along- and cross-isobath velocity components (Thompson, 1977). The properties of the topographically trapped baroclinic long waves forced by the translating storm estimated using the above-described methodology are described in the following sections.

5.2. Cross-correlation

Cross-correlation coefficients have been estimated for three pairs of points located along isobaths: points 2 and 6, 3 and 7, 4 and 8 (Figs. 4 and 9). Since the time series are strongly serially correlated, the correlation coefficients have been estimated using a random resampling technique (Ebisuzaki, 1997; Zwiers, 1990). The results are verified by calculating the cross-correlation coefficients employing Box–Jenkins procedure (Box and Jenkins, 1976). The lags of maximum cross-correlation coefficients agree for both approaches. It should be mentioned that time series along the deeper part of the slope (points 8 and 4) exhibit strong higher frequency oscillations resulting in a spiky cross-correlation function and less confidence in the time lag estimates.

From the theory of baroclinic topographic waves, the along-isobath component of the wavenumber vector is negative, meaning points on the right in Fig. 4 lead points to the left. Thus, only positive lags are shown in Fig. 9 which indicates propagation of the wave in the negative direction along isobaths, i.e., propagation with shallow water to the right. The distance between the points in a pair is 150 km. Knowing the distance and time of the phase propagation a phase speed can be calculated as follows. For point 2, the first maximum of the sample cross-correlation function at the positive lag is 54 h resulting in the along-isobath propagation rate of the wave of \( C_v = 0.77 \text{ m s}^{-1} \). This leads to the along-isobath wavenumber component \( k = -1.1 \times 10^{-5} \text{ rad m}^{-1} \). Estimates of \( k \) for other points have been derived in a similar fashion and are listed in Table 1.

Cross-isobath wavenumber components have been derived from the dispersion relation (4). The sign of the cross-isobath component is estimated from the cross-spectrum \( (S_{uv}) \) of the along-isobath \( (u) \) and upslope \( (v) \) velocity components, which is the momentum flux decomposed into frequency components. Following Hamilton (1984) and Thompson (1977), if the real part of the cross-spectrum \( (S_{uv}) \) of the velocity components at a given frequency is negative, the energy flux has an upslope
Also the correlations between Section (4.2.3). The momentum flux (been band-pass filtered similar to spectra described in and 4 are shown in Fig. 10. Both time series have the cross-spectra of the velocity components at points 2, 3, and 4 are shown. Averaging over nine frequency bands has been used to smooth the spectral estimates.

Fig. 10. The spectra (cm² s⁻²) of near-bottom momentum flux per unit mass at points 2, 3, and 4 (shown in Fig. 4) vs. frequency (rad h⁻¹) (solid gray). Also the correlations between \( u_r \) and \( v_r \) (\( \rho_{uv} \)) are shown. Averaging over nine frequency bands has been used to smooth the spectral estimates.

component and the wavenumber vector is directed downslope. And if the cross-spectrum is positive, the energy flux is downslope and the wavenumber vector points upslope. The cross-spectra of the velocity components at points 2, 3, and 4 are shown in Fig. 10. Both time series have been band-pass filtered similar to spectra described in Section (4.2.3). The momentum flux (\( S_{uv} \)) is strongly positive at the low-frequency bands of the cross-spectra. The correlation between \( u_r \) and \( v_r \) components at different frequencies is calculated as

\[
\rho_{uv} = \frac{\text{Re}(S_{uv})}{(S_{uv}S_{rv})^{0.5}}. \tag{8}
\]

The correlation indicates positive relation between the cross-isobath and along-isobath velocity components for the low-frequency band of topographic waves. This suggests that at the identified frequencies of the bottom-trapped oscillations the energy is downslope. This is a reasonable result because the only energy source in the model experiment is wind stress imposed at the ocean surface, which means that the energy flux should propagate downward. If the linear theory holds, the negative downslope orientation of the energy flux restricts the wavenumber vector to be directed upslope.

Using the estimates of the wave characteristics, the wavenumber vectors are plotted in Fig. 11 and listed in Table 1. The wavenumber vectors indicate the upslope propagation of the topographic waves. Examination of Fig. 11 shows that the wavenumbers have smaller components in the along-isobath direction compared to the cross-slope direction, i.e., the waves have shorter length scales in the cross-slope direction compared to the along-isobath direction. The wave phase propagates up the slope at \(-5^\circ\) angle from the upslope direction (\(-30^\circ\) from the \( y \)-axis). The group velocity has a larger component along the isobath. This suggests predominantly along-isobath energy flux by the topographic waves over the slope. In the upslope/downslope direction, the energy is propagated to the deep ocean.

A notable feature of the simulated waves is the different orientation and magnitude of the vector at point 2 relative to the vectors at points 3 and 4 where the vectors are almost collinear. At point 2, the wave is shorter (59 km) and has lower frequency (0.86 \( \times \) 10⁻⁵ rad s⁻¹). This probably stems from the fact that the vertical density gradient is larger at the \(-400 \) m isobath (point 2) than deeper on the slope where the gradient varies slowly with depth resulting in different wave characteristics. Waves at points 3 and 4 have similar properties: orientation of the wavenumber vector and frequency. The estimated length of the simulated wave at point 3 is slightly longer (87 km) than at point 4 (73 km) mostly due to the smaller time lag of the

<table>
<thead>
<tr>
<th>Points</th>
<th>( \omega^h ) (( \times ) 10⁻⁵ rad s⁻¹)</th>
<th>( T^b ) (h)</th>
<th>( k^c ) (( \times ) 10⁻⁵ rad m⁻¹)</th>
<th>( L^e ) (km)</th>
<th>( \theta^f ) (°)</th>
<th>( C^g ) (m s⁻¹)</th>
<th>( N^h ) (( \times ) 10⁻⁵ rad s⁻¹)</th>
</tr>
</thead>
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<tr>
<td>2</td>
<td>0.86</td>
<td>202</td>
<td>-1.1</td>
<td>10.5</td>
<td>59</td>
<td>119</td>
<td>0.08</td>
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<tr>
<td>3</td>
<td>1.14</td>
<td>153</td>
<td>-1.8</td>
<td>7</td>
<td>87</td>
<td>127</td>
<td>0.16</td>
</tr>
<tr>
<td>4</td>
<td>1.14</td>
<td>153</td>
<td>-2</td>
<td>8.6</td>
<td>73</td>
<td>126</td>
<td>0.13</td>
</tr>
</tbody>
</table>

\( a \) Wave frequency.
\( b \) Wave period.
\( c \) Along-isobath component of the wavenumber vector.
\( d \) Cross-isobath component of the wavenumber vector.
\( e \) Wavelength.
\( f \) Angle between the wavenumber vector and positive direction of \( x \)-axis.
\( g \) Phase speed.
\( h \) Cutoff frequency for topographic Rossby wave.
6. Energy of the internal waves

6.1. Energy propagation by the topographic waves

Baroclinic topographic Rossby waves propagate energy. Luyten (1974) has shown that the direction of the group velocity vector \( \vec{C}_g \) of the topographic Rossby wave corresponds to the direction of energy flux. Oey and Lee (2002) have demonstrated that (1) the component of the group velocity vector in the direction of the wavenumber vector \( \vec{K} \) must be zero, in other words these two vectors are orthogonal, and (2) \( \vec{C}_g \times \vec{K} = -N|\nabla h| \cos(\theta) \) (Fig. 1; note that in our case \( \nabla h \) is oppositely directed to Oey and Lee’s case). Thus, when the wavenumber vector is directed upslope the group velocity vector is directed downslope (case shown in Fig. 1). Note that along-isobath component of the group velocity of the baroclinic topographic wave is negative (for \( f > 0 \)). Hence, the energy is propagated by the topographic waves with the shallow water to the right. In the following section, the energetics of simulated waves trapped along the slope is analyzed.

6.2. Energy budget of a fixed volume on a slope

To demonstrate the energy propagation by the waves, the energy budget is computed for a volume element along the continental slope (Figs. 4 and 12). The volume sits on the slope and is bounded by side faces orthogonal to each other and a top face at \(-300\) m depth. The volume is oriented such that the back and front faces are parallel and the side faces are perpendicular to the isobaths. The back face coincides with the \(-500\) m isobath. The front face is \(19\) km away from the \(-500\) m isobath. The left and right faces are \(22\) km apart.

The rate of change of kinetic and potential energy within the volume can be calculated using Gauss’s Theorem by means of energy fluxes through the surfaces bounding the volume. Neglecting internal energy and energy dissipation the energy equation is (Gill, 1982)

\[
\frac{\partial}{\partial t} \left( \rho \left( \Phi + \frac{\vec{u} \cdot \vec{u}}{2} \right) \right) + \nabla \cdot \vec{F}_{\text{tot}} = 0, \tag{9}
\]

where \( \Phi \) is potential energy per unit mass associated with gravitational and centrifugal forces and \( \vec{F}_{\text{tot}} \) is the total energy flux vector given by

\[
\vec{F}_{\text{tot}} = \rho \vec{u} \left( \Phi + \frac{\vec{u} \cdot \vec{u}}{2} \right) + \rho \vec{u}. \tag{10}
\]

After integrating Eq. (10) over the volume and applying Gauss’s Theorem for the divergence term, the rate of change of the total energy is given by

\[
\frac{d}{dt}(K_e + P_e) = -\iint \vec{F}_{\text{tot}} \cdot \vec{n} \, dS, \tag{11}
\]

where \( K_e \) is the kinetic energy

\[
K_e = \iiint \frac{\rho \vec{u} \cdot \vec{u}}{2} \, dx \, dy \, dz, \tag{12}
\]

\( P_e \) is the potential energy

\[
P_e = \iiint \rho \Phi \, dx \, dy \, dz \approx \iint \rho g z \, dx \, dy \, dz, \tag{13}
\]

\( \vec{n} \) is norm to the surface of the volume element and \( dS \) is an element of area.

Calculations of the energy fluxes reveal that the greatest contribution to the energy within the volume comes through the side faces (Fig. 13). The area of the bounding faces is different except for the right and left faces. In order to analyze the intensity of the energy fluxes in different directions, the fluxes in Fig. 13a-c are normalized by the area of the faces. Vertical flux through the upper face normalized by the area of the face is small compared to the normalized horizontal fluxes through the sides. The energy fluxes through the faces oriented across the slope (“right” and “left” faces) have the largest magnitudes and are equal in magnitude and opposite in sign.
The largest energy flux into the volume (up to 2000 J s⁻¹ /C₀ m⁻¹ /C₀) is simulated through the right side approximately 40 h after the hurricane impacts the southern shelf slope. The energy flux through the front and back sides is less than one-half that of the across slope faces (<1000 J s⁻¹ /C₀ m⁻¹ /C₀). The large energy flux just after the hurricane masks details of variations later in the time series. Details of the time series of fluxes through the right/ left and front/back faces are shown in Fig. 13b and c, respectively. Dominant low-frequency oscillations are evident in the time series of the fluxes through the right and left faces. Note that the time series have not been filtered. The periods of these oscillations ranges from 130 to 200 h which agrees with the spectral estimates of the
frequencies of the topographic Rossby waves (Section 4.2.3). The time series of fluxes through the front and back faces are noisier and no evident oscillations in the frequency range of interest can be identified from visual analysis. This stems from the fact that the behavior of the cross-isobath velocity component is more irregular and complicated than the along-isobath component (this has been noted by Kundu et al. (1975) from analysis of observed velocity fields near the Oregon coast; see also Allen, 1980). Spectra (not presented) computed from these fluxes show concentration of energy at the topographic Rossby wave frequency band and higher frequencies. This result agrees with the analyses of the wavenumber vector calculations which show that the energy is predominantly propagated by the topographic waves along the isobaths. Although the fluxes through the front and back faces are not always opposite in sign there is an obvious tendency for them to be anti-correlated, i.e., the maximum in the flux through the back face corresponds minimum in the flux through the front face and vice versa. When the energy flux through the back face is positive and the energy flux through the front face is negative, the energy is propagated downslope. Presumably this is related to the topographic Rossby waves whose wavenumber vector estimates (Fig. 11) suggest that the energy is propagated downslope.

Integration over time of the energy fluxes \( \int_{t_0}^{t} F \, dt \) gives an estimate of the energy gain or loss in the volume. Fig. 13d presents fluxes integrated in time through the sides of the volume. Energy fluxes through the right and left faces of the volume dominate fluxes through the other faces. Immediately apparent are two regimes of the energy flux to the volume. At the beginning of the simulation, fluxes through the side faces are of the same magnitude as those through other faces. After approximately 200 h of integration, the magnitude of the side-face fluxes rapidly increases and stays at this level with minor oscillation through the remainder of the model run. The period of rapid increase of the fluxes is related to the moment when the first packet of topographic waves reaches the volume. A striking feature of these fluxes is their symmetry about zero, which is an indication of the along-isobath energy propagation. The sign of the fluxes proves that the energy is propagated in the direction it is expected to be transported by the topographic Rossby wave, i.e., with the shallow water to the right coinciding with the group velocity (Luyten, 1974; Pedlosky, 2003).

Fluxes through other faces of the volume are small compared to the fluxes through the side faces. The dashed bold black line (“Total”) indicates the total flux through the whole surface of the volume. Thus, this value represents energy gain or loss of the volume and it is an order of magnitude less than the transient fluxes through the side faces. This line stays close to zero, which means that the fluxes through the right and left surface do not contribute to the net energy in the volume. From 94% to 98% of the amount of energy brought by the wave through the right surface is carried out through the left side. The time series of the total flux indicates that shortly after the hurricane passes over the slope (after 40 h) the volume has gained energy and the energy in the volume remains mostly unchanged with some small low-frequency oscillations. When the hurricane is over the slope the energy flux through the upper face is, due to its larger surface area, up to eight times larger than through the front and back faces. (Note that having been normalized by the face area in Fig. 13a, the vertical flux is indistinguishably close to zero being two to four orders of magnitude smaller than fluxes through other faces.) After the hurricane passes, the vertical flux contributes little to the energy balance. The amount of energy in the volume is mostly maintained due to the unbalance between the cross-isobath energy fluxes.

7. Generation of topographic waves by a faster moving hurricane

The translation speed of the tropical cyclone determines the amplitude and frequency of the oscillations excited in the ocean. The response of the upper and deeper ocean to the passing storm is different. The horizontal currents excited by the wind stress curl are confined to the upper ocean within the Ekman layer. But the vertical velocity field extends throughout the depth of the ocean forcing the horizontal velocity field in the deeper ocean to adjust in order to conserve the mass balance (Greatbatch, 1984). Vertical velocities have strong dependence on the translation speed of the storm (Price, 1981; Greatbatch, 1984) and through them, the horizontal velocities as well. Fast hurricanes do not penetrate deep (Gill and Clarke, 1974). Storms of different intensities force stronger or weaker upslope/downslope motions of the water particles resulting in different rates and amplitudes of the change of relative vorticity, the driving force of the topographic Rossby wave propagation mechanism. In order to analyze how storm translation speed may affect the characteristics of generated topographically trapped waves, a twin model experiment is performed. The experiment is identical to that analyzed in the previous sections but the translation speed is 20 km h\(^{-1}\) (5.6 m s\(^{-1}\)). The wavelet decomposition of the time series still reveals low-frequency motions along the slope with periods at 150–200 h (right column in Fig. 6). However, the amplitude of these low-frequency motions is small and near-intertidal oscillations (the inertial period is 53 h) are clearly seen and are comparable in magnitude to the low-frequency oscillations. At all depths, inertial oscillations are well pronounced. This is in contrast to the experiment with the slowly moving hurricane in which these oscillations are barely distinguishable in the wavelet diagram (left column in Fig. 6).

The kinetic energy spectra (not presented) of the along-isobath component of the velocity vector at different depth levels have a broad peak in the low-frequency band with two maxima at \( \omega \sim 0.041 \) rad h\(^{-1}\) and at inertial frequency \( \omega \sim 0.11 \) rad h\(^{-1}\) which is anticipated from the wavelet
The energy is roughly equally partitioned between the low-frequency band and near-inertial peak. The spectra do not show energy trapping near the bottom. This is probably due to a very weak low-frequency signal which is poorly identified by the Fourier transformation. The presence of the inertial peak on the spectra is not surprising. As mentioned in Gill (1984), there is a tendency for weak inertial currents to be concentrated near the bottom. These oscillations are dominant over the slope during the model experiment with a faster moving storm. During the slower moving storm these oscillations were masked by more energetic topographic Rossby waves.

Energy fluxes through the sides of the volume (Fig. 14) differ noticeably from those calculated for the slower hurricane. As expected, the magnitude of fluxes is much smaller for the fast moving storm. The fluxes through the opposite faces are almost identical in magnitude but opposite in sign. At the end of the model run, the total energy change in the volume is mostly explained by the vertical energy flux oscillations with near-inertial period. This result differs from the first experiment where the vertical flux was negligibly small throughout the whole model run and no noticeable inertial signal has been detected in the time series of energy fluxes (which does not necessarily mean that the inertial oscillations do not exist, but rather that they are masked by more energetic sub-inertial motions). Finally, the energy fluxes through the front and back faces in this experiment predominantly remain positive and negative, respectively. This indicates upslope energy propagation, a result which differs from the experiment with a slower hurricane where the energy propagates primarily downslope.

These results suggest that wave motions in the wake of the fast moving cyclone are predominantly inertial oscillations. Topographic Rossby waves excited by the fast moving storm are weak. A possible rationale can be given based on the time scale for the passage of the storm (Greatbatch, 1984):

\[ k_s = \frac{U}{L_s f}. \]  

where \( U \) is the speed of the storm, \( L_s \) is the length scale of the storm, and \( k_s^{-1} \) measures the time scale for the passage of the storm in units of the local inertial time scale. Taking the length scale of the storm to be 350 km (Section 3), \( k_s = 0.14 \) for the slow-moving storm and \( k_s = 0.48 \) for the faster moving storm. The passing time for the slow-moving storm is seven local inertial time scales while for the faster moving storm it is 2. Simulated topographic Rossby waves have period \( > 3 \) local inertial time scales. The time scale of the faster moving storms is much shorter than the time scale of low-frequency topographic Rossby wave oscillations.

8. Summary

The analysis in this paper has focused on numerical simulation of low-frequency wave motions along the continental slope excited by a hurricane. The simulated region is analogous to the Nicaragua Shelf, chosen for the study due to its low-latitude location, geometry and history of tropical cyclone impact. A simulation is performed with a hurricane translating over the shelf at 6 km h\(^{-1}\). Examination of temperature and density fields reveals strong upwelling of deep dense water at the northern and
southern edges of the shelf as the hurricane passes over the region. The upwelled water parcels crossing the isobaths result in topographically trapped waves propagating along the continental slope with the shallow water on the right. The waves are apparent in visualizations of the temperature and density field evolution (Fig. 3).

Analysis of vertical structure of the model currents at different locations across the slope indicates strong decoupling between the upper ocean (above the pycnocline at \( \sim 200 \) m) and deeper layer. Currents in the deeper ocean reveal coherence in their vertical structure. Velocity vectors are predominantly oriented along isobaths. The magnitude of the vectors increases towards the bottom. The wavelet analysis of the time series of the along-isobath component of the near-bottom velocity identifies well-pronounced low-frequency oscillations (period of 150–200 h) along the slope at depths below the pycnocline. A prominent feature of the spectral diagrams is energetic motions in the low-frequency band below the cutoff frequency (that is, in the frequency range for topographic Rossby waves). The bottom trapping of energy is evident in the spectra of kinetic energy at different depths at locations which are deeper than the pycnocline. No energy trapping near the bottom is identified at the point which is located at the depth of the pycnocline base (point 1). The spectral peaks are broad (due to simulated oscillations being not exactly periodic) with maxima at 153–203 h.

Rotary spectra analysis of the currents at the same locations has been performed. This analysis has identified similar peaks in the low-frequency domain both for positive and negative components. Ellipses of the rotary constituents for identified frequencies suggest clockwise rotation of the velocity vector below the pycnocline. Orientations of the semi-major axes of the ellipses are nearly along isobath with a small cross-isobath component. The motions are almost rectilinear at the upper part of the slope and less rectilinear on the deeper part of the slope.

The findings of the time series analysis support the assumption that simulated oscillations along the slope at points 2, 3, and 4 (Fig. 4) are topographic Rossby waves. Based on the calculated oscillation periods, phase propagation and wavelengths, the simulated low-frequency motions trapped along the slope are interpreted in terms of a topographic Rossby wave model (Rhines, 1970). The identified frequencies of the topographic Rossby waves have been used to calculate the characteristics of simulated baroclinic topographic waves at points 2, 3, and 4 (Table 1). Wavenumber vectors (Fig. 11) indicate up-slope phase propagation. The wavenumber vector orientation agrees with the nearly along-isobath particle motions estimated from the rotary spectral analysis and indicates predominantly along-isobath energy propagation by the waves.

Energy fluxes calculated for a fixed volume placed on a slope demonstrate the ability of this class of wave to transport a large amount of energy along the slope over a long distance. The energy fluxes into the volume through the faces normal to the isobaths are nearly equal in magnitude but opposite in sign. Thus, the energy flux through the side faces almost cancels each other contributing little to the volume energy budget. In the time series of these energy fluxes, near-regular oscillations with a period of 150–200 h are clearly evident. The energy flux through the faces parallel to the isobaths shows that the energy propagation across the isobaths does change directions but is dominantly downslope. The downward propagation of energy can be attributed to the topographic waves as suggested by the wavenumber orientation. Energy flux through the upper face is negligibly small throughout the simulation. On average, energy flux through the upper face is into the volume indicating downward propagation of the energy. Simulated waves transport energy in the direction expected for topographic Rossby waves, which again supports the assumption that simulated topographically trapped motions are indeed topographic Rossby waves.

With a faster translation speed (20 km h\(^{-1}\)), the hurricane excites strong inertial oscillations and weak coastally trapped wave motions over the slope. The wavelet diagram for the near-bottom along-isobath velocity component highlights the differences between the very strong inertial oscillations and the weak low-frequency waves. Energy calculations indicate strong inertial fluctuations of the energy fluxes through a volume element on the slope. The different oceanic response to the faster translating hurricane is a result of time scale of the passing fast storm being much shorter than the time scale of the topographic Rossby waves supported by the system.

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