A mechanism of the Madden–Julian Oscillation based on interactions in the frequency domain

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SUMMARY

The surface and boundary-layer fluxes of moisture exhibit a large amplification as the waves in the Madden–Julian Oscillation (MJO) time-scales interact with synoptic time-scales of 2 to 7 days. This amplification is clearly seen when the datasets are cast in the frequency domain for computations of the respective fluxes. Those flux relations carry triple-product nonlinearities, and the fluxes on the time-scale of the MJO are evaluated using co-spectra of triadal frequency interactions. The trigonometric selection rules on interactions among these frequencies are largely satisfied by the time-scales of the MJO and two others that reside in the synoptic time-scales. Tropical instabilities provide a rich family of tropical disturbances that appear to be ready and waiting to interact with the MJO time-scales (since these satisfy the selection rules for non-vanishing interactions). A consequence of these nonlinear interactions in the frequency domain is a two- to three-fold amplification of the surface fluxes. Although this analysis does not address how a small signal in the sea surface temperature on the time-scale of the MJO arises in a coupled atmosphere–ocean model, we are able to show that its presence enables a large amplification of this time-scale vertically across the planetary boundary layer. Given a low-frequency ocean with many time-scales, this process amplifies the fluxes on the time-scale of the MJO; this amplification eventually feeds back to the ocean via amplified surface stresses, and an equilibrium state with a robust MJO in the coupled system is realized. The datasets for this study were derived from a coupled ocean–atmosphere model that was able to resolve a robust MJO in its simulations. This study also examines the character of sensible-heat fluxes and momentum within the same framework.

KEYWORDS: Climate modelling Intraseasonal oscillations Nonlinear dynamics of waves

1. INTRODUCTION

This paper explores the surface and PBL (Table 1 lists acronyms used) fluxes of moisture, sensible heat and momentum in the frequency domain with reference to the MJO. The pioneering work of Madden and Julian (1971) has seen a major thrust of research into intraseasonal oscillations in the last 30 years. The MJO is a planetary-scale wave that traverses from west to east in roughly 40 days. It has been identified as a global phenomenon with its largest amplitude in the equatorial tropics. Its signature is seen in most variables, such as sea-level pressure, zonal wind and divergent circulation. Since that finding, numerous studies have elucidated the importance of this phenomenon for its modulation of the monsoon activity and even typhoon behaviour in the western Pacific Ocean. Several recent studies have even portrayed the links of the MJO with the onset and decay of the ENSO.

In this study, the formulation for the surface and PBL fluxes is based on the FSU coupled atmosphere–ocean global spectral model (LaRow and Krishnamurti 1998; Krishnamurti et al. 2000; Cubukcu and Krishnamurti 2002). Triple-product nonlinearities convey interesting scale interactions. In the present study we show that the expressions for the surface fluxes in the constant-flux layer theory, and in the PBL flux theory of numerical models, carry such triple-product nonlinearities. Thus, it is possible to examine the fluxes on a certain time-scale as they arise due to interactions with other time-scales. If we designate the Madden–Julian time-scale (20 to 60 days) as a centre-piece for such computations, we see that a large number of possible interactions with the MJO time-scale arise from pairs of time-scales from within the synoptic

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time-scale, in the range of 2 to 7 days. Tropical instabilities arising from horizontal and vertical wind shear and convection, provide a rich environment for these tropical higher frequencies (waves, depressions and storms on time-scales of 2 to 7 days). Given a weak low-frequency signal in the SST on the time-scale of the MJO, it is possible to perceive frequency interactions such as to amplify the MJO signal. A demonstration of this is the goal of this paper. We offer a hypothesis that the surface and PBL fluxes invoke nonlinear interactions affecting the growth of the Madden–Julian time-scale. In the frequency domain, this contributes to the maintenance and amplification of the Madden–Julian time-scale in a coupled atmosphere–ocean system.

The inter-frequency interactions can contribute to either growth or decay of the Madden–Julian time-scale. This is determined by the sign of the co-spectra evaluated from the relevant frequencies. For quadratic nonlinearities, a selection rule for non-vanishing exchanges among frequencies requires that the two frequencies \( r \) and \( s \), say, are equal. For triple-product nonlinearities among member frequencies \( r, s \) and \( p \), the trigonometric selection rules require that \( p = r \pm s \), i.e. the growth of a time-scale within the Madden–Julian window (with frequency \( p \)) is exactly equal to \( r + s \) or \( r \mp s \). It turns out that \( p \) at the time-scale of the MJO generally prefers two high-frequency time-scales within the 2- to 7-day window for the augmentation of moisture fluxes.

Over the ocean the spectra of SSTs exhibit some small power on the MJO time-scale (Krishnamurti et al. 1988). Figures 1(a) and (b) show power spectra of the SST and zonal wind at 850 hPa, respectively, for the Indian, West Pacific and Central Pacific Oceans. These are based on the coupled-model run covering the period from March 1996 to February 1997 (Cubukcu and Krishnamurti 2002). This shows a plot of power \( x \) frequency against the logarithm of the frequency (along the abscissa). The SST shows a noticeable signal on the time-scale of the MJO, whereas on the synoptic time-scales the power is much smaller. We do see a semi-annual signal in the SST from the model output arising from the north–south seasonal migrations of the warm SST regions near the equator. The zonal winds in the atmosphere show a larger signal on the synoptic-scale, as is to be expected, and on the MJO time-scale. The interaction among these time-scales is the topic of this study. Given a number of synoptic-scale tropical disturbances on the time-scale of 2 to 7 days (e.g. Yanai and Nitta 1968; Reed and Recker 1971; Reed et al. 1977; Thompson et al. 1979; and several others), a unique opportunity is provided for interactions between these time-scales and the MJO time-scale in the frequency domain of the surface and PBLs.
In this paper we demonstrate the following aspects of the growth of the Madden–Julian time-scale over near equatorial latitudes for the oceanic basins, since the amplitude of the MJO is largest there (Krishnamurti and Gadgil 1985).

- The mutual interactions between the low-frequency behaviour of the SST and the higher-frequency synoptic disturbances of the tropical atmosphere over the atmospheric surface layer (the constant-flux layer).
The mutual interactions of the Madden–Julian time-scale with the synoptic-scale disturbances in the PBL.

The issue of climate model simulation of the MJO has been a topic of great interest in recent years. Summaries of recent contributions on the topic may be found in Kemball-Cook et al. (2002) and Maloney (2002). The consensus seems to be that models with prescribed SSTs are not able to simulate realistic features of the MJO, whereas the coupled atmosphere–ocean models are somewhat more successful in this respect. Maloney (2002) explored the effects of WISHE following Emanuel (1987) in the simulation of the MJO. He noted that a removal of WISHE in the NCAR-CCM3.6 model led to an improved simulation of the MJO, and he concluded that WISHE might contribute to the growth of modes outside of MJO frequencies. This is just one aspect of the details of a cumulus parametrization through which MJO sensitivity has been addressed. A more recent unpublished work from Colorado State University (David Randal, personal communication) points to the importance of cloud-resolving models where explicit, rather than parametrized, cumulus convection has been very successful in the mapping of the MJO. Some of the earlier work of Manabe et al. (1965) where moist convective adjustment was used has provided some evidence of intraseasonal oscillations. Further work is clearly warranted to identify the scope of models that resolve the MJO and its variability.

Simulation of the MJO is regarded as one of the most important components for the medium- and long-range forecasts of the Asian summer monsoon. In many GCM studies, it has been observed that the simulation of tropical intraseasonal oscillations highly depends on the choice of cumulus convection parametrization scheme (Wang and Schlesinger 1999). In some recent studies, Lee et al. (2001) emphasized the influence of cloud–radiation interaction and cumulus entrainment constraint in simulating tropical intraseasonal oscillations using the aqua–planet version of Seoul National University GCM (SNUGCM). Wang and Schlesinger (1999) pointed out the importance of the boundary-layer relative-humidity thresholds for producing realistic variability of the MJO in GCMs. In another study, Maloney and Hartman (2001) pointed out that convective downdraughts are important and control the variability of the tropical MJO. However, most AGCMs forced with a slowly varying SST annual cycle are unable to represent the eastward propagation of convection from the Indian Ocean to the West Pacific. This indicates that the atmospheric internal dynamics may contain mechanisms driving the MJO in AGCMs. The implication is that the ocean is necessary for generating the MJO in a model, and this remains an open question. Waliser et al. (1999) have compared the MJO in coupled and atmosphere-only versions of the same GCM and showed an improvement in many aspects of simulation of the MJO. Due to the complex interaction between large-scale dynamics and convection, and between convection and the ocean surface, representation of the MJO presents a challenge for a coupled model (Inness and Slingo 2002).

2. QUADRATIC AND TRIPLE-PRODUCT NONLINEARITIES AND COMPUTATIONS OF CO SPECTRA

We assume that a given time series of data for two variables \( u(t) \) and \( v(t) \) are cyclic and discrete in time. Since the FSU coupled model was run for one year, we have assumed a fundamental periodicity over this time span. Our aim is to examine the role of the MJO within this period. These series are represented by temporal Fourier series.
with discrete frequency \( n \) and Nyquist frequency \( N \) such as:

\[
\frac{2\pi}{N} nt + \frac{2\pi}{N} nt
\]

\( u(t) = \sum_{n=0}^{N} \left\{ C_n^u \cos \frac{2\pi}{N} nt + S_n^u \sin \frac{2\pi}{N} nt \right\}. \tag{1}
\]

In particular, \( C_n^u = u_o \) (time mean), see Sheng and Hayashi (1990a,b). The sample frequency co-spectrum \( P_n(u, v) \) is defined as:

\[
P_n(u, v) = \frac{1}{2}(C_n^u C_n^v + S_n^u S_n^v).
\]

In the wave number domain the co-spectrum would be defined by:

\[
P_n(u, v) = \frac{1}{2\pi} \int_0^{2\pi} \left\{ \left( C_n^u \cos \frac{2\pi}{N} n\lambda + S_n^u \sin \frac{2\pi}{N} n\lambda \right) \times \left( C_n^v \cos n\lambda + S_n^v \sin \frac{2\pi}{N} n\lambda \right) \right\} d\lambda.
\]

However over the frequency domain the co-spectrum would be expressed by:

\[
P_n(u, v) = \int_{-T/2}^{T/2} \left\{ \left( C_n^u \cos \frac{2\pi}{N} nt + S_n^u \sin \frac{2\pi}{N} nt \right) \times \left( C_n^v \cos \frac{2\pi}{N} nt + S_n^v \sin \frac{2\pi}{N} nt \right) \right\} dt / \int_{-T/2}^{T/2} dt.
\]

Thus, in principle, the frequency co-spectrum can be formally replaced by a wave number co-spectrum, Hayashi (1980).

Given triad interaction in the frequency domain, \( v(t) \) in Eq. (2) can be expressed as a product of two sets \( b(t) \) and \( c(t) \), i.e. \( v(t) = b(t)c(t) \).

Now, \( P_n(u, v) \) takes on the form:

\[
P_n(u, v) = P_n(u, bc) = L(n) = \sum_{r,s} LN(n, r, s).
\]

\( LN(n, r, s) \) in Eq. (3) denotes a contribution to \( L(n) \) due to the specified combination of frequencies \( r \) and \( s \), where \( r \) and \( s \) satisfy either of the selection rules \( r + s = n \) or \( |r - s| = n \). The explicit expression for \( L(n) \) in the frequency domain is given below, using Fourier transforms of \( u, b \) and \( c \) in the time domain.

Thus,

\[
L(n) = \int_{-T/2}^{T/2} \left\{ \left( C_n^u \cos \frac{2\pi}{N} nt + S_n^u \sin \frac{2\pi}{N} nt \right) \left( C_r^b \cos \frac{2\pi}{N} rt + S_r^b \sin \frac{2\pi}{N} rt \right) \right\} \times \left( C_s^c \cos \frac{2\pi}{N} st + S_s^c \sin \frac{2\pi}{N} st \right) dt / \int_{-T/2}^{T/2} dt.
\]

\( (4) \)
The above formalism has been extensively discussed by Hayashi (1980) in his pioneering studies. Thus we may write:

\[
L(n) = \frac{1}{T} \int_{-T/2}^{T/2} \left( \sum_n \sum_r \sum_s \frac{C_n^u C_r^b C_s^c}{2\pi N} nt \cos \frac{2\pi}{N} r t \cos \frac{2\pi}{N} s t \right. \\
+ C_n^u S_r^b C_s^c \cos \frac{2\pi}{N} nt \sin \frac{2\pi}{N} r t \cos \frac{2\pi}{N} s t \\
+ S_n^u C_r^b C_s^c \sin \frac{2\pi}{N} nt \cos \frac{2\pi}{N} r t \cos \frac{2\pi}{N} s t \\
+ S_n^u S_r^b C_s^c \sin \frac{2\pi}{N} nt \sin \frac{2\pi}{N} r t \cos \frac{2\pi}{N} s t \\
+ C_n^u C_r^b S_s^c \cos \frac{2\pi}{N} nt \cos \frac{2\pi}{N} r t \sin \frac{2\pi}{N} s t \\
+ C_n^u S_r^b S_s^c \cos \frac{2\pi}{N} nt \sin \frac{2\pi}{N} r t \sin \frac{2\pi}{N} s t \\
+ S_n^u S_r^b S_s^c \sin \frac{2\pi}{N} nt \sin \frac{2\pi}{N} r t \sin \frac{2\pi}{N} s t \bigg) dt.
\]

Considering the first term of this equation

\[
\frac{1}{T} \int_{-T/2}^{T/2} \left( \sum_n \sum_r \sum_s \frac{C_n^u C_r^b C_s^c}{2\pi N} nt \cos \frac{2\pi}{N} r t \cos \frac{2\pi}{N} s t \right) dt \\
= \frac{1}{2T} \int_{-T/2}^{T/2} \left[ \sum_n \sum_r \sum_s \frac{C_n^u C_r^b C_s^c}{2\pi N} nt \cos \frac{2\pi}{N} (r+s)t \cos \frac{2\pi}{N} (r-s)t \right] dt.
\]

This is an infinite integral and must be made finite for computational purposes. Using the orthogonal property of Fourier functions, this term takes the following form:

\[
\frac{1}{T} \int_{-T/2}^{T/2} \sum_{n=r+s} C_n^u C_r^b C_s^c \cos^2 \frac{2\pi}{N} nt dt + \sum_{n=r-s} C_n^u C_r^b C_s^c \cos^2 \frac{2\pi}{N} nt dt \\
+ \sum_{n=s-r} C_n^u C_r^b C_s^c \cos^2 \frac{2\pi}{N} nt dt
\]

(only these terms survive under the condition \(n = r + s, n = r - s, n = s - r\)).

Applying \(\cos^2 A = (1 + \cos 2A)/2\), this takes the form

\[
= \frac{1}{4} \left[ \sum_{n=r+s} C_n^u C_r^b C_s^c \right. \\
\left. + \sum_{n=r-s} C_n^u C_r^b C_s^c \right. \\
\left. + \sum_{n=s-r} C_n^u C_r^b C_s^c \right]
\]
Hence the above equation reduces to:

\[
L(n) = \frac{1}{4} \left[ + \sum_{r+s=n} C_n^u C_r^b C_s^c + \sum_{r-s=-n} C_n^u C_r^{b'} C_s^{c'} + \sum_{r-s=n} S_n^u C_r^b S_s^c + \sum_{r-s=-n} S_n^u C_r^{b'} S_s^{c'} \right]
\]

This is used to calculate the net gain or loss of energy (in our problem the fluxes) by a frequency \( n \) as it interacts with frequencies \( r \) and \( s \).

\[(a)\text{ Surface similarity theory} \]

Our use of surface similarity theory, based on the work of Businger et al. (1971), can be expressed following Krishnamurti et al. (1998).

The latent-heat flux is given by:

\[
L_h = \rho C_{\text{LH}} L |V| (Q_s - Q_a).
\]

Here \( \rho \) is the density of air, \( L \) is the latent heat of vaporization, \(|V|\) is the magnitude of the surface wind and \( Q_s \) and \( Q_a \) denote the specific humidities over the sea surface and at the anemometer level. \( C_{\text{LH}} \) denotes the exchange coefficient for latent heat. This is a stability-dependent time varying coefficient of the similarity theory.

The expression for sensible-heat flux is given by:

\[
L_s = \rho C_p C_s |V| (T_s - T_a)
\]

where \( C_p \) is the specific heat of air at constant pressure, \( C_s \) is the exchange coefficient for sensible heat, \( T_a \) is the temperature of air near the surface (at anemometer level) in Kelvin and \( T_s \) is the surface temperature. The expression for \( u \) momentum flux is given by:

\[
\tau^x = \rho C_M |V| (u_a - u_s)
\]

where \( u_a \) and \( u_s \) are zonal winds at anemometer and surface levels, respectively; \( C_M \) is the exchange coefficient for momentum. \( L_h \), \( L_s \) and \( \tau^x \) are the respective fluxes of latent heat, sensible heat and momentum at the top of the constant-flux layer. It is assumed that \( C_{\text{LH}} = C_s = C_M \), where \( C_{\text{LH}} \), \( C_s \) and \( C_M \) are the bulk exchange coefficients of latent heat, sensible heat and momentum respectively; these exchange coefficients are functions of stability and, as such, are space and time dependent. Daily values of \( L_h \), \( L_s \) and \( \tau^x \) on the left-hand side (l.h.s.) of Eqs. (6), (7) and (8), respectively, are obtained as model output based on surface similarity theory. On the right-hand side (r.h.s.) of these equations daily values of \(|V|\), \(|Q_s - Q_a|\), \((T_s - T_a)\) and \((u_a - u_s)\) are also obtained from model output datasets. Using all those values in Eqs. (6), (7), and (8), values of bulk coefficients are obtained as space- and time-dependent quantities. We used this methodology for computations of fluxes in the frequency domain.
Using Fourier time series for $C_{\text{LH}}, |V|$, and $L(Q_s - Q_a)$ and taking the co-spectrum among them on the r.h.s. of expression (6), with the help of formula (5) latent-heat flux in the frequency domain can be written as:

$$
\langle L_h(n) \rangle = \frac{\rho}{2} \left[ + \sum_{r+s=n} + \sum_{r-s=n} + \sum_{r-s=-n} \right] [LHC(n).MVC(r).DMC(s)]
$$

$$
+ \frac{\rho}{2} \left[ - \sum_{r+s=n} + \sum_{r-s=n} + \sum_{r-s=-n} \right] [LHC(n).MVS(r).DMS(s)]
$$

$$
+ \frac{\rho}{2} \left[ + \sum_{r+s=n} - \sum_{r-s=n} + \sum_{r-s=-n} \right] [LHS(n).MVC(r).DMS(s)]
$$

$$
+ \frac{\rho}{2} \left[ + \sum_{r+s=n} - \sum_{r-s=n} - \sum_{r-s=-n} \right] [LHS(n).MVS(r).DMC(s)], \quad (9)
$$

where $(LHC, LHS)$, $(MVC, MVS)$ and $(DMC, DMS)$ are the cosine and sine coefficients of $C_{\text{LH}}, |V|$, and $L(Q_s - Q_a)$ associated with frequencies $n, r$ and $s$ respectively.

Using Fourier time series for $C_{\text{LH}}, |V|$ and $(T_s - T_a)$ and taking the co-spectrum among them in the r.h.s. of expression (8), with the help of formula (5), sensible-heat flux in the frequency domain can be written as:

$$
\langle L_s(n) \rangle = \frac{\rho C_p}{2} \left[ + \sum_{r+s=n} + \sum_{r-s=n} + \sum_{r-s=-n} \right] [LHC(n).MVC(r).DTC(s)]
$$

$$
+ \frac{\rho C_p}{2} \left[ - \sum_{r+s=n} + \sum_{r-s=n} + \sum_{r-s=-n} \right] [LHC(n).MVS(r).DTS(s)]
$$
\[
\frac{\rho C_p}{2} \left[ + \sum_{r+s=n} - \sum_{r-s=-n} \right] [LHS(n).MVC(r).DTS(s)] \\
+ \frac{\rho C_p}{2} \left[ + \sum_{r+s=n} - \sum_{r-s=-n} \right] [LHS(n).MVS(r).DTC(s)], \quad (10)
\]

where \((DTC, DTS)\) are the cosine and sine coefficients of \((T_s - T_a)\) associated with frequency \(s\). Using Fourier time series for \(C_{LH}, |V|\) and \((u_a - u_s)\) and taking the co-spectrum among them on the r.h.s. of expression (10), with the help of formula (5), \(u\) momentum flux in the frequency domain can be written as:

\[
(\tau^u(n)) = \frac{\rho}{2} \left[ + \sum_{r+s=n} - \sum_{r-s=-n} \right] [LHC(n).MVC(r).DUC(s)] \\
+ \frac{\rho}{2} \left[ + \sum_{r+s=n} - \sum_{r-s=-n} \right] [LHC(n).MVS(r).DUS(s)] \\
+ \frac{\rho}{2} \left[ + \sum_{r+s=n} - \sum_{r-s=-n} \right] [LHS(n).MVC(r).DUS(s)] \\
+ \frac{\rho}{2} \left[ + \sum_{r+s=n} - \sum_{r-s=-n} \right] [LHS(n).MVS(r).DUC(s)], \quad (11)
\]

where \(DUC, DUS\) are the cosine and sine coefficients of \((u_s - u_a)\) associated with frequency \(s\).
\( \alpha_n^u = (C_n^u + S_n^u)^{1/2}, \quad \varphi_n^u = \tan^{-1}\left(-\frac{S_n^u}{C_n^u}\right). \)

If we substitute (12) into the Fourier expansion of \( u \) (Eq. (1)), we obtain:

\[
L(n) = \frac{1}{2} \alpha_n^u \alpha_r^b \alpha_s^c \times
\]

\[
\{ \cos(\varphi_n^u - \varphi_r^b - \varphi_s^c) \}, \quad r + s = n,
\]

\[
\{ \cos(\varphi_n^u - \varphi_r^b + \varphi_s^c) \}, \quad r - s = n,
\]

\[
\{ \cos(\varphi_n^u + \varphi_r^b - \varphi_s^c) \}, \quad s - r = n.
\]

Since the \( \alpha \)'s are positive definite, the maximum possible value of \( L(n) \) is obtained when the cosine contribution is unity, i.e., when:

\[
\varphi_r^b + \varphi_s^c = \varphi_n^u \pm 2k\pi, \quad r + s = n,
\]

\[
\varphi_r^b - \varphi_s^c = \varphi_n^u \pm 2k\pi, \quad r - s = n,
\]

\[
-\varphi_r^b + \varphi_s^c = \varphi_n^u \pm 2k\pi, \quad -r + s = n.
\]

or, in other words, when \( \varphi_r^b, \varphi_s^c \) relate to \( \varphi_n^u \) in the same way that \( r, s \) relate to \( n \). Figure 2(a) is a schematic illustration of the \( r, s, n \) frequency space. Here we highlight those regions of \( r + s \) and \( r - s \) where the synoptic-scale interacts with the MJO timescale. Outside of the two slanting zones in Fig. 2(a), the MJO does not interact with the synoptic timescales. Also shown in the bottom left is a region where \( r + s \) lies on the MJO time-scale. Those are the regions where \( r \) and \( s \) also individually lie on the MJO time-scale. Since a robust synoptic time-scale covering the period 2 to 7 days
Figure 2. (a) A schematic diagram illustrating regions on a frequency, $r$, versus frequency, $s$, space where the synoptic time-scales can interact with the Madden–Julian Oscillation (MJO) time-scales. The shaded areas denote where $s - r$ or $r - s$ can amplify the MJO time-scales via triad interactions. (b) A schematic diagram illustrating the lines along which three frequencies 9, 48 and 57 interact to amplify the lower frequency (with a period of 40 days). Since the relationship among frequencies is cyclical, this family of lines reappears at $\pm 2k\pi$. (c) An example of three frequencies 9, 48 and 57 showing where the triad interactions can amplify the MJO time-scale. The phase shifts of the synoptic-scale frequencies, 48 and 57, illustrated by thin dotted and solid lines, match the phase shift of the MJO, shown by the heavy solid line.
is known to exist over the tropics, strong interaction within the MJO time-scale is possible along the shaded region. To illustrate an example of an interaction leading to growth of the MJO time-scale, consider the contribution of the interaction of waves \( n = 9 \), \( r = 48 \) and \( s = 57 \) (\( n = s - r \)) to the energy of \( n = 9 \). According to Eq. (17), there will be maximum growth of energy at frequency 9 when \( -\varphi_{48} + \varphi_{57} = \varphi_{9} \pm 2k\pi \). The solutions to this equation are displayed as a diagram in Fig. 2(b). An example of a triad of waves satisfying this equation is shown in Fig. 2(c).

Given the SST, \( T_s \), and the surface specific humidity \( Q_s \), if they carry a small signal on the MJO time-scale we ask whether that frequency can amplify from triad interactions at the top of the constant-flux layer. The selection rules tell us that higher-frequency tropical waves can, in principle, provide such a possibility. A question not addressed in this study is whether the temporal variations of the exchange coefficients can also contribute to the growth or decay of the MJO via triad interactions.

3.3 Disposition of fluxes in the PBL

Here, again, our interest is in the specific algorithm that was used in our coupled model for the vertical disposition of surface fluxes within the PBL. Our formulation of the PBL follows that of Louis (1979). The vertical flux convergence is expressed by a vertical diffusion of surface fluxes, the general expression is given by:

\[
\frac{\partial \tau}{\partial t} = \ldots + \frac{g^2}{p_s^2} \frac{\partial}{\partial \sigma} \left( \rho^2 K \frac{\partial \tau}{\partial \sigma} \right),
\]

where \( p_s \) is the surface pressure, \( \sigma \) is the vertical sigma coordinate (\( p/p_s \)), and \( \tau \) is a basic dependent variable such as momentum, heat or moisture. The exchange coefficient, \( K \), is determined from a mixing-length theory that involves stability-dependence via the bulk Richardson number \( R_{IB} \) (Louis 1979; Krishnamurti et al. 1998).

In our model the exchange coefficients are expressed by:

\[
K_H = K_Q = \ell^2 \frac{\partial |V|}{\partial z} F_H R_{IB},
\]

\( (19) \)
and

$$K_M = \ell^2 \frac{\partial |V|}{\partial z} F_m R_{IB},$$  \hspace{1cm} (20)$$

where $F_h$ ($F_m$) denotes the non-dimensional heat (momentum) flux, $\ell$ is the mixing length (Blackadar 1962), expressed by $\kappa z / (1 + \kappa z / \lambda)$ where $\kappa$ is the Von Karman constant, $z$ is the height of the relevant computational level in the PBL, and $\lambda$ is a constant which denotes an asymptotic mixing length whose values for heat and moisture fluxes are set to 450 m, and to 150 m for momentum.

Following Louis (1979), the analytical formulae for the surface similarity theory, $F_h$ and $F_m$ are given as:

$$F_h = F_m = \frac{1}{(1 + 5R_{IB})^2}, \quad R_{IB} \geq 0,$$  \hspace{1cm} (21)$$

for the stable case, and

$$F_h = \frac{1 + 1.286|R_{IB}|^{1/2} - 8R_{IB}}{1 + 1.286|R_{IB}|^{1/2}}, \quad R_{IB} < 0,$$  \hspace{1cm} (22)$$

$$F_m = \frac{1 + 1.746|R_{IB}|^{1/2} - 8R_{IB}}{1 + 1.746|R_{IB}|^{1/2}}, \quad R_{IB} < 0,$$  \hspace{1cm} (23)$$

for the unstable case. Here, the bulk Richardson number over an atmospheric layer is given by:

$$R_{IB} = \frac{g}{\theta \partial \theta / \partial z} \left[ \frac{\partial |V|}{\partial z} \right]^2.$$  \hspace{1cm} (24)$$

The heat flux is given by:

$$F_H = \rho C_p K_H \frac{\partial \theta}{\partial z} = \rho C_p l^2 \frac{\partial |V|}{\partial z} F_h R_{IB} \frac{\partial \theta}{\partial z} = \rho C_p \frac{\kappa^2 z^2}{(1 + \kappa z / \lambda)^2} \frac{\partial |V|}{\partial z} \frac{R_{IB}}{(1 + 5R_{IB})^2} \frac{\partial \theta}{\partial z},$$

for the stable case, and

$$F_H = \rho C_p \frac{\kappa^2 z^2}{(1 + \kappa z / \lambda)^2} \frac{\partial |V|}{\partial z} \left( R_{IB} + \frac{1.286R_{IB} R_{IB}^{1/2} - 8R_{IB}^2}{1 + 1.286|R_{IB}|^{1/2}} \right) \frac{\partial \theta}{\partial z}$$

for the unstable case.

Using Fourier time series for the term involving $R_{IB}$, $\partial \theta / \partial z$ and $\partial |V| / \partial z$, and by taking co-spectra among them in the r.h.s. of expression (25) with the help of formula (5), the heat flux in the frequency domain is given by:

$$\langle F_H(n) \rangle = \rho C_p \left[ \frac{\kappa^2 z^2}{2 (1 + \kappa z / \lambda)^2} \right] + \sum_{r+s=n} [RIC(n) \cdot PTC(r) \cdot VTC(s)]$$

$$+ \rho C_p \left[ \frac{\kappa^2 z^2}{2 (1 + \kappa z / \lambda)^2} \right] + \sum_{r-s=n} [RIC(n) \cdot PTS(r) \cdot VTS(s)]$$

$$+ \sum_{r-s=-n} [RIC(n) \cdot PTC(r) \cdot VTC(s)]$$

$$+ \sum_{r-s=-n} [RIC(n) \cdot PTS(r) \cdot VTS(s)]$$
for the stable case, and

$$F_Q = \rho K_Q \frac{\partial q}{\partial z} = \rho l^2 \frac{\partial |V|}{\partial z} \frac{\partial q}{\partial z} = \rho \frac{\kappa^2 z^2}{(1 + \kappa z/\lambda)^2} \frac{\partial |V|}{\partial z} \frac{R_{IB}}{(1 + 5 R_{IB})^2} \frac{\partial q}{\partial z},$$

for the stable case, and

$$F_M = \rho K_M \frac{\partial v}{\partial z} = p l^2 \frac{\partial |V|}{\partial z} \frac{\partial v}{\partial z} = \rho \frac{\kappa^2 z^2}{(1 + \kappa z/\lambda)^2} \frac{\partial |V|}{\partial z} \frac{R_{IB}}{(1 + 5 R_{IB})^2} \frac{\partial v}{\partial z},$$

for the unstable case. Using Fourier time series for the terms involving $R_{IB}$, $\partial q/\partial z$ and $\partial |V|/\partial z$, and taking the co-spectrum among them on the r.h.s. of expression (27) with the help of formula (5), the expression of moisture flux in the frequency domain, $F_Q(n)$ is the same as $F_H(n)$ except that $PTC$ and $PTS$ are to be replaced by $PQC$ and $PVS$, the Fourier temporal coefficients of $\partial q/\partial z$.

The momentum flux is given by:

$$F_M = \rho K_M \frac{\partial v}{\partial z} = p l^2 \frac{\partial |V|}{\partial z} \frac{\partial v}{\partial z} = \rho \frac{\kappa^2 z^2}{(1 + \kappa z/\lambda)^2} \frac{\partial |V|}{\partial z} \frac{R_{IB}}{(1 + 5 R_{IB})^2} \frac{\partial v}{\partial z},$$

for the stable case, and

$$F_M = \rho \frac{\kappa^2 z^2}{(1 + \kappa z/\lambda)^2} \frac{\partial |V|}{\partial z} \frac{R_{IB}}{(1 + 5 R_{IB})^2} \frac{\partial v}{\partial z},$$

for the unstable case. Using Fourier time series for the terms involving $R_{IB}$, $\partial v/\partial z$ and $\partial |V|/\partial z$, and taking co-spectra among them on the r.h.s. of expression (28) with the help of formula (5), the expression for $F_M(n)$ in the frequency domain is the same as that for $F_H(n)$ except that $PTC$ and $PTS$ are replaced by $PVC$ and $PVS$, the Fourier temporal coefficients of $\partial v/\partial z$.

The frequency interactions in the PBL are determined entirely by the variation of the exchange coefficient $K$. Our premise is that these fluxes in the frequency domain build up within the PBL, thus enhancing the amplitude of the Madden–Julian time-scale via the triad interactions. Thus we expect that a small oceanic signal in the SST on the Madden–Julian time-scale is enhanced at the top of the atmospheric constant-flux layer, and a further enhancement occurs across the PBL. These are illustrated in the results of computations presented in the next section.
3. **FSU coupled ocean–atmosphere model and the MJO**

(a) **Description of the coupled model**

(i) **Ocean model.** The ocean model employed here is a modified version of the Max Planck Institute ocean model (Sterl 1991; Latif et al. 1994). The model has 17 irregularly spaced vertical layers. The upper part of the ocean is of greater interest since turbulent activity due to external forcing is effective there; therefore, the upper 300 m have finer vertical resolution with the thickness of layers from the top being 20, 20, 20, 20, 20, 30, 30, 40, 50, 50, 100, 200, 500, 1000, 1500 and 2000 m, respectively. There is variable resolution in the meridional direction where: within ±10° of the equator the resolution is constant at 0.5°; between ±10° and ±20° it increases to 1°; and outside of ±50° it is set to 5°. Constant resolution of 5° is used in the zonal direction. The equations of motion are solved using finite-differencing techniques on a horizontally staggered E-grid scheme (Arakawa and Lamb 1977).

The time integration of the full system with a time step of 120 minutes is carried out by the method of fractional steps. This method allows for each individual equation to be separated into components which are integrated individually and then combined to give the solution of the full system. For turbulent diffusion, a constant mixing coefficient of 1000 m² s⁻¹ is used in order to calculate the horizontal mixing processes, whereas in the vertical a formula dependent on the Richardson number is used. Surface forcing is represented by the exchange of heat and momentum at the air–sea interface.

(ii) **Atmospheric model.** The numerical weather prediction model used in this study is the FSUGSM described in Krishnamurti et al. (1998). The horizontal and vertical resolution of the model are flexible; for this study we use a horizontal resolution truncated at wave number 42 (T42) which gives an approximate grid mesh of 2.8° latitude/longitude and a vertical resolution of roughly 0.5 km described by 14 sigma-levels between roughly 50 and 1000 hPa. A semi-implicit time integration scheme is used with a time step of 20 minutes to represent the time derivatives in the model equations. The high-frequency gravity-wave oscillations are suppressed by semi-implicit time differencing wherever they appear in the model equations. The initial data sources for the FSUGSM comprise the global analysis of ECMWF and the SSTs from NCEP. The prognostic model variables are: vorticity, divergence, dew-point depression, surface pressure, and a variable which combines the geopotential height and logarithm of the surface pressure. The model physics includes:

- modified Kuo scheme for cumulus parametrization (Krishnamurti et al. 1983);
- shallow convective adjustment (Tiedke 1984);
- dry convective adjustment;
- large-scale condensation;
- surface-layer parametrization by similarity theory (Businger et al. 1971);
- PBL parametrization includes a diffusive formulation based on mixing length theory dependent on Richardson number (Manobianco 1989);

The salient elements of this coupled model are outlined in Fig. 3.

(b) **Datasets**

The datasets used in the present study are global daily values for an entire year (March 1996 to February 1997, inclusive) extracted from the coupled-model output.
(Cubukcu and Krishnamurti 2002). The variables relevant to the study of fluxes include winds, temperature, bulk coefficients and humidity in the atmosphere, and the ocean temperatures of the coupled model. Here area averaging is carried out in areas: between latitudes $\pm 11^\circ$ and longitudes $130^\circ$ and $150^\circ$E for the warm pool; between longitudes $180^\circ$ and $160^\circ$W for the central tropical Pacific; and between $60^\circ$ and $110^\circ$E for the Indian Ocean. The same algorithms that were used within the coupled-model forecasts are used for the computation of fluxes in the frequency domain. It should be noted that the total fluxes of the coupled model’s output are not used directly, since the computation of fluxes in the frequency domain calls for the explicit computation of the co-spectra of the triad interactions.

At each transform grid point of the spectral model, where the physical processes are evaluated, we construct a time series for these triple-product variables. These are 360-day time series, one entry per day for each variable, denoting the value at the top of the constant-flux layer for the period from March 1996 to February 1997. This is based entirely on the model output datasets from the coupled model. This database is used to construct the frequency co-spectra for the triads described earlier.

(c) The MJO in the FSUCGSM

The coupled model has previously been applied to numerous climatological problems (e.g. LaRow and Krishnamurti 1998; Krishnamurti et al. 2000; Cubukcu and Krishnamurti 2002). The results have demonstrated that the FSUCGSM is an excellent
Figure 4. (a) A coupled-model result based on the eastward passage of the Madden–Julian Oscillation. Shown are the 200 hPa zonal wind anomalies (m s$^{-1}$) on a Hovmöller diagram for a 90-day period during the summer of 1996 in the equatorial belt 5°S to 5°N. (b) Power spectra of the 200 hPa zonal wind anomalies for the tropical Indian, West Pacific and Central Pacific Oceans.
tool for climatological studies. Cubukcu and Krishnamurti (2002) showed the model’s performance in simulating the tropical intra-annual oscillations such as the MJO to be quite successful.

The entire context of this paper is based on the same numerical simulation used by Cubukcu and Krishnamurti (2002). A simple example of MJO signal as simulated by the FSUCGSM is shown in Fig. 4(a), which displays the time–longitude cross-section of tropical 200 hPa zonal wind anomalies for three consecutive months towards the end of the time integration (October–December 1996). The anomalies are relative to a one-year mean of the model output; positive values denote westerly anomalies. The zonal wind at 200 hPa is often used in various climate diagnostics bulletins to illustrate the eastward passage of the MJO—a phenomenon that was captured quite nicely in our one-year coupled-model run. The model output, as illustrated, shows a robust MJO, with the largest westerly wind anomalies of the order of 20 m s$^{-1}$. The power spectra of 200 hPa winds shown in Fig. 4(b) highlights the peak signals in the time range of 25–60 days that represent the MJO. From Figs. 1(a) and (b), and 4(b), a strong signal in the power spectra of the SSTs, and the zonal winds at 850 hPa and 200 hPa levels, can be seen for several frequencies within the 20- to 60-day time-scales. It does not show a strong band covering the entire MJO time-scale as noted by Salby and Hendon (1994) and Hendon and Salby (1994). However, several spectral peaks covering the entire range from 25 to 60 days can be seen on these figures. In this study we have limited our examination to 30–50 days, primarily based on the SST spectra and our previous studies.

4. Computation of Fluxes in the Frequency Domain

The results presented in this paper are based on the following computations of the fluxes. The letters are used to label corresponding panels of plots displayed in Figs. 5 to 11.

(a) Total fluxes on the time-scale of the MJO for the sensible heat, latent heat and momentum across the constant-flux layer.

(b) Total fluxes in the constant-flux layer on the time-scale of the MJO arising from interaction of the MJO with the synoptic time-scale of from 2 to 7 days.

(c) Fluxes contributed by salient (strongest contributing) triad interactions in the surface layer.

(d) Salient triad interaction frequencies contributing to (a) in the constant-flux layer.

(e) Total fluxes on the time-scale of the MJO for the sensible heat, latent heat and momentum in the PBL at the 850 hPa level.

(f) Total fluxes in the PBL on the time-scale of the MJO arising from interaction of the MJO time-scale with synoptic time-scale of 2 to 7 days.

(g) Fluxes contributed by the salient triad interactions in the PBL.

(h) Salient triad interaction frequencies contributing to (e) in the PBL.

In this section fluxes of latent and sensible heat are expressed in W m$^{-2}$, and momentum fluxes in N m$^{-2}$. We consider three domains for illustrating these fluxes in the frequency domain:

- Indian Monsoon: 11.2°S to 11.2°N, 61.88°E to 109.69°E;
- Central Pacific Ocean: 11.2°S to 11.2°N, 160.31°W to 180°W;
- Western Pacific Ocean: 11.2°S to 11.2°N, 151.88°E to 132.19°E.

We did not include a domain over the Atlantic since the MJO signal was weak in that sector.
Figure 5. Latent-heat fluxes (W m$^{-2}$) over the Indian Ocean region. (a) Total latent-heat fluxes on the time-scale of the Madden–Julian Oscillation (MJO) across the constant-flux layer. (b) Total fluxes of latent heat across the constant-flux layer on the time-scale of the MJO arising from interaction of the MJO with the synoptic time-scale of 2 to 7 days. (c) Fluxes of latent heat contributed by salient (strongest contributing) triad interactions in the surface layer. (d) Salient triad interaction frequencies contributing to latent-heat fluxes on the time-scale of the MJO across the constant-flux layer. (e) Total latent-heat fluxes on the time-scale of the MJO in the planetary boundary layer (PBL) at 850 hPa. (f) Total latent-heat fluxes in the PBL on the time-scale of the MJO arising from interaction of the MJO time-scale with the synoptic time-scale of 2 to 7 days. (g) Salient triad interaction frequencies contributing to latent-heat fluxes on the time-scale of the MJO in the PBL. (h) Salient triad interaction frequencies contributing to latent-heat fluxes on the time-scale of the MJO in the PBL at 850 hPa.
The results for latent-heat fluxes in the frequency domain are illustrated in Figs. 5, 6 and 7. These cover the results from our computations for the Indian, West Pacific and Central Pacific Oceans. Each illustration comprises eight panels (a) to (h) corresponding to the flux computations listed at the beginning of this section.

Overall the results of these latent-heat fluxes over the Indian Ocean show that the total surface fluxes (Fig. 5(a)) on the time-scale of the MJO are largest in the trade wind belt of the southern hemisphere and the south-west monsoon flow over the Arabian Sea. These maximum total fluxes are of the order of 10 W m\(^{-2}\). The fluxes over the equatorial belt are smaller. In general, the fluxes over land areas between 90° and 110°E
Figure 7. Same as Fig. 5 but for the Central Pacific region.

are small. Around 30 to 50% of these total fluxes (on the MJO time-scale) come from the triad interaction of the MJO time-scale with two other frequencies on synoptic time-scales, Fig. 5(b). The single salient time-scale that contributes the most to the latent-heat fluxes on the MJO time-scale is found to be in the synoptic time-scales of 2 to 7 days. This seems to be true over the entire Indian Ocean domain. These fluxes and the salient triads are shown in Figs. 5(c) and (d).

The total fluxes of latent heat within the PBL at 850 hPa on the time-scale of the MJO are shown in Fig. 5(e). These total fluxes are almost twice those at the surface. A surprising aspect is the lack of continuity of the 850 hPa fluxes with respect to those at the surface. Over the active monsoon region east of 80°E the fluxes at 850 hPa are quite large, and the pattern of fluxes bears little resemblance to those at the surface.
level, especially over land areas. Steady input of moisture laterally into the monsoon region from the Oceans can contribute to the excessive fluxes at the 850 hPa level over the surface level. Around 50 to 75% of these total fluxes (on the time-scale of the MJO) arise from the interaction of the MJO time-scale with high-frequency motions (i.e. on the 2- to 7-day time-scale), Fig. 5(f). Nearly 10% of these total fluxes can be attributed to a contribution from the single local salient triad, Fig. 5(g). These salient triads are displayed in Fig. 5(h). A typical salient triad of frequencies here is 9, 57, 48, roughly corresponding to 40, 6 and 7 days, respectively. It is interesting to note again that the interactions between members of the MJO time-scale and the synoptic time-scale (2 to 7 days) carry these triads that contribute the largest to the fluxes on the time-scale of the MJO. This is true over the entire Indian Ocean domain. The data

Figure 8. Same as Fig. 5 but for sensible-heat fluxes (W m$^{-2}$) over the Indian Ocean region.
Figure 9. Same as Fig. 5 but for sensible-heat fluxes (W m$^{-2}$) over the West Pacific region.

considered for the present study cover the period from March 1996 to February 1997, and hence the underlying thermodynamical effects of El Niño may have come into play in possibly shifting the latent-heat flux maxima from the West Pacific to the Indian Ocean. This maximum of latent-heat flux reaches 26.5 W m$^{-2}$ at (11.2°N, 75.94°E). These results convey some of the most important results of this study. A good proportion of the total latent-heat flux (on the time-scale of the MJO) arises from the interaction of the MJO time-scale with the disturbances at the synoptic time-scale (2 to 7 days). A small signal in the SST on the time-scale of the MJO is successively amplified via these triad interactions, first over the constant-flux layer and next within the PBL.
Figure 10. Same as Fig. 5 but for sensible-heat fluxes (W m$^{-2}$) over the Central Pacific region.

Tropical disturbances arising from the tropical instabilities abound on the synoptic time-scale, and the presence of a non-zero SST fluctuation on the MJO time-scale facilitates the rapid amplification of the MJO via these triad interactions.

The results of these same computations for the Central and the West Pacific Ocean are presented in Figs. 6 and 7. These domains are both essentially oceanic except over Australia. Overall we find quite similar results over the three ocean basins. The southern hemisphere trades carry the largest moisture and latent-heat fluxes on the MJO time-scale over the western Pacific Ocean. For moisture fluxes on the MJO time-scale the largest contribution comes from triad interaction with the synoptic time-scale over all tropical ocean basins. The salient triads such as 7, 57, 50 (which translate roughly to
52, 6 and 5 days) contribute about 25% of the total fluxes of moisture on the MJO time-scale over a specific region. We also note that, overall, the near equatorial belt is not a large contributor to fluxes on the MJO time-scale. These salient triads are also quite active over the date line, where they again seem to contribute nearly 15% of the total fluxes (on the MJO time-scale) in the PBL (at 850 hPa level). The significant nonlinear inter-time-scale triad interactions in latent-heat flux involve a pair of frequencies around 40 and 57, corresponding to high-frequency oscillations on the MJO time-scale at the two layers and over the three regions considered, (6, 57, 51), (9, 49, 40) and (9, 49, 40) exhibit the most prominent-scale interactions having values of 1.25, 0.82 and 1.78 W m\(^{-2}\) over the Central Pacific, West Pacific and Indian Oceans, respectively. This amplification of the MJO over the western Pacific is consistent with the robust

Figure 11. Same as Fig. 5 but for momentum fluxes (N m\(^{-2}\)) over the Indian Ocean region.
MJO seen over this region on the familiar $x - t$ Hovmöller diagrams of the zonal winds (Fig. 4).

(b) Sensible-heat fluxes

Figures 8, 9 and 10 illustrate the results for sensible-heat flux over the Indian, Central and West Pacific Oceans, respectively, each comprising eight panels. Panel (a) shows the total sensible-heat flux in the constant-flux layer on the time-scale of 20 to 60 days. The largest values of these fluxes are around 3 to 10 W m$^{-2}$. These total surface fluxes are largest in the trade wind belt of the southern hemisphere, the southwest monsoon flows of the Arabian Sea and the monsoon trough over India. The near equatorial belt has a minimum of sensible-heat flux on the MJO time-scale. Panels (b) show the contributions to the fluxes on the MJO time-scale arising from all possible interactions with high-frequency motion (i.e. 1- to 10-day time-scale). Roughly 30 to 50% of the total sensible-heat flux on the MJO time-scale appears to come from the interaction with these high-frequency motions. The pattern of the total fluxes and the MJO time-scale fluxes resulting from high-frequency interaction are similar, and the maxima of the fluxes generally occur over regions of strong surface winds. The flux contribution by the single largest triad and the specific salient triads are illustrated in panels (c) and (d). A typical triad is 8, 18 and 10, here the three frequencies 8, 18 and 10 interact, where 8 = 18 − 10 satisfies the selection rule. The time-scales of these are roughly 45, 20 and 36 days. This is clearly not one of the triads where the MJO time-scale interacts with the synoptic time-scale. If we look across the Indian Ocean, we find that the salient triads contributing to an enhancement of the fluxes on the MJO time-scale are not in the high frequencies. This is not surprising since the thermal structure on the scale of tropical disturbances is quite flat and the Bowen ratio quite small over the tropics. In this sense the decomposition of fluxes on the time-scale of the MJO are quite different for latent- and sensible-heat fluxes. Panels (e) illustrate the total flux of sensible heat within the PBL at the 850 hPa level. The PBL fluxes are about 25 to 50% larger than the surface fluxes, due to lateral convergence of fluxes on the MJO time-scale within the PBL. The contribution to the 850 hPa total fluxes that arises from triad interaction with the synoptic-scale are shown in panels (f). Again, we note that roughly 50% of the total fluxes on the time-scale of the MJO are contributed by the interaction with the high-frequency motions on the time-scale of 2 to 7 days, even though these are not the salient triads. Panels (g) and (h) show the sensible-heat fluxes, within the PBL at 850 hPa, arising from single salient triads and the specific triads respectively. Here again we note that the strongest sensible-heat fluxes are not contributed from the interaction of the MJO time-scale with the synoptic time-scale. Frequencies much lower than the synoptic time-scales provide the salient triads for interaction with the MJO time-scales. In general the sensible-heat fluxes in the frequency domain call for interactions of the MJO time-scale with another member of the MJO time-scale and a much lower frequency. These fluxes are small, and do not convey anything important in respect of the growth of the MJO fluxes across the PBL. We have also examined the results of the sensible-heat flux computations over the western and eastern Pacific. In all cases the maximum fluxes occur over the region of strongest surface and PBL winds. The results for the three ocean basins were quite similar in terms of the composition of salient triads.

The fluxes of sensible heat are found to be at a maximum over the Central Pacific, where they are of the order of 16 and 25 W m$^{-2}$ at (11.2°N, 160.31°W) in the surface layer, and between the surface layer and the free atmosphere, respectively. Similarly, the values of the fluxes of sensible heat over West Pacific are of the order of 2 and 6 W m$^{-2}$
at (11.2°N, 151.88°E) in the above mentioned two layers. The maximum values of the sensible-heat flux in the respective layers over Indian Ocean are of the order of 15 and 23 W m^{-2}. Thus we notice systematic strengthening of fluxes of sensible heat on the MJO time-scale from the surface layer to the free atmosphere through the PBL. At almost all grid points this trend has been maintained.

(c) Momentum fluxes

The results for momentum fluxes on the time-scale of the MJO for the Indian Ocean are presented in Figs. 11(a) to (h). The largest upward flux of momentum is over the southern trade winds and in the south-westerly monsoon flow. Between the surface level and the PBL these fluxes are approximately doubled. Overall, the patterns of fluxes at the surface and the PBL (850 hPa level) are quite similar. The most interesting aspects of the momentum fluxes are the distribution and composition of the salient triads at the surface and the PBL. A number of these triads show interactions between two members of the MJO family and a lower frequency (i.e. lower than the MJO time-scale). For example, 8, 9, 1 denotes an interaction among 44- and 45-day oscillations and the annual cycle. Overall, this entire pattern of salient triads is very different from that for the moisture fluxes. The contribution of the high frequencies interacting with the MJO (Fig. 11(b)) is larger. What this means is that higher frequencies, i.e. the synoptic time-scales, do contribute significantly to the overall total even though they are not a part of the salient triads. This is also true for the sensible-heat fluxes. In that sense the moisture fluxes contributed by the high frequencies at the surface and in the PBL are quite unique, in that they readily interact with the MJO to amplify the latter. The results for the Central and West Pacific Oceans are quite similar to those over the Indian Ocean and are not displayed here.

(d) Total MJO time-scale fluxes versus total fluxes including all time-scales

The fluxes discussed so far in this paper are those on the time-scale of the MJO. It is important to know how those contributions on the time-scale of the MJO compare with the total annual-mean surface fluxes (from March 1996 to February 1997 inclusive), where all time-scales are included. The total surface flux of latent heat for the three oceanic basins, Indian, West Pacific and Central Pacific are shown in Figs. 12(a), (b) and (c), respectively. Overall, the largest oceanic fluxes are of the order of 140 W m^{-2}, and occur near the borders of these domains where the surface winds are strongest. The total latent-heat fluxes on the time-scale of the MJO are only around 10 W m^{-2}. The total fluxes shown here include contributions for all time-scales; a substantial portion are contributed on the synoptic time-scales. These total fluxes invariably increase between the surface and 850 hPa. These are illustrated in Figs. 12(d), (e) and (f) for the three ocean basins. The increase of fluxes is largest over the land areas of the monsoon region over the Indian Ocean, where there appears to be a doubling of fluxes between the surface and 850 hPa. Figures 12(g), (h) and (i) show the increase of fluxes between the surface and 850 hPa for the contributions of the three oceanic areas on the MJO time-scale. It is clear that a large proportion (approximately 60%) of the increase of total fluxes between the surface level and the PBL arises from the contribution on the MJO time-scale. This increase in the MJO time-scale is largely attributed to the triad interactions with the synoptic time-scales. It is this increase of the original signal on the MJO time-scale, starting from the SSTs over the ocean, which appears quite striking in these datasets.
Figure 12. (a) Total flux of latent heat (W m\(^{-2}\)) over the Indian Ocean; (b) and (c) as (a) but for the West Pacific and Central Pacific Ocean basins, respectively. (d), (e), (f) As (a), (b) and (c) but differences in total fluxes (W m\(^{-2}\)) between the 850 hPa and surface levels. (g), (h), (i) As (a), (b) and (c) but flux contributions on the time-scale of the Madden–Julian Oscillation.

5. **Concluding Remarks**

Tropical instabilities have been enumerated by many scientists: Yanai and Nitta (1968) on barotropic shear flow instabilities; Lindzen (1974) on Wave CISK*; Rennick (1977) on combined barotropic–baroclinic instabilities plus CISK; Moorthi and Arakawa (1985) on vertical shear and convection; and several others. The focal point of these studies is the mechanisms of growth of tropical wave disturbances whose time-scale is of the order of 2 to 7 days (Reed *et al.* 1977). These disturbances can be found over

* Conditional Instability of the Second Kind.
most of the tropical oceanic basins. They are, so to speak, waiting to interact with the MJO time-scale of 20 to 60 days. A number of these tropical wave disturbances satisfy the trigonometric selection rule, Fig. 2(a):

\[ n = r \pm s \]  

(the tropical wave frequencies).

This relationship appears to be more readily met by the pre-existing tropical wave disturbance and a weak MJO signal over the tropical oceanic SSTs, or in the surface-layer flows. From Eq. (9) we can see that the intraseasonal frequency, \( n \), is associated with exchange coefficient \( C_{\text{LH}} \), which is a function of stability and therefore a function of Richardson number \( (R_{\text{IB}}) \). \( R_{\text{IB}} \) depends on surface potential temperature and wind stress for computation of latent-heat fluxes in the surface layer. SST affects these two parameters very close to the sea surface, therefore, the intraseasonal variation of \( C_{\text{LH}} \) is involved with SST. The humidity fluxes are affected much more by the SST oscillations (Krishnamurti et al. 1988). The term \( (\bar{Q}_s - \bar{Q}_a) \) in Eq. (6) does have variations at intraseasonal time-scales, but synoptic time-scales involved with it are found to be important in nonlinear interactions between time-scales as far as their contributions to the MJO time-scale are concerned. The signal in intraseasonal SST in the surface layer associated with stability-dependent \( C_{\text{LH}} \) gets amplified in the PBL, though \( C_{\text{LH}} \) in the PBL is independent of SST.

It is this feature that makes it possible for the surface fluxes of moisture to amplify over the constant-flux layer, and it is this same feature that allows for the further amplification of the MJO over the PBL. Our parametrization of cumulus convection carries, at most, quadratic nonlinearities, which can convey in-scale (MJO to MJO) information from the cloud base to the cloud top. Thus, an MJO signal of the SST can be passed on through the deep convective cloud base to the tropical upper troposphere. It is our premise that a similar extension of this analysis over the deep convective layers would reveal the passage of this signal from the cloud base to most of the tropics via divergent circulations. Over tropical regions of convection the mid-troposphere vertical motion shows a signal on the time-scale of the MJO; the same feature was also noted over subtropical regions of descent. The present work is, of course, based entirely on model output from a coupled model that happened to simulate a somewhat realistic MJO signal. The model output was better suited for demonstrating the boundary-layer amplification of the MJO signal. This is an internally consistent dataset where the large-scale variables, especially in the PBL, were part of known model equations. Those equations were used, with the same model algorithms, to address the triad interactions in the frequency domain. What is presented here does not preclude other existing theories on the MJO, most of which are based on climate diagnostics and reanalysis datasets.

According to Fjortoft (1960), cascading of energy can take place either way, i.e. from larger-scales to smaller-scales (temporal and spatial) or vice versa. In the present study interactions of oscillations slower than the MJO, with the MJO and higher frequencies, are not considered. Enhanced fluxes of latent heat do feed convection on the MJO-scale. Propagation of clouds on this scale was noted by Yasunari (1980). Steady import of moisture laterally into the monsoon region from the oceans as well as from the southern trades carrying the largest moisture into Pacific are important features in the surface layer. Tropical disturbances arising from the tropical instabilities abound on the synoptic time-scale, and the presence of a small SST signal on the MJO timescale facilitates the rapid amplification of the MJO through nonlinear triad interactions.

We have also looked at the issues of sensible-heat and momentum transports in the frequency domain. While these results are most revealing and complementary to
the above findings, we feel that the amplification of moisture fluxes is the central issue for this MJO theory. Further work is clearly needed to understand the scale interaction of sensible-heat and momentum fluxes. The role of the ocean has not been adequately addressed here. A starting point in our analysis was the given fact that a small signal on the time-scale of the MJO exists in the SST over the upper ocean. If a MJO frequency in SST were to be absent (identically zero) then the triad interactions with the tropical SST that undergoes amplification over the atmosphere? How does that oceanic signal come about? The answer to this question requires further work. Oceanic behaviour is inherently of lower frequency than the atmosphere. If we accept that, and allow for some non-vanishing signal (however small) in the SST fields on the time-scale of the MJO to be present, given a plethora of tropical disturbances on the synoptic time-scale of 2 to 7 days in the lower troposphere we can provide a simple explanation. These disturbances arise from prevalent tropical instabilities. These time-scales provide rich possibilities for triad interactions with a MJO time-scale because they are able to satisfy the selection rules for triad interactions. This permits the MJO time-scale to amplify in the PBL. The resulting winds acquire a stronger signal on the MJO time-scale; these in turn convey the MJO time-scale to the ocean via surface stresses to amplify the SST signal. This positive feedback can establish an equilibrium state in the ocean and in the atmosphere, with both exhibiting the MJO signal. Thus we perceive the presence of the synoptic-scale disturbances in the tropics, on time-scales of 2 to 7 days, as an essential element for the excitation of the MJO.

We have not considered the changes in the atmospheric basic states as such, as described in Molinari et al. (1997), Hartmann and Maloney (2001) and Maloney and Hartmann (2001). In the frequency domain even the so-called basic-state’s variation can be part of the MJO. That could have been revealed from an analysis in the ‘wave number–frequency domain’ where wave number zero (evidently the basic state) would have exhibited variations. That is beyond the scope of this study as we have concentrated here only on the ‘frequency–frequency’ interactions.

It may be premature to consider the scenario portrayed here as a theory for the MJO. Further work is clearly needed to isolate the effects of the respective capabilities of coupled versus uncoupled models in the simulation of the MJO. Given a picture, such as Fig. 4, where for many months an intraseasonal wave traverses the globe, it seems that the oceanic contributions may have to be important. An AGCM, with prescribed SST, can carry a MJO signal primarily by the aforementioned scenario for a long period of time, simply from the inertia of the initial state of the surface layer and PBL where the proposed mechanism can operate for a few months. Evidently, lacking an oceanic support on the MJO time-scale, this signal could weaken in the AGCMs. Atmospheric internal dynamics may generate the MJO, but the role of ocean may be important for its enhancement. Computations of fluxes over land in the frequency domain may give more clues about the physical and dynamical mechanisms of the origin of the MJO. This problem will be addressed in our future work.

Computation of energy fluxes in the frequency domain is a powerful tool for isolating the role of an important time-scale such as the MJO and the ENSO. In a number of recent studies, Sheng and Hayashi (1990a,b) explored global energetics in the frequency domain. They focused on the maintenance of the eddy kinetic energy on the time-scale of the MJO. In these studies several frequencies covering ENSO, annual, MJO, and storm-scales (1–10 days) were examined using 6 years of daily global datasets. A major finding of this study was that the high-frequency (2- to 7-day) time-scales were a major source of energy for the MJO. That transfer mainly occurs from
three-component triad interactions, i.e. kinetic to kinetic energy exchanges invoking nonlinear dynamics. A smaller gain of energy for the MJO occurs from the in-scale vertical overturnings of potential to kinetic energy. The roles of the annual cycle and larger time-scales in the maintenance of the MJO were smaller. The main message from these studies is that tropical high-frequency disturbances play a major role in the maintenance of the MJO. The present study of the PBL fluxes in the frequency domain complements these results.

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