Development of a reduced space adjoint data assimilation technique for numerical simulation of oceanic circulation

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Numerical simulation has become one of the most important tools in studying geophysical problems, such as those encountered in physical oceanography and meteorology. The launch of many scientific satellites over recent years has provided scientists access to enormous volumes of data, presenting new opportunities for finding innovative ways to use them. The assimilation of these data into numerical simulations has become a topic of much interest in geophysics. The main objective of this study is to develop and apply a new technique for efficiently using the adjoint method to assimilate satellite altimeter data into a modern fully three-dimensional ocean general circulation model.

There are a number of different methods for data assimilation, classified as function fitting methods, statistical interpolation methods, nudging data assimilation, variational (adjoint) methods and the Kalman filter. An extensive review of these methods can be found in Ghil and Malanotte-Rizzoli (1991) and Le Dimet and Navon (1988). Being one of the best, but also computationally demanding, the adjoint method minimizes the distance between a model solution and the observations, which is called the cost function. This can be defined so that the solution does not have to obey the dynamics of the model exactly by adding a term that measures the model error as in the representer method. The minimization of the cost function gives rise to the weak constraint minimization problem. When the solution is required to satisfy the model exactly, it is referred to as strong constraint minimization. This is done by adjusting the state of the control variables of the model (e.g. initial conditions). An iterative technique is used in which the ocean model is integrated forward in time, followed by a backward integration of an adjoint of the system forced by the observations. By writing the cost function in the form of an inner product, the result of the backward integration of the adjoint model to the initial time will be the gradient of the cost function with respect to the control variables. Thus, an optimization method can be applied to reduce the cost function by changing the control variables at each iteration.

In this study, the adjoint method will be applied to the Navy Coastal Ocean Model (NCOM) in the Gulf of Mexico (Fig. 1). The NCOM is a three dimensional, hydrostatic, primitive equation ocean model with a hybrid sigma (terrain following)/ z-level vertical coordinate recently developed at the U.S. Naval Research Laboratory. The model domain includes the whole Gulf of Mexico and the western Caribbean Sea, from 98.15°W to 80.60°W and from 15.55°N to 31.50°N, with model equations discretized on a 1/20-degree grid, in latitude and longitude (Morey et al., 2003). The model uses 20 sigma layers uniformly distributed in the upper 100 meters and 40 unevenly distributed z-level layers below 100m. Sea surface height data from the Topex/Poseidon satellite altimeter will be assimilated into the model. The computational cost for a complicated model like the NCOM is very expensive, and the adjoint of this model would be even more resource intensive. In order for the technique to be practical for operational use, it is crucial to find a way to reduce the computational cost. For a wide class of oceanographic phenomena, the vertical dimension of the ocean structure can be very well approximated by a single active depth-averaged layer over an infinitely deep layer at rest. This formulation is often called a reduced gravity model, and is much less computationally expensive. For this reason, a backward model based on a reduced gravity formulation will be used as the adjoint, instead of integrating the adjoint of the NCOM directly. This novel approach, termed a "reduced space adjoint technique", will be developed and tested for the first time in a highresolution model of the Gulf of Mexico. After the forward integration of the NCOM at each time, the first baroclinic mode will be extracted using vertical normal mode decomposition. Since the vertical normal mode decomposition has to be conducted on a non-flat bottom, conventional methods will not be used here. However, two options are taken into consideration: the first one is to compute the vertical modes at each point in the x-y space; the second is to compute the normal modes in a region that is relatively flat compared to the other areas. Cyclostationary Empirical Orthogonal Analysis (Kim, 1997) will then be applied to the output of the model, and the regression method will be used to obtain the first baroclinic mode. Then the adjoint of the corresponding reduced

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gravity model will be run. The validity and efficiency of this technique will be examined with idealized cases and realistic simulations of the Gulf of Mexico ocean circulation.



Figure 1. The model domain of the Gulf of Mexico numerical simulation using the Navy Coastal Ocean Model (NCOM). The black lines are Topex/Poseidon tracks.

It is anticipated that this new variation of the adjoint data assimilation technique will dramatically improve the convergence speed and reduce the computational cost so that the technique may be more practically applied to operational oceanography and research in ocean numerical simulation.

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